Review: The FLAG working group

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The FLAG working group reviews lattice results relevant for pion and kaon physics with the aim of making them easily accessible to the particle physics phenomenology community. The set of quantities considered so far comprises light quark masses, kaon and pion form factors, the kaon mixing parameter, and low energy constants of $SU(2)_L \times SU(2)_R$ and $SU(3)_L \times SU(3)_R$ chiral perturbation theory.

1 Introduction

Strong claims like

- "We find a (2-3)σ tension in the unitarity triangle" [1]
- "... confirming CKM unitarity at the permille level" [2]
- "... we find evidence of new physics in both B_d and B_s systems" [3]
- "Possible evidence for the breakdown of the CKM-paradigm of CP-violation" [4] ,

have recently been made based on lattice QCD results. This illustrates the role lattice QCD predictions are playing for the phenomenology of the Standard Model (SM). To this end the FLAG working group has formed with the aim of allowing also to an outsider a judgement of the quality and *state-of-the-art-fulness* of lattice results.

The quantities FLAG is currently considering comprise the light quark masses m_u, m_d, m_s, the ratio of the leptonic kaon and pion decay constant, f_K/f_π, the semi-leptonic $K \to \pi$ form factor at vanishing momentum transfer $f_+(0)$, the kaon mixing parameter B_K and a number of low energy constants of $SU(2)_L \times SU(2)_R$ and $SU(3)_L \times SU(3)_R$ chiral perturbation theory.

The FLAG report provides all relevant formulae and notation, a detailed quality assessment of every computation, where FLAG considers appropriate an average or recommended range and a lattice dictionary for non-experts and details of every single lattice simulation. After having made a start FLAG is currently discussing extending the set of quantities (e.g. charm and beauty). Periodic updates of the results are planned. Note also the similar effort by [1].

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2 Lattice QCD

While perturbation theory is an invaluable tool at weak coupling it cannot make predictions for bound state observables like the proton mass or the properties of hadron decays. While potential models and sum rules clearly allow for studying these properties, only lattice QCD has the potential to make systematically improvable predictions.

QCD is a theory with three colour charges, six quarks in the fundamental representation with broken iso-spin in an infinite space-time continuum with the charged pion mass being 139.6MeV. In order to regularise the theory and make it accessible to simulations by means of a Monte-Carlo integration of its defining path integral, in lattice QCD one reduces its dynamical flavour content to $N_f = 2$ (degenerate up- and down-quark), 2+1 (plus a strange quark) or 2+1+1 (plus a charm quark), keeps the iso-spin symmetry exact, varies the pion mass ideally close to the physical point, considers only a finite volume of around 3-4fm and discretises space time with a lattice spacing of typically down to 0.05fm.

Clearly lattice QCD is thus affected by a number of systematic effects which need to be understood theoretically and controlled in the numerical simulation. For each of the major systematic effects FLAG has laid our quality criteria which are explained in detail in the document. In order to allow the reader a rough screening of available results, summaries are provided in terms of a colour coding:

- ★ when the systematic error has been estimated in a satisfactory manner and convincingly shown to be under control;
- ● when a reasonable attempt at estimating the systematic error has been made, although this could be improved;
- ■ when no or a clearly unsatisfactory attempt at estimating the systematic error has been made.

This colour coding has been set up for judging the quality of the chiral extrapolation, the continuum extrapolation, finite size effects, renormalisation, renormalisation scale running and the publication status. Where applicable, results only enter averages if they carry no red tag for any of the criteria. This also applies to the publication status - only published and peer-reviewed results (or simple updates of previously published data and analyses) qualify for inclusion into any average. FLAG treats results with different flavour content N_f separately.

3 FLAG summaries

In this section a selection of results of the FLAG document with a focus on the determination of the CKM-matrix element V_{us} are highlighted. Note that the summaries are based on the FLAG document [2]. Results that have appeared in the meantime will be included in future updates of the document.

3.1 CKM first-row unitarity - $|V_{us}|$

The determination of $|V_{us}|$ proceeds as follows: On the one hand, one experimentally measures the rate of a flavour changing process $s \to u$, where s is the strange quark and u the up quark. On the other hand one computes the SM prediction for the same process whose amplitude is proportional to the CKM-matrix element $|V_{us}|$ and which receives contributions from the electromagnetic, the weak and the strong interactions. While the former two are treated in perturbation theory for the processes considered here, the contribution from the latter needs to be computed in lattice QCD. Eventually, $|V_{us}|$ is determined by equating the experimental result with the SM-prediction.
For kaon/pion leptonic decays the relation between experiment and theory in the SM was computed by Marciano [5] and using the latest analysis of experimental results [6] it yields the correlation

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.2758(5)\,. \tag{1}$$

Since lattice QCD can provide f_K/f_π one obtains a prediction for $|V_{us}/V_{ud}|$. For semi-leptonic kaon decays the latest summary of experimental results together with SM-contributions yields [6]

$$|V_{us}| f_+(0) = 0.2163(5)\,, \tag{2}$$

and $|V_{us}|$ is readily extracted provided a prediction of $f_+(0)$.
Figure 1 exemplifies FLAG's compilation of results showing the lattice data with 2, 2+1 and 2+1+1 flavours of dynamical fermions for both $f_+(0)$ and f_K/f_π. As an example we show the underlying data for $f_+(0)$ together with the FLAG assessment in table 1. The results for f_K/f_π and for $f_+(0)$ for $N_f = 2$, $N_f = 2+1$ and $N_f = 2+1+1$, respectively, are all mutually compatible. The observation that the simulation and analysis techniques that lead to all these results differ significantly amongst the quoted collaborations causes confidence in the approach. The effect of adding the dynamical strange quark (and the charm quark) does not lead to any visible effects beyond the current level of precision.
The following averages/recommended values are explained in detail in the FLAG document: $f_K/f_\pi|_{N_f=2+1} = 1.193(5)$ (average over BMW, MILC and HPQCD/UKQCD), $f_K/f_\pi|_{N_f=2} = 1.210(18)$ (ETM) and, $f_+(0)|_{N_f=2+1} = 0.959(5)$ (RBC+UKQCD) and $f_+(0)|_{N_f=2} = 0.956(8)$ (ETM).
On the one hand these results can be used for making predictions for $|V_{ud}|$, $|V_{us}|$, $f_+(0)$ and f_K/f_π based on the experimental results (1) and (2) and on the assumption of CKM first row unitarity $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ (at the current level of precision $|V_{ub}|$ is too small to play any significant role). On the other hand, when using only the experimental result as input the first row unitarity can be tested. This is summarised in the plot in figure 2.
Given that KLOE-2 is aiming at reducing the uncertainty in their experimental determination for $|V_{us}| f_+(0)$ by a factor of about two in the near future [36, 37] it is fair to ask about prospects on the theory side. Recent progress for $f_+(0)$ [7, 8, 38] has allowed to remove

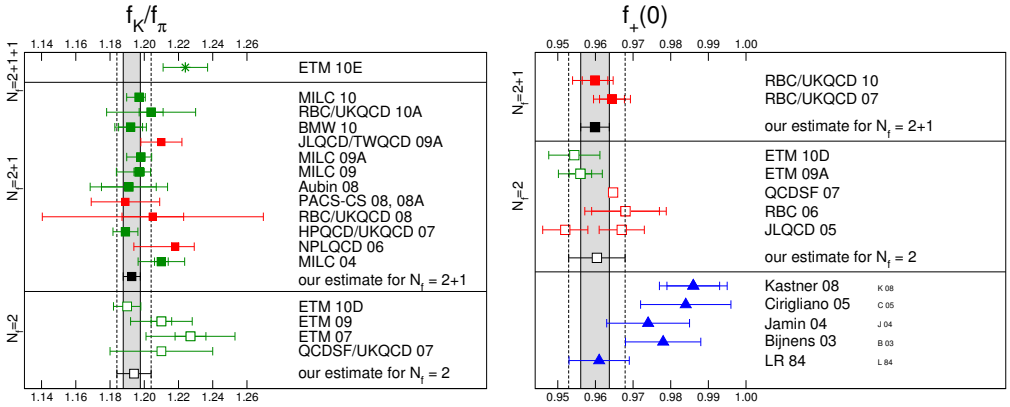

Figure 1: Scatter plots of the lattice data on the ratio of leptonic decay constants f_K/f_π and the semi-leptonic $K \to \pi$ vector form factor at vanishing momentum transfer, $f_+(0)$. Green symbols identify results that are free of *red tags* according to FLAG's assessment. The vertical bands correspond to FLAG's average/recommended range for $N_f = 2$ and $N_f = 2+1$, respectively.

Collaboration	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	$f_+(0)$
RBC/UKQCD 10	2+1	A	●	■	★	$0.9599(34)(^{+31}_{-47})(14)$
RBC/UKQCD 07	2+1	A	●	■	★	0.9644(33)(34)(14)
ETM 10D	2	C	●	★	●	$0.9544(68)_{stat}$
ETM 09A	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	2	C	■	■	★	$0.9647(15)_{stat}$
RBC 06	2	A	■	■	★	0.968(9)(6)
JLQCD 05	2	C	■	■	★	0.967(6), 0.952(6)

Table 1: Quality assessment of the semi-leptonic $K \to \pi$ vector form factor at vanishing momentum transfer, $f_+(0)$. FLAG classifies the publication status as A published or plain update of published results, P preprint or C conference contribution.

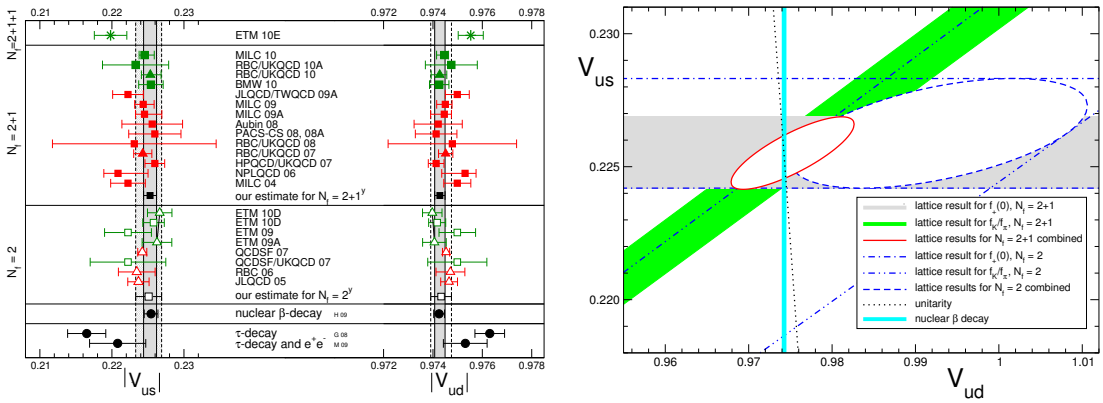

Figure 2: Left: Lattice predictions for $|V_{us}|$ and $|V_{ud}|$ assuming SM unitarity. Right: FLAG's illustration of lattice results in the $|V_{us}|$-$|V_{ud}|$-plane [2]. The ellipse represent the combined unitarity analysis for $N_f = 2+1$ flavours (solid red) and $N_f = 2$ flavours (dashed blue) while the black dashed line represents SM-unitarity. According to this analysis all results are compatible with first row unitarity.

one of the two most dominant uncertainties (momentum resolution in lattice simulations). The remaining dominant uncertainty is the one due to the chiral extrapolation which will disappear once results appear for physical pion masses. Cut-off effects in this observable will remain a sub-dominant uncertainty for a while: flavour symmetry implies that if the average light quark mass m_q is set equal to the strange quark mass m_s, the lattice data yield $f_+(0) = 1$, irrespective of the lattice spacing or the size of the box and for any value of m_s. Cut-off effects can therefore only affect the difference $1 - f_+(0)$, which turns out to be about 0.04. For f_K/f_π the error due to the chiral extrapolation will also disappear once all collaborations simulate directly at the physical point. The statistical error can be reduced by simulating longer (naively it reduces with $1/\sqrt{N}$ where N is proportional to the Monte Carlo time).

3.2 Neutral Kaon mixing

The mixing of neutral pseudoscalar mesons plays an important role in the understanding of the physics of CP-violation. Here we present the summary of lattice data for neutral kaon mixing which provides a probe for indirect CP-violation. The plot in figure 3 shows the status of lattice computations for $\hat{B}_K$. There has been tremendous progress over the last, say, five years. In particular the utilisation of chirally symmetric lattice fermion formulations [39, 40] has allowed to circumvent the problem of operator mixing of the 4-fermion operator. This reduces systematic effects considerably. The FLAG recommended values for the renormalisation group independent B-parameter which are indicated by the vertical bands in the plot in figure 3 are $\hat{B}_K = 0.738(20)$ ($N_f = 2+1$, Aubin 09, RBC/UKQCD 10B) and $\hat{B}_K = 0.792(25)(17)$ ($N_f = 2$, ETM 09D).

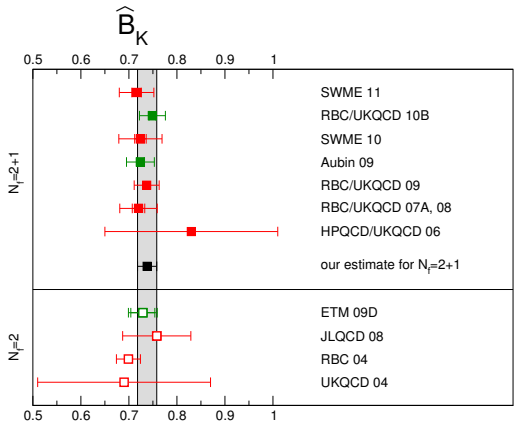

Figure 3: Scatter plot for $N_f = 2$ and $N_f = 2+1$ results for the neutral kaon mixing parameter $\hat{B}_K$.

3.3 Quark masses

The bare parameters of the $N_f = 2(2+1)$ Lagrangian are the gauge coupling and the up- and down- (and strange) quark masses. Since lattice QCD is simulated in the iso-spin limit, it is sufficient to provide two (three) hadronic quantities from experiment to tune the parameters of the Lagrangian to their physical values. There exist well defined procedures for how to determine renormalised quark masses in a second step. Various collaborations have carried out programmes to determine the light quark masses and the scatter plot in figure 4 gives an impression for the precision which can by now be achieved in lattice QCD. The FLAVIA recommended values indicated by the vertical bands in the plots are $m_{ud} = 3.43(11)$MeV, $m_s = 94(3)$MeV for $N_f = 2+1$ (MILC 09A, RBC/UKQCD 10A and HPQCD 10) and $m_{ud} = 3.6(2)$MeV, $m_s = 95(6)$MeV for $N_f = 2$ (ETM 10B). In fact, the precision is so good that QCD and QED iso-spin breaking effects play a significant role and need to be taken into account in any reliable estimate of the systematic uncertainties. The combined statistical and systematic errors in the FLAG average for $N_f = 2+1$ are around 3% for both the average up- and down-quark mass and the strange quark mass. FLAG also provides estimates of the individual up- and down-quark masses and found, based on phenomenological estimates of the quark's electro-magnetic self-energies that the up-quark mass to be different from zero by 15 standard deviations.

3.4 Low energy constants

On the one hand lattice practitioners are putting a lot of effort into improving algorithms and computers in order to be able to simulate QCD for physical quark masses, in this way avoiding systematic-affected extrapolations to the physical point. That this is now possible has been demonstrated only recently [72]. However, many simulations are carried out with

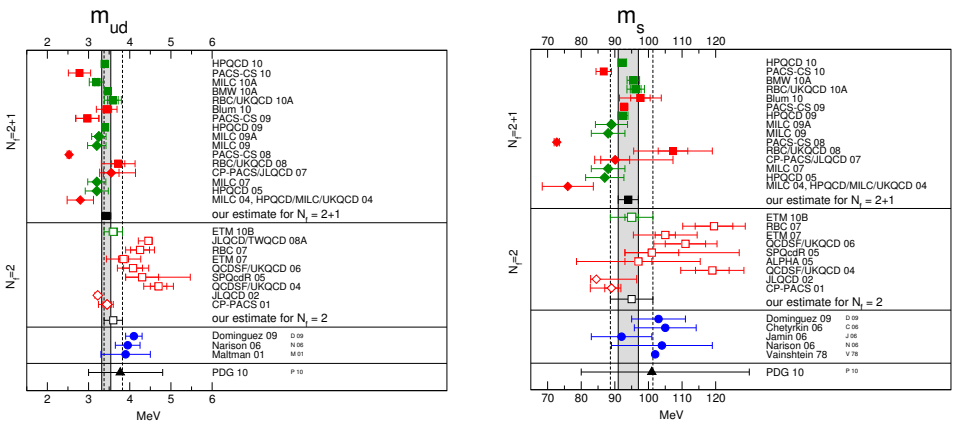

Figure 4: Scatter plot for the average up- and down-quark masses and the strange quark mass.

heavier than physical quark masses and chiral perturbation theory is employed to provide ansätze for the mass dependence of the observable under consideration. The low-energy constants of chiral perturbation theory can be determined from fits of these ansätze to the lattice data. The FLAG document also presents an overview and analysis over the status of determinations of these low-energy constants.

4 Outlook

FLAG has set out to provide the (Beyond) Standard Model phenomenologist with predictions for low-energy parameters and observables of QCD. The aim is to screen and judge all available lattice data from a specialist's point of view and to provide a summary of the status and a quality assessment of each single result for the non-specialist and where appropriate also an average or recommended range.

FLAG is envisaging regular updates of the summary table and scatter plots and also of the averages. In addition FLAG is working on extending the set of data considered so far towards observables and parameters related to charm and bottom quarks.

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