News on hadrons in a hot medium

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Some of the modifications that a thermal medium, of the type generated in heavy ion collision experiments at the LHC, may impose on the properties of hadrons, are reviewed. The focus is on hadrons containing at least one heavy quark (charm or bottom or their antiparticles).

1 Introduction

The title that I was given is quite broad, so I chose to interpret it in a particular way. Denoting by T the temperature and by μ the baryonic chemical potential, the part "hot medium" will be taken to indicate the temperature range 50 MeV $\ll T \ll$ 1000 MeV, perhaps reachable in the current generation of heavy ion collision experiments; the medium is also assumed *not* to be "dense", meaning that the chemical potential is small, $|\mu| \ll \pi T$. The most drastic cut concerns "hadrons": in this talk only those which more or less maintain their identity in the temperature range considered are discussed, meaning that at least one heavy quark with a mass $M \gg \pi T$ should be present. The final cut is related to the word "news": I will try to concentrate on recent works from 2010/2011, not tracing literature any further back than to 2004 (with one exception).

With these excuses, the outline is that I will start by reviewing "open c, b", i.e. the fate of D and B mesons in a hot medium. Subsequently recent developments concerning quarkonium physics, or "bound $c\bar{c}, b\bar{b}$", i.e. J/ψ and Υ mesons, are discussed. A final short section concerns what I refer to as "thermal $c\bar{c}, b\bar{b}$", standing for quark-antiquark pairs generated from thermal fluctuations. I would like to apologize once more for the exclusion of lighter hadrons, on which many new insights have been reported at this conference and elsewhere and but on which I possess no expertise.

2 Open c, b

Jets forming around energetic single heavy quarks are among the basic events in a hadronic collision, and the gluon-fusion amplitude that is responsible for them, illustrated at leading

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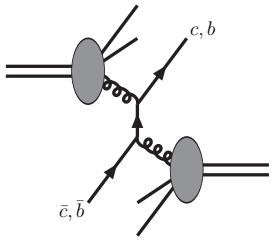
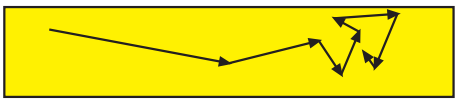
(1) 

is theoretically relatively well understood [1]. For thermal physics, the interesting question is what happens after the initial production. The idea is that apart from the heavy quarks, a lot of "soft stuff" gets generated as well, which could rapidly reach a state of thermal equilibrium. Then the heavy quarks need to propagate through this "medium" (shown with yellow below), a process that can be sketched as

(2)  .

What is illustrated here is that the heavy quark jets tend to get slowed down and eventually stopped, by bremsstrahlung as well as by elastic scatterings. In the latter case some gluons can be off-shell and soft, and this can lead to large infrared effects from processes like

(3)  .

Therefore we may expect, with some hindsight already folded in, that despite their large inertia the heavy quarks do interact strongly with the medium. In fact, experimentally, charm quark jets get "quenched" practically as effectively as light jets; this is shown in fig. 1, and is among the reasons that the medium generated in heavy ion collisions is nowadays conceived to constitute a "strongly interacting quark-gluon plasma", or sQGP.

It is a challenge for the theoretical finite-temperature community to quantitatively explain, starting from the basic rules of QCD and statistical physics, the experimentally observed strong interactions felt by the heavy quarks. (Although inspiring, AdS/CFT-based studies for analogous theories are not discussed here for lack of space.) Indeed many different lines of research have been pursued in this vain. For instance, a parameter that characterizes the strength of interactions felt by the heavy quarks, called the "momentum diffusion coefficient", has been computed at leading [3] and next-to-leading [4, 5] order in the weak-coupling expansion. Given that the weak-coupling expansion shows questionable convergence, various model studies at $T \gg 200$ MeV also appear well-motivated as an intermediate stage [6–8]. In the end, though, it is important to come up with a non-perturbative first-principles formulation within QCD [9, 10] and move towards a numerical lattice determination of the quantities in question [11–13]. (For charm this is also being attempted

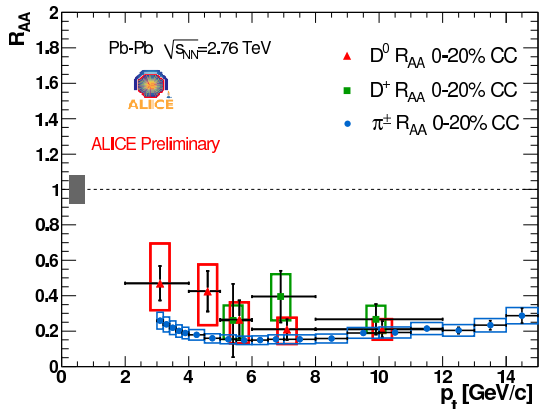


Figure 1: The "nuclear modification factor", R_{AA}, as a function of the transverse momentum, from recent ALICE data [2]. The fact that $R_{AA} < 1$ indicates that jets get quenched, and the data show that D-mesons stop about as effectively as pions despite their much larger inertia. In fact even B-mesons appear to interact strongly with the medium [2].

through a "brute force" approach [14].) Another line is that at low temperatures, $T \ll 200$ MeV, the same physics can be addressed analytically through the use of chiral effective theories or extensions thereof [15–18]. There are also many works in continuous progress, with a basic philosophy that can be traced back to ref. [3], that aim to implement theoretical results of the type mentioned in a combined hydrodynamics and Langevin simulation, in order to obtain results that can be directly compared with experiment.

Let me highlight a couple of these developments in a bit more detail. A perhaps most economical non-perturbative formulation of the problem, valid at least for the bottom case, is to make use of Heavy Quark Effective Theory in order to remove the heavy quark mass scale from the problem, leaving over a purely gluonic correlator [10]:

$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re}\,\mathrm{Tr}\Big[U_{\beta;\tau}\, gE_i(\tau,\mathbf{0})\, U_{\tau;0}\, gE_i(0,\mathbf{0})\Big]\right\rangle}{\left\langle \mathrm{Re}\,\mathrm{Tr}[U_{\beta;0}]\right\rangle}\,, \tag{4}$$

where gE_i stands for the (bare) colour-electric field and $U_{\tau_2;\tau_1}$ is a straight timelike Wilson line in Euclidean signature. The "transport coefficient" corresponding to this correlator yields the momentum diffusion coefficient alluded to above, often denoted by κ. In fig. 2 results are shown from the direct next-to-leading order perturbative computation of κ and from a measurement of the correlator defined in eq. (4). It should become clear how large corrections (left) could be pretty much hidden on the Euclidean lattice (right), whose results at first sight follow closely a next-to-leading order prediction. This indicates that a very high resolution is needed in the lattice measurement; therefore no final results are available to date, but the problem appears (relatively) well-posed, and work continues.

As another highlight, consider the hadronic phase, $T \ll 200$ MeV. The momentum diffusion coefficient mentioned above can be related (in the limit $M \gg \pi T$) to the "usual" diffusion

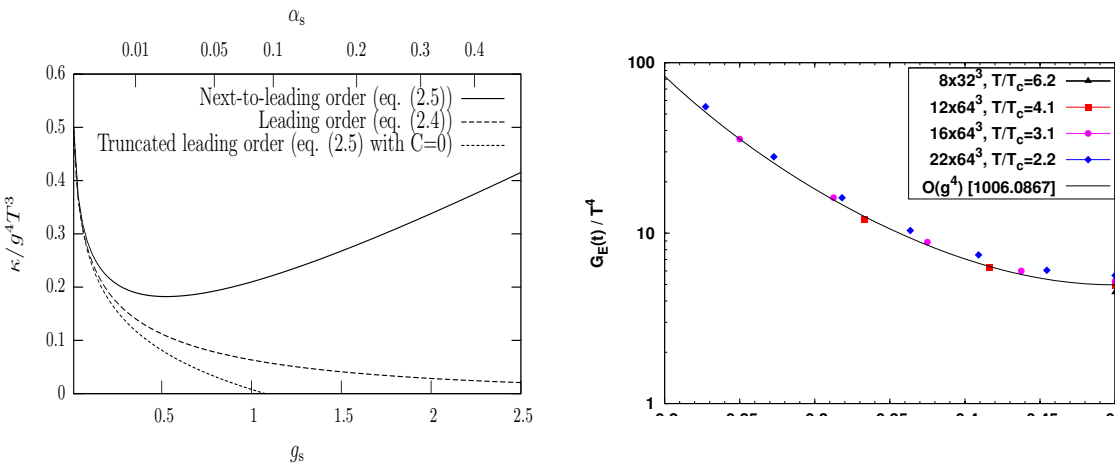


Figure 2: Left: Leading and next-to-leading order results for the heavy-quark momentum diffusion coefficient, κ, as a function of a renormalized gauge coupling (from ref. [5]). Right: Lattice results for the corresponding Euclidean correlator (from ref. [12]), compared with a next-to-leading order determination of the same quantity (from ref. [11]).

coefficient, let us denote it by D, through the Einstein relation, $D = 2T^2/\kappa$. If interactions are strong, then κ is large (cf. fig. 2 left) and therefore D is small. In fig. 3, results are shown from various works, concentrating particularly on the hadronic phase; the results have been obtained by making use of Heavy Meson Chiral Perturbation Theory, hadronic models, or some "intermediate" framework. The main observations are that, first of all, various works differ signficantly from each other so that they probably contain non-negligible systematic uncertainties; but that, second, it seems conceivable that strong interactions are *not* immediately switched off in the hadronic phase. This might play a role in the analysis of heavy ion collision data as well.

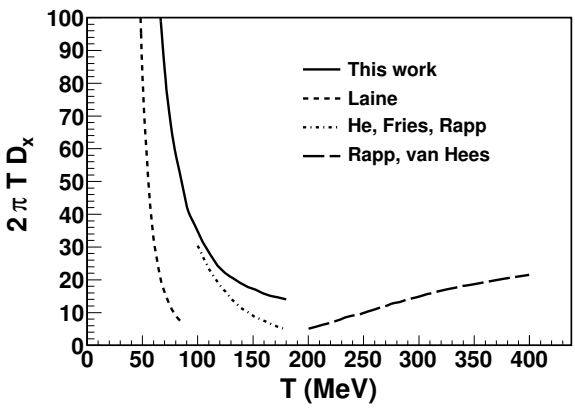
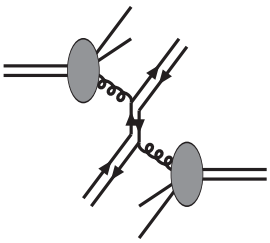


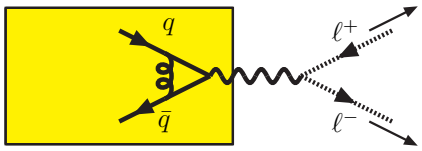
Figure 3: The charm quark diffusion coefficient from ref. [18], with results from refs. [6, 15, 16] also shown for comparison.

3 Bound $c\bar{c}, b\bar{b}$

Although quarkonium physics is in some sense a more traditional and popular probe for quark-gluon plasma formation than the physics related to heavy quark jets discussed in the previous section, it is also clear that this is a more challenging topic, both theoretically and experimentally. The difficulty becomes apparent already by drawing a simple example for an initial hard process in which quarkonium can get generated:

(5)  .

Clearly the cross section must be smaller than that in eq. (1), and furthermore the corresponding rate is more difficult to compute reliably. On a more positive note, the final signal is simple, given that quarkonium can decay into a dilepton pair, say $\mu^+\mu^-$, which is easily identified and whose invariant mass can be measured with a relatively good resolution:

(6) 
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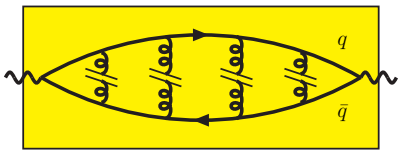
Indeed a possible thermal modification of the bottomonium spectral shape, illustrated in fig. 4, is among the most spectacular early results from the LHC heavy ion program.

Apart from experimental news, there has also been some theoretical progress on thermal quarkonium physics in recent years. I would like to imagine that a slight paradigm shift is taking place: whereas it was originally envisaged that quarkonium remains a coherent quantum-mechanical bound state in a thermal medium, with only the potential that binds it together getting modified by Debye screening [23], a process that could perhaps be illustrated as

(7) 
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a "modern" view is that there might be an additional effect also taking place: coherence may be (partly) lost due to random kicks from a heat bath. Technically, this implies that a suitably defined static potential may develop an imaginary part. The corresponding physics could be thought of for instance in terms of the following diagram, obtained from eq. (7) by

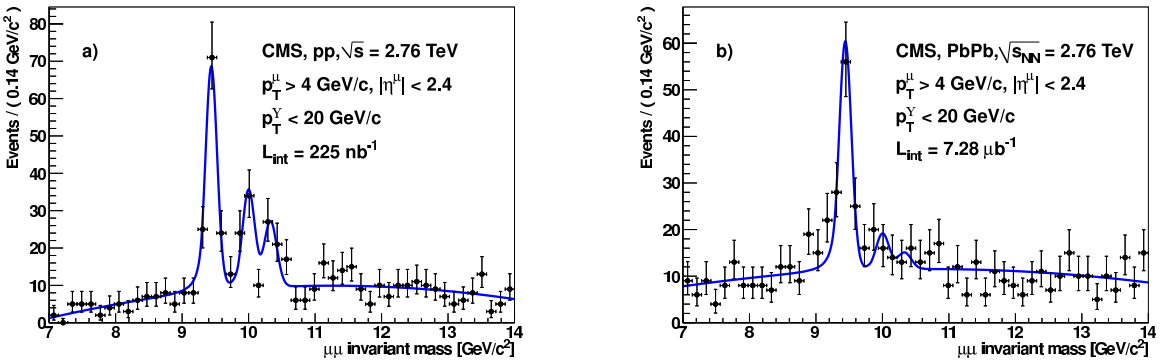
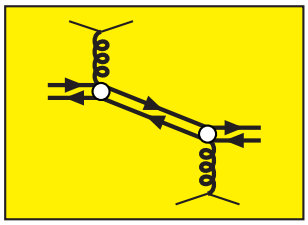


Figure 4: The disappearance of spectral features from bottomonium resonances when going from *pp* to *PbPb* collisions, according to CMS (from ref. [19]). Qualitatively the pattern appears to be in accordance with a classic "sequential suppression" scenario [20] (see also ref. [21]); a quantitative interpretation has been put forward in ref. [22].

twisting two of the gluon lines outwards:

(8)  .

The full body of literature concerning quarkonium at high temperatures is too vast to fit in this review, so I restrict to mentioning representative theoretical works that contain elements in this "modern" direction. Various computations resulting in a complex real-time static potential have been reported e.g. in refs. [24–34] (actually many inequivalent definitions of a "static potential" are being used). These results can be given a systematic role at least in the weak-coupling regime where a number of momentum scales can be identified and, if there is a hierarchy between them, a full-fledged effective field theory approach can be formulated (termed "PNRQCD$_{\rm HTL}$" in a particular regime) [35–38]. On the side of applications, at high temperatures where quarkonium "melts" the quantities that can still be defined are the spectral function and the thermal part of the dilepton production rate, $\mathrm{d}N_{\mu^-\mu^+}/\mathrm{d}^4x\,\mathrm{d}^4Q$ [8, 39–45]. On the other hand, if we stay below the melting temperature, which may be phenomenologically relevant particularly for some of the bottomonium resonances (cf. fig. 4), then it is meaningful to speak of the "binding energy" of a bound state as well as of its "width"; these are simpler objects than the complete spectral function, and can be computed in rather explicit form [46], including even their velocity dependence [47] (see also ref. [48]). Ultimately, of course, a lattice investigation is needed [49–52], but even those might preferably be formulated within a non-relativistic effective theory approach [53–55], particularly in the bottomonium case.

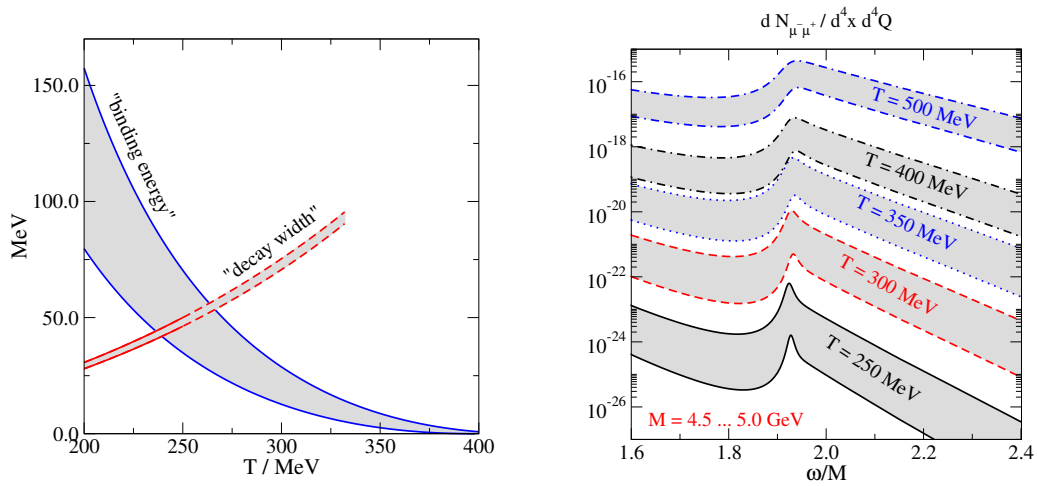


Figure 5: Left: A thermal "decay width" that decoherence due to random kicks from the heat bath could bestow upon a quantum-mechanical bound state (from ref. [24]). Right: A thermal component in the dilepton production rate, with a resonance peak at low temperatures going over into a smooth threshold at high temperatures (from ref. [43]).

Let me again highlight a couple of these developments in a bit more detail. The conceptual change from an "on-off" melting picture towards a gradual evolution of the spectral shape of a quarkonium resonance is illustrated on the qualitative level in fig. 5, for the bottomonium case. More quantitatively, at low temperatures, far below "melting" ($T \ll 250$ MeV in terms of a situation illustrated in fig. 5(left)), the width shows a specific T-dependence [46] and, as mentioned, its velocity-dependence is also computable [47].

As a second highlight, let me mention lattice computations within effective theories. For lattice QCD, a "scale hierarchy" (say, between m_π and m_B) is always a serious challenge, because for a reliable representation of continuum physics the lattice spacing should be shorter than the smallest physical length scale of the system, whereas for an exclusion of finite-volume effects the lattice extent should exceed the largest physical length scale; these complementary requirements easily lead to a prohibitively large number of lattice sites. On the other hand, for effective field theories, the existence of a clear scale hierarchy is a blessing. This suggests the idea of combining the two approaches; indeed a study of the bottomonium system within the so-called NRQCD approach has recently been launched [55] (there are well-known issues related to the existence of a continuum limit but these should be no worse than at zero temperature). Another direction is to directly determine a real-time static potential, perhaps to be used within a "PNRQCD$_{\rm HTL}$"-type framework, through a spectral analysis of an imaginary-time Wilson loop [53, 54], as sketched in fig. 6.

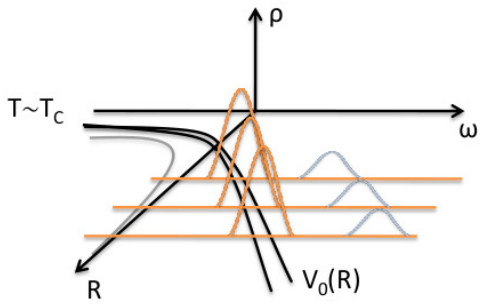


Figure 6: A sketch for how a spectral analysis of an imaginary-time Wilson loop could reveal a real part of a static potential, from the position of a peak, and an imaginary part, from its width (from a talk by A. Rothkopf, illustrating the work reported in refs. [53, 54]).

4 $c\bar{c}$, $b\bar{b}$ from thermal fluctuations

It is interesting to ask under which conditions heavy quarks or mesons can "chemically equilibrate", i.e. be part of the heat bath with an entropy density determined by T, rather than by some non-thermal initial process such as those illustrated in eqs. (1), (5). Naively, one would think that this is the case only for $T \gg 2M$, so that there is no Boltzmann suppression, $\exp(-\frac{2M}{T})$, hindering the rate of pair creation of a quark-antiquark pair. However, trying to be more quantitative, one may ask e.g. how the heavy quark mass M should be interpreted; say, as an $\overline{\rm MS}$ scheme mass, $M \to m_c^{\overline{\rm MS}}(3\,{\rm GeV}) \approx 1$ GeV; as a more "physical" pole mass, $M \to m_c^{\rm pole} \sim (1.5 - 2.0)$ GeV; or as something else? Furthermore, the Boltzmann weight comes with a prefactor which might partially compensate for the exponential suppression, and in fact anyone familiar with the "imaginary-time" formalism of thermal field theory would from the outset suggest a comparison of $2M$ with $2\pi T$ rather than T, which is a significant difference. All in all, it seems that the issue is non-trivial.

In any case, sample results from computations of how chemically equilibrated charm quarks would contribute to thermodynamic observables are shown in fig. 7, both from the weak-coupling expansion and from the lattice. It seems that there could be effects visible at surprisingly low temperatures; this might be relevant for the initial stages of hydrodynamics at the LHC, where current phenomenological studies tend to ignore charm quarks altogether. (For another argument in a related direction, see ref. [60].)

5 Conclusions

In heavy ion collisions at the LHC, heavy-quark related observables are becoming increasingly important: the existence of the heavy mass scale makes experimental signals clearly identifiable, and also facilitates theoretical analyses by allowing for the use of modern effective theory methods. That said, much further work is needed for quantitative conclusions.

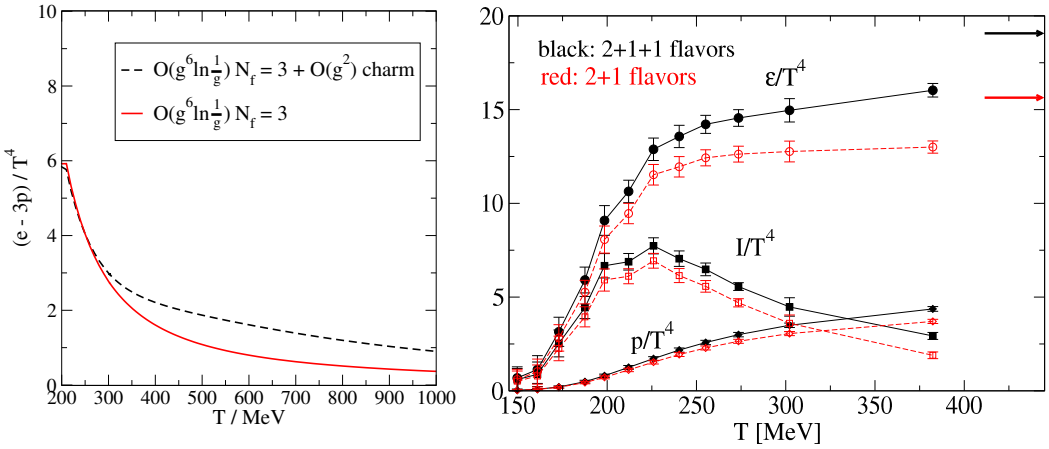


Figure 7: Left: Perturbative results for the "trace anomaly", with and without charm quarks (from ref. [56]). Right: Lattice estimates for the same quantity, denoted here by I (from ref. [57]; see refs. [58, 59] for similar works by other groups).

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