Partial restoration of chiral symmetry and pion in nuclear medium

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We discuss partial restoration of chiral symmetry in nuclear medium, which is realized as an effective reduction of the quark condensate in nuclear medium. We derive the in-medium Weinberg-Tomozawa relation. We also give a brief calculation of the in-medium quark condensate based on chiral perturbation theory. We conclude that the density dependence of the quark condensate beyond the linear density comes from the vertex correction of the chiral field insertion and nucleon correlations.

1 Introduction

One of the goals of the contemporary nuclear physics is to figure out the QCD vacuum structure at finite density and/or temperature. Especially, the fate of dynamical breaking of chiral symmetry in finite baryonic density is phenomenologically interesting, because we could obtain experimental evidence of partial restoration of chiral symmetry in the nuclear medium by investigating in-medium properties of meson in nuclei. Recently, precise measurements of the spectra of deeply bound pionic atoms were performed [1,2], and with these data the pion optical potential parameters were determination in detail. Especially, the repulsive enhancement of the isovector π^--nucleus interaction was accurately extracted as $b_1^{\rm free}/b_1 = 0.78 \pm 0.05$ at around $\rho \sim 0.6\rho$ [2]. The b_1 repulsive enhancement was also seen in low-energy pion-nucleus scatterings [3]. With help of the theoretical discussion [4,5], the b_1 parameter is converted to the in-medium pion decay constant F_t, and we concluded that the experimental finding of the b_1 enhancement is to be a signal of the reduction of the pion decay constant in nuclear matter. Further, the exact relation between the in-medium pion decay constant and quark condensate was also found at the linear density approximation in Ref. [5]. Now we have arrived at the qualitative confirmation of the partial restoration of chiral symmetry in nuclear medium and should go a step further to make the argument more quantitative [7,8]. In this paper we briefly discuss the sum rule for the in-medium quark condensate and the in-medium Weinber-Tomozawa relation. We also show a calculation of the in-medium quark condensate based on chiral perturbation theory beyond the linear density.

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2 Sum rule for the in-medium quark condensate

To connect the phenomenological consequences extracted from experimental observation to the quark condensate, we need theoretical consideration which makes bridge between hadronic description and quark language. In Ref. [5], an exact sum rule for a symmetric nuclear matter was derived in the chiral limit:

$$\sum_\alpha \mathrm{Re}\left[F_{t,\alpha} G_\alpha^{*1/2}\right] = -\langle \overline{q}q\rangle^* \tag{1}$$

where the summation is taken over all of pionic zero modes in nuclear matter which have the same quantum number with the pion in nuclear matter and whose energy is zero measured from the nuclear matter ground state, and the matrix elements of $F_{t,\alpha}$ and $G_\alpha^{1/2}$ are given in the nuclear matter rest frame by

$$\langle \Omega | A_0^a | \Omega_\alpha^b(k)\rangle = i\delta^{ab}\omega_\alpha F_{t,\alpha} \tag{2}$$

$$\langle \Omega_\alpha^b | \phi_5^a | \Omega\rangle = \delta^{ab} G_\alpha^{*1/2} \tag{3}$$

with the axial current A_μ^a, the pseudoscalar density ϕ_5^a, the ground state of the symmetric nuclear matter $|\Omega\rangle$ and the pionic zero mode state $|\Omega_\alpha\rangle$.

One of the most important consequence of this sum rule is that to obtain the in-medium chiral condensate one has to sum up all of the pionic zero modes. This means that one should not have to separate out the in-medium pion properties from complicated dynamics of pion and nuclear matter. This sum rule is valid for all densities and derived by current algebra as a low energy theorem. Instead, we need description of dynamics of in-medium pion and nuclear matter for actual calculation of matrix elements. This sum rule is also available for experimental confirmation of partial restoration of chiral symmetry, once the matrix elements are extracted from experimental observation.

In the linear density approximation, the sum rule can be simplified to $F_t G_\pi^{*1/2} = -\langle \overline{q}q\rangle^*$. By taking its ratio to the in-vacuum Glashow-Weinberg relation $F_\pi G_\pi^{1/2} = -\langle \overline{q}q\rangle$ [9], we obtain the scaling law

$$\frac{F_t}{F_\pi} Z_\pi^{1/2} = \frac{\langle \overline{q}q\rangle^*}{\langle \overline{q}q\rangle} \ . \tag{4}$$

where $Z_\pi^{1/2}$ is the in-medium wavefunction renormalization, which can be extracted at the linear density from the πN scattering data. The in-medium reduction of the pion decay constant was obtained in the pionic atom.

3 In-medium Weinberg-Tomozawa relation

As discussed in Ref. [5], the in-medium Weinberg-Tomozawa relation can be derived by considering the chiral-limit correlation function of the axial current A_μ^a in the asymmetric nuclear matter: $\Pi^{ab}_\nu(q) = \int d^4x e^{iq\cdot x}\partial^\mu \langle \Omega' | T[A^a_\mu(x) A^b_\nu(0)] | \Omega' \rangle$, where $|\Omega'\rangle$ is the ground state of the asymmetric nuclear matter normalized as $\langle \Omega' | \Omega' \rangle = 1$ and specified by the isoscalar density $\rho = \rho_p + \rho_n$ and the isovector density $\delta\rho = \rho_p - \rho_n$. In the soft limit, using the axial current conservation $\partial \cdot A = 0$, we obtain

(5) $$\Pi^{ab}_{\nu=0}(0) = \int d^3x [A^a_0(x), A^b_0(0)] = i\epsilon^{ab3} \langle \Omega' | V^3_0 | \Omega' \rangle,$$

where we have used the commutation relation $[Q^a_5, A^b_\nu] = i\epsilon^{abc} V^c_\nu$. The matrix element in the right hand side of Eq. (5) implies a spacial average of the isospin density in the nuclear matter and counts the z component of the isospin of the nuclear matter state. For the ground state of nuclear matter, at the linear $\delta\rho$, the matrix element is written as

(6) $$\Pi^{ab}_0(0) \simeq i\epsilon^{ab3} \frac{1}{2} \delta\rho.$$

On the other hand, inserting the hadronic complete set into the correlation function, we obtain the hadronic description of the correlation function in the soft limit where we take $\vec{q} \to 0$ first as

(7) $$\Pi^{ab}_0(0) = \lim_{\omega\to 0} i\omega (\omega F^{a\alpha}_t) \frac{1}{\omega^2 \delta^{\alpha\beta} - \Sigma^{\alpha\beta}} (\omega F^{\beta b}_t),$$

where $\Sigma^{\alpha\beta}$ is the self-energy of the zero mode and F^{ab}_t is the matrix element of the axial current for the ground state $|\Omega'\rangle$ and the pionic zero modes in the asymmetric nuclear matter $|\Omega'_\alpha\rangle$ given by $\langle \Omega' | A^a_0 | \Omega'_\alpha \rangle = i\omega F^{a\alpha}_t$. Thus we obtain a sum rule

(8) $$\sum_{\alpha,\beta} \lim_{\omega\to 0} \omega F^{a\alpha}_t \left(\delta^{\alpha\beta} - \frac{1}{2\omega} \frac{\partial \Sigma^{\alpha\beta}}{\partial \omega} \right)^{-1} F^{\beta b}_t = \epsilon^{ab3} \frac{1}{2} \delta\rho.$$

The explicit expression of the $F^{a\alpha}_t$ and $\Sigma^{\alpha\beta}$ depends on the description of the complete set, but the sum is not dependent on the description. Now we expand $F^{\alpha\beta}_t$ and $\Sigma^{\alpha\beta}$ in terms of the isovector density $\delta\rho$, and find

(9) $$\Sigma^{\alpha\beta} = \epsilon^{\alpha\beta 3} \frac{\omega}{F^2_t} \delta\rho,$$

where F_t is the decay constant in the symmetric nuclear matter given in Eq. (2). This is just a consequence of the isospin symmetry

4 A model calculation of the quark condensate

The sum rule (1) was derived by considering the correlation function of the axial current and pseudoscalar density in symmetric nuclear matter at the chiral limit: $\Pi^{ab}_5(q) =$

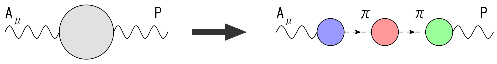

Figure 1: Separation of $\langle A^a_\mu(q)P^b(0)\rangle^*$.

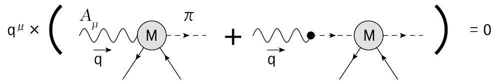

Figure 2: Conservation of axial current

$\int d^4x e^{iq\cdot x}\partial^\mu \langle\Omega|T[A^a_\mu(x)\phi^b_5(0)]|\Omega\rangle$. Taking the soft limit and using the Ward-Takahashi identity, we obtain

(10) $$\lim_{q\to 0}\int d^4x e^{iq\cdot x}\partial^\mu\langle\Omega|T[A^a_\mu(x)\phi^b_5(0)]|\Omega\rangle = -i\delta^{ab}\langle\bar{q}q\rangle^*$$

In this section, we directly calculate the left hand side based on in-medium chiral perturbation theory. Here we take, for example, the formulation developed in Refs. [10], in which the generating functional of the SU(2) chiral Lagrangian in non-interacting nucleon gas environment is calculated in expansions of Fermi-see insertion and chiral order counting. The Fermi momentum k_f is also regarded as a small expansion parameter as well as the pion momentum and mass in the in-vacuum chiral perturbation theory.

We calculate the in-medium Green function $\langle A^a_\mu(q)P^b(0)\rangle^*$ in the chiral limit [12]. First of all, it is possible to separate the Green function into the three parts, the medium corrections of the decay constant, pion wavefunction and the pseudo-scalar coupling (Fig. 1), which are connected with the in-vacuum pion propagators. Among three parts, there is cancellation between the corrections for the decay constant and wavefunction at the soft limit $q_\mu \to 0$ thanks to the axial vector current conservation in the chiral limit, $\partial^\mu A^a_\mu(x) = 0$. This is a generalized Goldberger-Treiman relation (Fig. 2). Therefore, only the density dependence of the pseudo-scalar coupling contributes to the in-medium quark condensate.

The medium correction for the quark condensate starts from $O(p^4)$ of the chiral expansion, but the contribution of this order vanishes at the chiral limit due to the cancelation mentioned above. The leading order correction is given by the diagram shown in Fig. 3(a) with the $O(p^2)$ πPNN vertex. This contribution reproduces the well-known result of the linear density approximation [11]:

(11) $$\langle\bar{u}u + \bar{d}d\rangle^* = \langle\bar{u}u + \bar{d}d\rangle_0(1 + \frac{8c_1\rho}{f^2_\pi}),$$

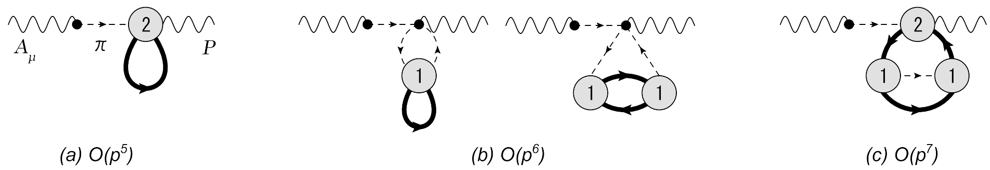

Figure 3: Diagrams for the medium corrections of the quark condensate in the chiral limit. $O(p^n)$ means the order of the chiral expansion.

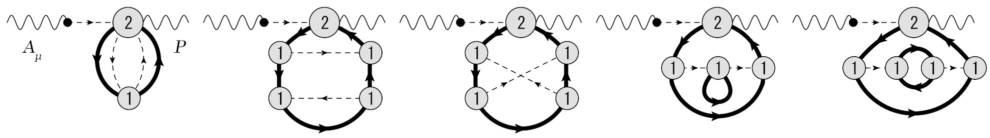

Figure 4: Further higher order contributions. These diagrams represent the two-pion exchange and nucleon-nucleon correlations.

where $\langle \overline{u}u + \overline{d}d \rangle_0$ is the quark condensate in vacuum, ρ is the nuclear matter density and c_1 is one of the low energy constants (LEC) in the πN chiral Lagrangian at $O(p^2)$, which is related to the πN sigma term. With $C_1 = -0.58\ \mathrm{GeV}^{-1}$, $f_\pi = 92.4$ MeV and the normal nuclear density $\rho_0 = 0.17\ \mathrm{fm}^{-3}$, we have $\langle \overline{u}u + \overline{d}d \rangle^* = \langle \overline{u}u + \overline{d}d \rangle_0 (1 - 0.35\rho/\rho_0)$. This suggests 30 percent reduction of the quark condensate at the nuclear density.

Further density corrections beyond the linear density can be calculated with the diagrams shown in Fig. 3(b) and(c). They are counted as $O(p^6)$ and $O(p^7)$ of the chiral expansion and give k_f^4 and higher power contributions. In this way, one can perform the systematic expansion in terms of the chiral counting with this formulation. Since, as discussed above, the finite density contribution to the quark condensate comes only from the medium corrections of the pseudo-scalar coupling, further higher order corrections come from diagrams shown in Fig. 4. This means that the density corrections of the quark condensate are given by multi-pion exchanges between nucleons and multi-nucleon correlations in the nuclear matter. Therefore, in order to perform the realistic calculation of the in-medium quark condensate, one has to first formulate realistic nuclear matter. Indeed, the in-medium chiral perturbation theory has the systematic scheme for counting the order of the chiral expansion and this is good for theoretical analyses. Nevertheless, only the pion-nucleon dynamics described by the chiral perturbation theory may not describe realistic nuclear matter, and more phenomenological descriptions are necessary to obtain realistic nuclear matter having the saturation properties. Thus, we may have to go beyond the chiral counting scheme in order to calculate the quark condensate in nuclear matter.

5 Conclusion

The deeply bound pionic atoms are the most successful systems to investigate the in-medium pion properties, because we have certainly the bound states with so narrow widths that we can perform detailed spectroscopy. From the observed spectra we can extract in-medium pion properties. With these quantities we have concluded that partial restoration of chiral symmetry takes place in nucleus with help of theoretical arguments, which complete the story from the observation to QCD. Now we are going to the next stage to determine the in-medium quark condensate quantitatively. We have also performed brief calculation of the in-medium quark condensate based on chiral perturbation theory. This calculation shows that to obtain the realistic quark condensate in a nuclear medium one needs realistic description of nuclear matter.

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