

Heavy Baryon Spectrum and New Heavy Exotics

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We discuss several highly accurate theoretical predictions for masses of baryons containing the b quark which have been recently confirmed by experimental data. Several predictions are given for additional properties of heavy baryons. We also discuss the two charged exotic resonances Z_b with quantum numbers of a $(b\bar{b}u\bar{d})$ tetraquark, very recently reported by Belle in the channel $[Y(nS)\pi^+, n = 1, 2, 3]$. Among possible implications are deeply bound $I = 0$ counterparts of the Z_b -s and existence of a $\Sigma_b^+\Sigma_b^-$ dibaryon, a *beauteron*.

1 Introduction

QCD describes hadrons as valence quarks in a sea of gluons and $\bar{q}q$ pairs. At distances above $\sim 1 \text{ GeV}^{-1}$ quarks acquire an effective *constituent mass* due to chiral symmetry breaking. A hadron can then be thought of as a bound state of constituent quarks. In the zeroth-order approximation the hadron mass M is then given by the sum of the masses of its constituent quarks m_i , $M = \sum_i m_i$. The binding and kinetic energies are “swallowed” by the constituent quarks masses. The first and most important correction comes from the color hyper-fine (HF) chromo-magnetic interaction,

$$(1) \quad M = \sum_i m_i + \sum_{i<j} V_{ij}^{HF(QCD)}; \quad V_{ij}^{HF(QCD)} = v_0 (\vec{\lambda}_i \cdot \vec{\lambda}_j) \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} \langle \psi | \delta(r_i - r_j) | \psi \rangle$$

where v_0 gives the overall strength of the HF interaction, $\vec{\lambda}_{i,j}$ are the $SU(3)$ color matrices, $\sigma_{i,j}$ are the quark spin operators and $|\psi\rangle$ is the hadron wave function. This is a contact spin-spin interaction, analogous to the EM hyperfine interaction, which is a product of the magnetic moments, $V_{ij}^{HF(QED)} \propto \vec{\mu}_i \cdot \vec{\mu}_j = e^2 \vec{\sigma}_i \cdot \vec{\sigma}_j / (m_i m_j)$. In QCD, the $SU(3)_c$ generators take place of the electric charge. From eq. (1) many very accurate results have been obtained for the masses of the ground-state hadrons. Nevertheless, several caveats are in order. First, this is a low-energy phenomenological model, still awaiting a rigorous derivation

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from QCD. It is far from providing a complete description of the hadronic spectrum, but it provides excellent predictions for mass splittings and magnetic moments. The crucial assumptions of the model are: (a) HF interaction is considered as a perturbation which does not change the wave function; (b) effective masses of quarks are the same inside mesons and baryons; (c) there are no 3-body effects.

2 Effective masses of quarks

Constituent quark mass differences depend strongly on the flavor of the spectator or "neighbor" quark [1]. For example, $m_s - m_d \approx 180$ MeV when the spectator is a light quark but the same mass difference is only about 90 MeV when the spectator is a b quark. Since these are *effective masses*, we should not be surprised that their difference is affected by the environment, but the large size of the shift is quite surprising and its quantitative derivation from QCD is an outstanding challenge for theory. We can extract the ratio of the constituent quark masses from the ratio of the the hyperfine splittings in the corresponding mesons. The hyperfine splitting between K^* and K mesons is given by

$$(2) \quad M(K^*) - M(K) = v_0 \frac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} [(\vec{\sigma}_u \cdot \vec{\sigma}_s)_{K^*} - (\vec{\sigma}_u \cdot \vec{\sigma}_s)_K] \langle \psi | \delta(r) | \psi \rangle = 4v_0 \frac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \langle \psi | \delta(r) | \psi \rangle,$$

and similarly for hyperfine splitting between D^* and D with $s \rightarrow c$ everywhere. From (2) and its D analogue we then immediately obtain

$$(3) \quad \frac{M(K^*) - M(K)}{M(D^*) - M(D)} \approx \frac{m_c}{m_s}$$

2.1 Color hyperfine splitting in baryons

As an example of hyperfine splitting in baryons, let us now discuss the HF splitting in the Σ (uds) baryons. Σ^* has spin $\frac{3}{2}$, so the u and d quarks must be in a state of relative spin 1. The Σ has isospin 1, so the wave function of u and d is symmetric in flavor. It is also symmetric in space, since in the ground state the quarks are in a relative S -wave. On the other hand, the u - d wave function is antisymmetric in color, since the two quarks must couple to a $\mathbf{3}^*$ of color to neutralize the color of the third quark. The u - d wave function must be antisymmetric in flavor \times spin \times space \times color, so it follows it must be symmetric in spin, i.e. u and d are coupled to spin one. Since u and d are in spin 1 state in both Σ^* and Σ their HF interaction with each other cancels between the two and thus the u - d pair does not contribute to the $\Sigma^* - \Sigma$ HF splitting,

$$(4) \quad M(\Sigma^*) - M(\Sigma) = 6v_0 \frac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \langle \psi | \delta(r_{rs}) | \psi \rangle$$

we can then use eqs. (2) and (4) to compare the quark mass ratio obtained from mesons and baryons:

$$(5) \quad \left(\frac{m_c}{m_s}\right)_{Bar} = \frac{M_{\Sigma^*} - M_{\Sigma}}{M_{\Sigma_c^*} - M_{\Sigma_c}} = 2.84; \quad \left(\frac{m_c}{m_s}\right)_{Mes} = \frac{M_{K^*} - M_K}{M_{D^*} - M_D} = 2.81$$

$$(6) \quad \left(\frac{m_c}{m_u}\right)_{Bar} = \frac{M_{\Delta} - M_p}{M_{\Sigma_c^*} - M_{\Sigma_c}} = 4.36; \quad \left(\frac{m_c}{m_u}\right)_{Mes} = \frac{M_{\rho} - M_{\pi}}{M_{D^*} - M_D} = 4.46$$

We find the same value from mesons and baryons $\pm 2\%$.

The presence of a fourth flavor gives us the possibility of obtaining a new type of mass relation between mesons and baryons. The $\Sigma - \Lambda$ mass difference is believed to be due to the difference between the $u - d$ and $u - s$ hyperfine interactions. Similarly, the $\Sigma_c - \Lambda_c$ mass difference is believed to be due to the difference between the $u - d$ and $u - c$ hyperfine interactions. We therefore obtain the relation

$$(7) \quad \left(\frac{\frac{1}{m_u^2} - \frac{1}{m_u m_c}}{\frac{1}{m_u^2} - \frac{1}{m_u m_s}}\right)_{Bar/Mes} = \frac{M_{\Sigma_c} - M_{\Lambda_c}}{M_{\Sigma} - M_{\Lambda}} = 2.16 \approx \frac{(M_{\rho} - M_{\pi}) - (M_{D^*} - M_D)}{(M_{\rho} - M_{\pi}) - (M_{K^*} - M_K)} = 2.10$$

The meson and baryon relations agree to $\pm 3\%$. We can write down an analogous relation for hadrons containing the b quark instead of the s quark, obtaining the prediction for splitting between Σ_b and Λ_b :

$$(8) \quad \frac{M_{\Sigma_b} - M_{\Lambda_b}}{M_{\Sigma} - M_{\Lambda}} = \frac{(M_{\rho} - M_{\pi}) - (M_{B^*} - M_B)}{(M_{\rho} - M_{\pi}) - (M_{K^*} - M_K)} = 2.51$$

yielding $M(\Sigma_b) - M(\Lambda_b) = 194$ MeV [1,2]. This splitting was measured by CDF [3], with isospin-averaged mass difference $M(\Sigma_b) - M(\Lambda_b) = 192$ MeV. There is also the prediction for the spin splittings, good to 5%

$$(9) \quad M(\Sigma_b^*) - M(\Sigma_b) = \frac{M(B^*) - M(B)}{M(K^*) - M(K)} \cdot [M(\Sigma^*) - M(\Sigma)] = 22 \text{ MeV}$$

to be compared with 21 MeV from the isospin-average of CDF measurements [3]. The challenge is to understand how and under what assumptions one can derive from QCD the very simple model of hadronic structure at low energies which leads to such accurate predictions.

3 Magnetic Moments of Heavy Quark Baryons

In Λ , Λ_c and Λ_b baryons the light quarks are coupled to spin zero. Therefore the magnetic moments of these baryons are determined by the magnetic moments of the s , c and b quarks,

respectively. The latter are proportional to the chromomagnetic moments which determine the hyperfine splitting in baryon spectra. We can use this fact to predict the Λ_c and Λ_b baryon magnetic moments by relating them to the hyperfine splittings in the same way as given in the original prediction [5] of the Λ magnetic moment. We obtain

$$(10) \quad \mu_{\Lambda_c} = -2\mu_{\Lambda} \cdot \frac{M_{\Sigma_c^*} - M_{\Sigma_c}}{M_{\Sigma^*} - M_{\Sigma}} = 0.43 \text{ n.m.}; \quad \mu_{\Lambda_b} = \mu_{\Lambda} \cdot \frac{M_{\Sigma_b^*} - M_{\Sigma_b}}{M_{\Sigma^*} - M_{\Sigma}} = -0.067 \text{ n.m.}$$

We hope these observables can be measured in foreseeable future and view the predictions (10) as a challenge for the experimental community.

4 Predicting the Masses of b-Baryons

On top of the already discussed Σ_b with quark content bqq , $q = u, d$. there are two additional ground-state b -baryons, Ξ_b and Ω_b :

Ξ_b : the quark content is bsq . Ξ_b can be obtained from an "ordinary" Ξ (ssd or ssu) by replacing one of the s quarks by a b , with one important difference. In the ordinary Ξ , Fermi statistics dictates that two s quarks must couple to spin-1, while in the ground state of Ξ_b the (sq) diquarks have spin zero. Consequently, the Ξ_b mass is given by the expression: $\Xi_b = m_b + m_s + m_u - 3v\langle\delta(r_{us})\rangle/m_u m_s$. The Ξ_b mass can thus be predicted using the known Ξ_c baryon mass as a starting point and adding the corrections due to mass differences and HF interactions:

$$(11) \quad \Xi_b = \Xi_c + (m_b - m_c) - 3v(\langle\delta(r_{us})\rangle_{\Xi_b} - \langle\delta(r_{us})\rangle_{\Xi_c})/(m_u m_s)$$

Since the Ξ_b and Ξ_c baryons contain a strange quark, and the effective constituent quark masses depend on the spectator quark, the optimal way to estimate the mass difference $(m_b - m_c)$ is from mesons which contain both s and b or c quarks:

$$(12) \quad m_b - m_c = \frac{1}{4}(3B_s^* + B_s) - \frac{1}{4}(3D_s^* + D_s) = 3324.6 \pm 1.4 .$$

On this basis we predicted [7] $M(\Xi_b) = 5795 \pm 5$ MeV. Our paper was submitted on June 14, 2007. The next day CDF announced the result [9], $M(\Xi_b) = 5792.9 \pm 2.5 \pm 1.7$ MeV, following up on an earlier D0 measurement, $M(\Xi_b) = 5774 \pm 11 \pm 15$ MeV [8].

Ω_b : for the spin-averaged Ω_b mass we have

$$(13) \quad \frac{1}{3}(2M(\Omega_b^*) + M(\Omega_b)) = \frac{1}{3}(2M(\Omega_c^*) + M(\Omega_c)) + (m_b - m_c)_{B_s-D_s} = 6068.9 \pm 2.4 \text{ MeV}$$

For the HF splitting we obtain

$$(14) \quad M(\Omega_b^*) - M(\Omega_b) = (M(\Omega_c^*) - M(\Omega_c)) \frac{m_c \langle\delta(r_{bs})\rangle_{\Omega_b}}{m_b \langle\delta(r_{cs})\rangle_{\Omega_c}} = 30.7 \pm 1.3 \text{ MeV}$$

leading to the following predictions:

$$(15) \quad M(\Omega_b) = 6052.1 \pm 5.6 \text{ MeV}; \quad M(\Omega_b^*) = 6082.8 \pm 5.6 \text{ MeV}$$

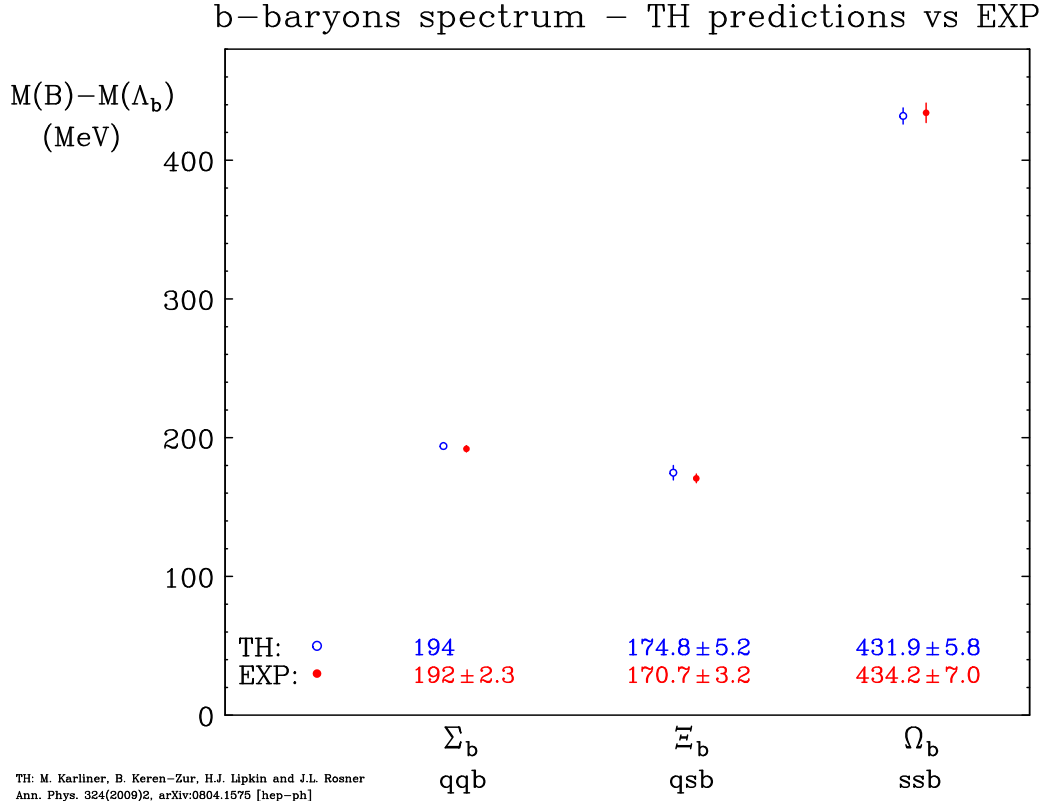


Figure 1: Masses of b -baryons – theoretical predictions [7, 10] vs. experiment.

About four months after our prediction (15) for Ω_b mass [10], D0 collaboration published the first measurement of Ω_b mass [11]: $M(\Omega_b)_{D0} = 6165 \pm 10(stat.) \pm 13(syst.)$ MeV. The deviation from the central value of our prediction was huge, 113 MeV. Understandably, we were very eager to see the CDF result. CDF published their result about nine months later, in May 2009 [12]: $M(\Omega_b)_{CDF} = 6054 \pm 6.8(stat.) \pm 0.9(syst.)$ MeV. Fig. 1 shows a comparison of our predictions for the masses of Σ_b , Ξ_b and Ω_b baryons with the CDF experimental data. We have made additional predictions [7, 10] for some excited states of b -baryons. Our results are summarized in Table 10 of Ref. [10].

The sign in our prediction $M(\Sigma_b^*) - M(\Sigma_b) < M(\Omega_b^*) - M(\Omega_b)$, appears to be counterintuitive, since the color hyperfine interaction is inversely proportional to the quark mass. This reversed inequality is not predicted by other recent approaches [13–15], but it is also seen in the charm data, $M(\Sigma_c^*) - M(\Sigma_c) = 64.3 \pm 0.5$ MeV $<$ $M(\Omega_c^*) - M(\Omega_c) = 70.8 \pm 1.5$ MeV. This suggests that the sign of the $SU(3)$ symmetry breaking gives information about the form of the potential. It is of interest to follow this clue theoretically and experimentally.

5 Heavy exotics

Ordinary hadrons contain either a $q\bar{q}$ pair or 3 quarks. The possible color representations of quark combinations are then completely determined by confinement. In a meson the $q\bar{q}$ pair *must* couple to a color singlet and in a baryon any two quarks *must* couple to an anti-triplet of color, to neutralize the color charge of the third quark. The situation is very different in exotic hadrons which contain both qq and $q\bar{q}$ pairs, eg. a tetraquark with two heavy quarks Q and two light quarks q , $Q\bar{Q}q\bar{q}$. Such states have important color-space correlations that are completely absent in ordinary mesons and baryons [16]. One also needs to keep in mind that the $q\bar{q}$ interaction is much stronger than q - q interaction. The result is emergence of color structures that are totally different from those in normal hadrons. In turn, this leads to some very unusual experimental properties of such states. Until May 2011 the leading candidate has been the $X(3872)$, which is most likely either a $c\bar{c}q\bar{q}$ or a threshold bound state of D and \bar{D}^* . Given that $X(3872)$ exists, it is fascinating to explore possible analogues containing b quarks. General considerations suggest that such states should be more strongly bound, since the attraction due to color forces is the roughly same, but the repulsion due to kinetic energy is smaller, as $E_k \sim p^2/m_Q$. Using a simple model, we have suggested that $b\bar{b}q\bar{q}$ might be below the $B\bar{B}$ threshold and $b\bar{c}q\bar{q}$ might be below the $B\bar{D}$ threshold. A crucial difference vs. ordinary mesons is that $(Qq)(\bar{Q}\bar{q})$ can form a $\bar{\mathbf{6}}\mathbf{6}$ color configuration which has much stronger binding than $\bar{\mathbf{3}}\mathbf{3}$. Some of these states have exotic electric charge, e.g. $bd\bar{c}\bar{u} \rightarrow J/\psi\pi^-\pi^-$. Their decays have striking experimental signatures: monoenergetic photons and/or pions, e.g. $bq\bar{c}\bar{q}$ with $I=0$ above $B_c\pi$ threshold can decay into $B_c\pi$ via isospin violation, or electromagnetically into $B_c\gamma$, both very narrow.

Hadrons containing two b quarks, such as double-bottom baryons bbq or $b\bar{b}q\bar{q}$ and $b\bar{b}q\bar{q}$ tetraquarks have a unique and a spectacular decay mode with two J/ψ -s in the final state. To see this, recall that a b quark can decay via the hadronic mode $b \rightarrow \bar{c}cs \rightarrow J/\psi s$. If both b quarks in a double-bottom hadron decay this way, for a bb baryon we get $(bbq) \rightarrow J/\psi J/\psi(ssq) \rightarrow J/\psi J/\psi \Xi$, and similarly for a tetraquark: $(b\bar{b}q\bar{q}) \rightarrow J/\psi J/\psi(\bar{s}s\bar{q}q) \rightarrow J/\psi J/\psi KK$, etc., with all final state hadrons coming from the same vertex. This unique signature is however hampered by a very low rate expected for such a process, especially if one uses dimuons to identify the J/ψ -s. It is both challenge and a opportunity for LHCb [16].

Exotic double-bottom hadrons Z_b : theoretical prediction and discovery by Belle

In 2008 Belle reported [17] anomalously large (by two orders of magnitude) branching ratios for the decays $Y(5S) \rightarrow Y(mS)\pi^+\pi^-$, $m = 1, 2$. In [18] we suggested that the enhancement is due to an intermediate state of a tetraquark $T_{bb}^- = (\bar{b}bu\bar{d})$ and a pion, mediating the two-step process

$$Y(5S) \rightarrow T_{bb}^\pm \pi^\mp \rightarrow Y(mS) \pi^+ \pi^-$$

We proposed looking for the $(\bar{b}bu\bar{d})$ tetraquark in these decays as peaks in the invariant mass of $Y(1S)\pi^+$ or $Y(2S)\pi^+$ systems.

Very recently Belle collaboration confirmed this prediction, announcing [19] the observation of two charged bottomonium-like resonances Z_b as narrow structures in $\pi^\pm Y(nS)$ ($n = 1, 2, 3$) and $\pi^\pm h_b(mP)$ ($m = 1, 2$) mass spectra that are produced in association with a single charged pion in $Y(5S)$ decays.

The measured masses of the two structures averaged over the five final states are $M_1 = 10608.4 \pm 2.0$ MeV, $M_2 = 10653.2 \pm 1.5$ MeV, both with a width of about 15 MeV.

Interestingly enough, the two masses M_1 and M_2 are about 3 MeV above the respective $B^*\bar{B}$ and $B^*\bar{B}^*$ thresholds. This strongly suggests a parallel with $X(3872)$, whose mass is almost exactly at the $D^*\bar{D}$ threshold. It also raises the possibility that such states might have a complementary description as deuteron-like "molecule" of two heavy mesons quasi-bound by pion exchange [20,21].

The attraction due to π exchange is 3 times weaker in the $I=1$ channel than in the $I=0$ channel. This is because for $I=1$ only π^0 contributes, whereas for $I=0$ both π^0 and π^\pm contribute. Consequently, in the charm system the $I=1$ state is far above the $D^*\bar{D}$ threshold and only the $I=0$ $X(3872)$ is bound 2 MeV below the average of the isospin-related D^+D^{*-} and $D^0\bar{D}^0$ thresholds. The situation is likely to be different in the bottom system. This is because the attraction due to π exchange is essentially the same, but the B mesons are much heavier than D mesons, so the kinetic energy is much smaller by a factor of $\sim m(B)/m(D) \approx 2.8$. Therefore the net binding is much stronger than in the charm system. This raises two very interesting possibilities:

(a) the Z_b states are almost bound (or quasi bound) $B^*\bar{B}$ and $B^*\bar{B}^*$ $I=1, J^P = 1^+$ states near threshold; the neutral members of their isomultiplets have $C=-1, G=+1$;

(b) since the binding in the $I=0$ channel is much stronger than in the $I=1$ channel, if we neglect effects other than π exchange we expect the corresponding $I^G=0^+, J^{PC} = 1^{++}$ states to be up to 40-50 MeV below the thresholds [22]. The $I=0$ states would then be expected close in mass to the $Y(4S)$. Their expected decay modes are

$$Z_b(I=0) \rightarrow Y(mS)\pi^+\pi^- \quad \text{and} \quad Z_b(I=0) \rightarrow Y(mS)\gamma,$$

as well as

$$Z_b(I=0) \rightarrow B\bar{B}\gamma \quad \text{via} \quad B^* \rightarrow B\gamma, \quad E_\gamma = 46 \text{ MeV};$$

which might well be within the reach of LHCb.

A $(\Sigma_b^+\Sigma_b^-)$ beauteron dibaryon?

The discovery of the Z_b states and their probable interpretation as $B^*\bar{B}$ and $B^*\bar{B}^*$ bound by pion exchange raises an interesting possibility that a strongly bound $\Sigma_b^+\Sigma_b^-$ deuteron-like state might exist, a *beauteron*. This is because Σ_b is about 500 MeV heavier than B^* and having $I=1$, it couples more strongly to pions than B and B^* which have $I = \frac{1}{2}$. The opposite

electric charges of Σ_b^+ and Σ_b^- provide an additional attraction. A possible decay mode of the beauteron is

$$(\Sigma_b^+ \Sigma_b^-) \rightarrow \Lambda_b \Lambda_b \pi^+ \pi^-$$

which might be observable in LHCb. If the beauteron exists, it should also be seen in lattice QCD.

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