# Unprejudiced Look at Effective Continuum Thresholds in Borel Dispersive Sum Rules


Wolfgang Lucha[1,a], Dmitri Melikhov[a,b,c], and Silvano Simula[d]

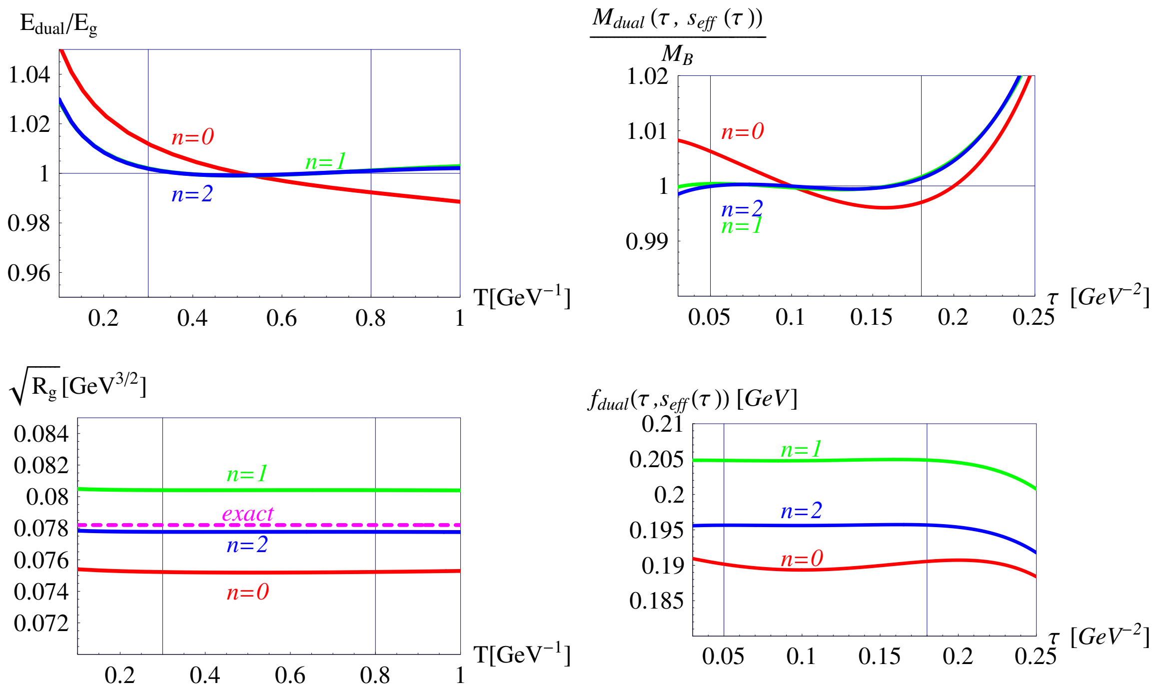
[a] *HEPHY, Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050 Vienna, Austria*

[b] *Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria*

[c] *SINP, Moscow State University, 119991 Moscow, Russia*

[d] *INFN, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Roma, Italy*


*Dispersive sum rules* represent long-standing tools for extracting hadron features from QCD; they are constructed by evaluating matrix elements of suitable operators (e.g. time-ordered products of quark currents) at the level of both hadron and QCD degrees of freedom. One's ignorance of hadronic excitations and continuum is circumvented by *'quark–hadron duality'*: beyond an *'effective threshold'* hadron and QCD contributions are *assumed* to cancel. We [1,2] estimate the error induced by such approximation and improve the accuracy of predictions by elevating our thresholds from constants to functions of momenta and a parameter $T$ or $\tau$ entering upon *Borel transformation* [3]. This move enables us to define *dual correlators*, where the QCD member, truncated at effective threshold $s_{\rm eff}$, *exactly* counterbalances the hadronic ground-state member; the form of $s_{\rm eff}$ can be determined by fitting known hadron features.





[1] Wolfgang.Lucha@oeaw.ac.at

To scrutinize the applicability of our proposed modified sum-rule algorithm to QCD [4] we confront extractions of ground-state decay constants $\sqrt{R_\mathrm{g}} \equiv |\Psi(\mathbf{0})|$ in quantum mechanics in terms of related wave functions $\Psi(\mathbf{x})$ with like extractions of heavy pseudoscalar-meson decay constants in QCD. The plots depict *dual* energy $E_\mathrm{dual}$ over true $E_\mathrm{g}$ and decay constant $\sqrt{R_\mathrm{g}}$ resulting from the funnel potential describing heavy-quark bound states [5] (left), and $B$-meson mass $M_\mathrm{dual}$ over its experimental value $M_B$ and decay constant $f_\mathrm{dual}$, predicted by QCD (right), vs. the associated Borel parameter: Adopting polynomial Ansätze of degree $n$ for the effective continuum thresholds, the band delimited by our $n = 1$ and $n = 2$ findings will provide an 'educated guess' of the intrinsic errors of bound-state features such as $\sqrt{R_\mathrm{g}}$. The similarity of the procedures in quantum mechanics and QCD gives us great confidence that our sum-rule alterations will prove to be successful also in hadron phenomenology [6].

## Acknowledgments

D.M. acknowledges support by the Austrian Science Fund (FWF) under Project No. P22843.

## References

[1] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **76**, 036002 (2007); in *QCD@Work 2007*, eds. P. Colangelo *et al.*, AIP Conf. Proc. **964** (AIP, Melville, New York, 2007), p. 296; Phys. Atom. Nucl. **71**, 1461 (2008); Phys. Lett. B **657**, 148 (2007); in *Hadron 07*, eds. L. Benussi *et al.*, Frascati Phys. Ser. **46** (INFN, Laboratori Nazionali di Frascati, 2007), p. 1109; PoS Confinement8, 180 (2009).

[2] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. B **671**, 445 (2009); PoS Confinement8, 106 (2009); D. Melikhov, Phys. Lett. B **671**, 450 (2009).

[3] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **79**, 096011 (2009); J. Phys. G **37**, 035003 (2010); W. Lucha, D. Melikhov, H. Sazdjian, and S. Simula, Phys. Rev. D **80**, 114028 (2009).

[4] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. B **687**, 48 (2010); Phys. Atom. Nucl. **73**, 1770 (2010); in *QCD@Work 2010*, eds. L. Angelini *et al.*, AIP Conf. Proc. **1317** (AIP, Melville, New York, 2010), p. 316; in *QCHS IX*, eds. F. J. Llanes-Estrada and J. R. Peláez, AIP Conf. Proc. **1343** (AIP, Melville, New York, 2011), p. 624.

[5] W. Lucha, F. F. Schöberl, and D. Gromes, Phys. Rep. **200**, 127 (1991); W. Lucha and F. F. Schöberl, Int. J. Mod. Phys. A **7**, 6431 (1992).

[6] W. Lucha, D. Melikhov, and S. Simula, in *QCD@Work 2010*, eds. L. Angelini *et al.*, AIP Conf. Proc. **1317** (AIP, Melville, New York, 2010), p. 310; PoS ICHEP 2010, 210 (2010); J. Phys. G **38**, 105002 (2011); in *QCHS IX*, eds. F. J. Llanes-Estrada and J. R. Peláez, AIP Conf. Proc. **1343** (AIP, Melville, New York, 2011), p. 379; PoS QFTHEP2010, 058 (2010); Phys. Lett. B **701**, 82 (2011).