

Pion Elastic Form Factor in a Rather Broad Range of Momentum Transfers from Local-Duality QCD Sum Rule

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Revisiting the relevance of local duality for the pion elastic form factor gives rise to optimism.

Recently, several analyses of the pion form factor $F_\pi(Q^2)$ at momentum transfer Q^2 around $Q^2 \approx 4 - 50 \text{ GeV}^2$ have appeared [1] which claim that $F_\pi(Q^2)$ remains much larger than the pQCD result even at $Q^2 \approx 50 \text{ GeV}^2$ (see Fig. 1a). These studies obtain a much larger $F_\pi(Q^2)$ than our result from the local-duality sum rule [2]. They imply that the LD limit is strongly violated even at rather large Q^2 . QCD sum rules utilizing nonlocal condensates [3] arrive at more moderate claims but also observe a large local-duality violation at $Q^2 = 10 - 20 \text{ GeV}^2$. A careful inspection reveals that all these analyses involve explicit or implicit assumptions. Consequently, in a recent study [4] we scrutinized the LD model and its accuracy, by taking advantage of the case of quantum mechanics: there hadronic features, such as form factors, may be found independently of the sum-rule method by solving the Schrödinger equation.

1 Local-Duality Sum Rules

Local-duality (LD) sum rules [6] are nothing but dispersive N -point sum rules in the limit of infinitely large Borel-mass parameter \widetilde{M} , that is, for $\tau \equiv 1/\widetilde{M}^2 \rightarrow 0$. In this limit, all power corrections vanish. The *assumption of quark-hadron duality* claims that, above some effective continuum threshold s_{eff} , the contributions of excited and continuum states at hadron level are dual to the high-energy region of the perturbative diagrams arising from QCD. Under this assumption, in the chiral limit the LD sum rules of interest for the present analysis read

$$(1) \quad f_\pi^2 = \int_0^{\bar{s}_{\text{eff}}} ds \rho_{\text{pert}}(s), \quad F_\pi(Q^2) f_\pi^2 = \int_0^{s_{\text{eff}}(Q)} \int_0^{s_{\text{eff}}(Q)} ds_1 ds_2 \Delta_{\text{pert}}(s_1, s_2, Q).$$

The spectral densities $\rho_{\text{pert}}(s)$ and $\Delta_{\text{pert}}(s_1, s_2, Q)$ are given by QCD perturbation theory [7]. All details of nonperturbative dynamics are encoded in the effective continuum thresholds \bar{s}_{eff} and $s_{\text{eff}}(Q)$. Fixing these, pion decay constant f_π and form factor $F_\pi(Q^2)$ can be derived.

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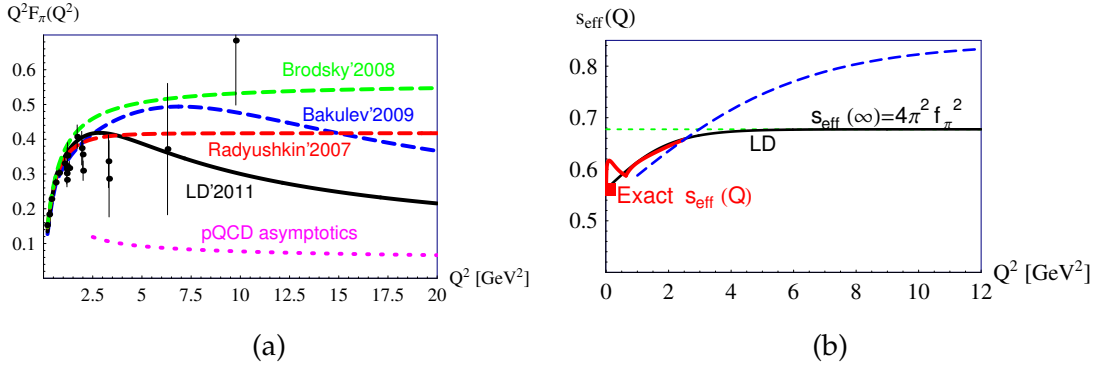


Figure 1: Dependence of both pion elastic form factor $F_\pi(Q^2)$ (a) and effective continuum threshold $s_{\text{eff}}(Q)$ (b) on the momentum transfer Q . In (b) the red line is the exact threshold $s_{\text{eff}}(Q)$ as reconstructed from experimental $F_\pi(Q^2 \leq 2.5 \text{ GeV}^2)$ data [5], the blue refers to a sum-rule study using nonlocal condensates [3], the black is our LD model (2) for $s_{\text{eff}}(Q)$.

One should be aware that any effective continuum threshold is different from the physical continuum threshold: the latter is a constant determined by the masses of the lowest-lying hadronic excitations whereas the effective continuum threshold is just an ingredient of the sum-rule method related to a specific implementation of quark–hadron duality. Therefore, effective thresholds are not constant but depend on the external kinematic variables [8, 9].

Taking into account the properties of the perturbative 2- and 3-point spectral functions, one may formulate an approximate LD model for the effective threshold $s_{\text{eff}}(Q)$: our model is based on some smooth interpolation between the behaviour of $s_{\text{eff}}(Q)$ for $Q \rightarrow 0$, determined by a Ward identity, and for $Q \rightarrow \infty$, determined by factorization properties of $\Delta_{\text{pert}}(s_1, s_2, Q)$. Remembering the well-measured pion elastic form factor in the region near $Q^2 \approx 2.5 \text{ GeV}^2$, we propose [4], in terms of the strong coupling constant $\alpha_s(Q^2)$, the simple parametrization

$$(2) \quad s_{\text{eff}}(Q) = \frac{4\pi^2 f_\pi^2}{1 + \alpha_s(0)/\pi} \left[1 + \tanh\left(\frac{Q^2}{Q_0^2}\right) \frac{\alpha_s(0)}{\pi} \right], \quad Q_0^2 = 2.02 \text{ GeV}^2.$$

For small Q^2 , following [2] we assume a freezing of $\alpha_s(Q^2)$. Note that $s_{\text{eff}}(Q)$ approaches its LD limit, viz., $s_{\text{eff}} = 4\pi^2 f_\pi^2$, already in the region $Q^2 > 4 - 5 \text{ GeV}^2$ (Fig. 1b). Accordingly, the only essential nonperturbative input for the LD model (2) is the pion decay constant f_π .

Figure 1a depicts the corresponding prediction for the pion elastic form factor $F_\pi(Q^2)$. Our LD model provides a perfect description of all the available experimental data in the region $Q^2 = 1 - 2.5 \text{ GeV}^2$. For $Q^2 \geq 3 - 4 \text{ GeV}^2$, the LD model reproduces well all the data except for the single point $Q^2 = 10 \text{ GeV}^2$; there our prediction is off the actual experimental value (which, in any case, is affected by a rather large error) by roughly two standard deviations. Interestingly enough, in the range $Q^2 \geq 3 - 4 \text{ GeV}^2$ the LD model yields significantly lower predictions than the findings of the different theoretical approaches presented in Refs. [1, 3].

A closer inspection of Fig. 1 easily reveals that it is virtually impossible to construct models compatible with all experimental findings in $Q^2 = 2.5 - 10 \text{ GeV}^2$: those approaches that hit the data at $Q^2 = 10 \text{ GeV}^2$ overestimate the data points of better quality at $Q^2 \approx 2 - 4 \text{ GeV}^2$. By construction, the LD model (2) is but an approximate model which involves too few free parameters to be able to take into account some subtle details of the confinement dynamics. Nevertheless, we would like to estimate the uncertainties of hadron-parameter predictions we might expect for the momentum-transfer range $Q^2 \geq 3 - 4 \text{ GeV}^2$. The obvious place to study this and to get an idea of the order of magnitude of the errors is quantum mechanics: there solving Schrödinger's equation numerically [10] gives the exact bound-state features.

2 Local-Duality Effective-Threshold Model in Quantum Mechanics

The main ingredient that constrains the formulation of our LD model (2) is the factorization of hard form factors. Consequently, this model may be tested in quantum mechanics (QM) for potentials containing both Coulomb and confining interactions. For definiteness, we [4] consider a set of power-law confining potentials: $V_{\text{conf}}(r) \propto r^n$, $n = 2, 1, \frac{1}{2}$. We adopt model parameters suitable for hadron physics and fix the strengths of all our $V_{\text{conf}}(r)$ such that for each of them the Schrödinger equation yields the *same value* $\psi(0)$ of the configuration-space bound-state wave function $\psi(r)$ at the origin and hence the same QM LD threshold model. We identified an important *universal* behaviour, which does not depend on the details of the confining interaction: the accuracy of the LD model for both effective continuum threshold and elastic form factor increases with Q in the range $Q \geq 2 \text{ GeV}$. Accordingly, we may infer that, if in the region $Q^2 \approx 4 - 8 \text{ GeV}^2$ the LD setup provides a satisfactory description of the experimental data, the accuracy of our predictions will not be worse for larger values of Q^2 .

3 Summary, Conclusions, and Outlook

We investigated the pion elastic form factor $F_\pi(Q^2)$ by means of an LD model, which can be formulated in any theory where hard exclusive amplitudes satisfy a factorization theorem (in essence, any theory where the interactions behave Coulomb-like at small distances and confining at large distances). Figure 1 and our QM studies lead us to our main conclusions:

1. For $Q^2 \leq 4 \text{ GeV}^2$, our exact effective threshold $s_{\text{eff}}(Q)$ exhibits a rapid variation with Q . This observation implies that the accuracy of the LD model for these momentum transfers depends on subtle details of the confining interactions and cannot be predicted in advance.
2. For $Q^2 \geq 4 \text{ GeV}^2$, irrespective of any details of the underlying confining interactions, the *maximum* deviations of the LD-model predictions from the exact elastic form factor occur in the range $Q^2 \approx 4 - 8 \text{ GeV}^2$. For Q^2 beyond this interval, our LD model's accuracy increases very fast. Our QM toy model with power-law potentials shows that, for arbitrary confining interactions, our LD model entails rather accurate numerical results for $Q^2 \geq 20 - 30 \text{ GeV}^2$.

3. Very precise data [5] on $F_\pi(Q^2)$ indicate that the LD limit $s_{\text{eff}}(\infty) = 4\pi^2 f_\pi^2$ of the effective threshold is reached already at comparatively low values $Q^2 = 5 - 6 \text{ GeV}^2$; therefore, large deviations from the LD limit at $Q^2 = 20 - 50 \text{ GeV}^2$, as obtained in [1], appear to us unlikely. Moreover, we expect the pion form factor at $Q^2 = 10 - 20 \text{ GeV}^2$ to be considerably lower than the prediction of an approach based on a sum rule involving nonlocal condensates [3].

Our analysis is not meant to constitute a proof of but rather to provide an argument for the accuracy of the LD model in QCD and the expected behaviour of the pion elastic form factor at large Q^2 . Thus, the accurate measurement of F_π in the region $Q^2 = 4 - 10 \text{ GeV}^2$ will have important implications for the behaviour of F_π at larger Q^2 , up to asymptotically large Q^2 .

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