# Studies of the $X(3872)$ as a mixed molecule-charmonium state in QCD Sum Rules 

Carina M. Zanetti ${ }^{1}$<br>Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil


#### Abstract

We use QCD sum rules to investigate the nature of the meson $X(3872)$, assumed to be a mixture between charmonium and exotic molecular $[c \bar{q}][q \bar{c}]$ states with $J^{P C}=1^{++}$. We discuss the mass of the state, decay widths of the channels $J / \psi+$ pions and $J / \psi \gamma$, and the branching ratio for the production of the state in the decay $B \rightarrow X(3872) K$.


The state $X(3872)$ has been first observed by the Belle collaboration in the decay $B^{+} \rightarrow$ $X(3872) K^{+} \rightarrow J / \psi \pi^{+} \pi^{-} K^{+}$[1], and was later confirmed by CDF, D0 and BaBar [2]. The current world average mass is $m_{X}=(3871.4 \pm 0.6) \mathrm{MeV}$, and the total decay width is $\Gamma<2.3 \mathrm{MeV}$ at $90 \%$ confidence level. Further studies from Belle, Babar and CDF strongly favor the quantum numbers $J^{P C}=1^{++}$or $2^{-+}$[3].
The $X(3872)$ was the first state of an increasing number of candidates for exotic hadrons discovered recently. The mass of this state is smaller than the one predicted by the constituent quark model predictions [7]. Moreover, the decay rates of processes $X(3872) \rightarrow$ $J / \psi \pi^{+} \pi^{-} \pi^{0}$ and $X(3872) \rightarrow J / \psi \pi^{+} \pi^{-}$are comparable [3]:

$$
\begin{equation*}
\frac{\Gamma\left(X \rightarrow J / \psi \pi^{+} \pi^{-} \pi^{0}\right)}{\Gamma\left(X \rightarrow J / \psi \pi^{+} \pi^{-}\right)}=1.0 \pm 0.4 \pm 0.3 \tag{1}
\end{equation*}
$$

which may indicate a strong isospin and $G$ parity violation, which is incompatible with a $c \bar{c}$ structure for $X(3872)$. These are strong evidences that the $X(3872)$ is not a conventional $q \bar{q}$ state, and attempts to treat this state as a multiquark state are being pursued, e.g. molecules or tetraquarks states. The QCD sum rules (QCDSR) technique is an important tool in the study the structure of hadronic states. In Ref. [4] the QCDSR calculations were performed to study the tetraquark structure $[c q][\overline{c q}]$, and the mass obtained was $m_{X}=(3.92 \pm 0.13) \mathrm{GeV}$. With a current for the mesonic molecule of the type ( $D^{* 0} \bar{D}^{0}-\bar{D}^{* 0} D^{0}$ ) the QCDSR calculation reported in [5] obtained the mass $m_{X}=(3.87 \pm 0.07) \mathrm{GeV}$ in a better agreement with the experimental mass. Although the QCDSR results for the mass of the $X(3872)$ are good, it is not possible to describe the decays data with the same success [6].
There are also evidences that seem to indicate the existence of a $c \bar{c}$ component in the $X(3872)$ structure, as discussed in Refs. [8,9]. Also, the recent observation, reported by BaBar [10],

[^0]of the decay $X(3872) \rightarrow \psi(2 S) \gamma$ at a rate: $\mathcal{B}(X \rightarrow \psi(2 S) \gamma) / \mathcal{B}(X \rightarrow \psi \gamma)=3.4 \pm 1.4$, is much bigger than the molecular prediction $\Gamma(X \rightarrow \psi(2 S) \gamma) / \Gamma(X \rightarrow \psi \gamma) \sim 4 \times 10^{-3}$ [11].

In what follows we present some results of the QCD sum rules (QCDSR) calculations on the $X(3872)$, which we assume to be a mixed charmonium-molecular state. This is implemented following the prescription suggested in [12] for the light sector. The mixing is done at the level of the currents and is extended to the charm sector. In a different context (not in QCDSR), a similar mixing was suggested already some time ago by Suzuki [9]. Physically, this corresponds to a fluctuation of the $c \bar{c}$ state where a gluon is emitted and subsequently splits into a light quark-antiquark pair, which lives for some time and behaves like a molecule-like state

The the mass of a hadronic state can be calculated in QCDSR [13] from the two-point correlator using a current that creates the states with the quantum number of the hadron. Considering the $X(3872)$ as a $J^{P C}=1^{++}$state we can construct a mixed charmoniummolecule current, following [12]. The $D^{0} D^{* 0}$ molecule is interpolated by (with $q=u$ ):

$$
\begin{equation*}
J_{\mu}^{(4 q)}(x)=\frac{1}{\sqrt{2}}\left[\left(\bar{q}_{a}(x) \gamma_{5} c_{a}(x) \bar{c}_{b}(x) \gamma_{\mu} q_{b}(x)\right)-\left(\bar{q}_{a}(x) \gamma_{\mu} c_{a}(x) \bar{c}_{b}(x) \gamma_{5} q_{b}(x)\right)\right] \tag{2}
\end{equation*}
$$

As in ref. [12] we define the normalized two-quark current as

$$
\begin{equation*}
J_{\mu}^{(2 q)}(x)=\frac{1}{6 \sqrt{2}}\langle\bar{u} u\rangle \bar{c}_{a}(x) \gamma_{\mu} \gamma_{5} c_{a}(x), \tag{3}
\end{equation*}
$$

and from these two currents we build the following mixed charmonium-molecular current for the $X(3872)$ :

$$
\begin{equation*}
J_{\mu}^{q}(x)=\sin (\theta) J_{\mu}^{(4 q)}(x)+\cos (\theta) J_{\mu}^{(2 q)}(x) \tag{4}
\end{equation*}
$$

where $\theta$ is the mixing angle between the two-quarks and four-quarks states.
The mass of the $X$ (3872) was obtained in Ref. [14], using the current (4). Taking into account the variation of the mixing angle $\theta$ in the region $5^{\circ} \leqslant \theta \leqslant 13^{\circ}$, the QCDSR calculation of the mass gives:

$$
\begin{equation*}
m_{X}=(3.77 \pm 0.18) \mathrm{GeV}, \tag{5}
\end{equation*}
$$

which is in a good agreement with the experimental value. The value obtained for the mass grows with the value of the mixing angle $\theta$, but for $\theta \geqslant 30^{\circ}$ it reaches a stable value being completely determined by the molecular part of the current.

In order to study the decays of the $X(3872)$ it is necessary to calculate the three-point function of the effective vertex for such decay. As a result of the calculation we can obtain the coupling constant that can be used to compute partial widths.

The width for the decay $X \rightarrow J / \psi V \rightarrow J / \psi F$ where $F=\pi^{+} \pi^{-}\left(\pi^{+} \pi^{-} \pi^{0}\right)$ for $V=\rho(\omega)$ is calculated in [14], introducing a small admixture of $D^{+} D^{*-}$ and $D^{-} D^{*+}$ components, then the $X$ current is written as:

$$
\begin{equation*}
J_{\mu}^{X}(x)=\cos \alpha J_{\mu}^{u}(x)+\sin \alpha J_{\mu}^{d}(x), \tag{6}
\end{equation*}
$$

with $J_{\mu}^{u}(x)$ and $J_{\mu}^{d}(x)$ given by Eq.(4). The mixing angle between the different light flavors are determined through the ratio (1) to be $\alpha \simeq 20^{\circ}$, which was obtained previously in [6,15].
The QCDSR analysis of the three-point function of the vertex $X-J / \psi-V$ was performed with the same range of the mixing angle $5^{\circ} \leqslant \theta \leqslant 13^{\circ}$ used to determine the mass. The partial width obtained was:

$$
\begin{equation*}
\Gamma\left(X \rightarrow J / \psi \pi^{+} \pi^{-} \pi^{0}\right)=(9.3 \pm 6.9) \mathrm{MeV} . \tag{7}
\end{equation*}
$$

in complete agreement with the experimental upper limit.
The work done in Ref. [14] showed the possibility of the description of the $X(3872)$ as a mixed state in QCDSR, fixing the mixing angles to be $5^{\circ} \leqslant \theta \leqslant 13^{\circ}$ and $\alpha \sim 20^{\circ}$. In Refs. [16] and [17] it was investigated further consequences of this analysis, calculating the radiative decay and the production in $B$ decays, using the mixing angles determined in [14].
Belle Collaboration reported the radiative decay mode $X(3872) \rightarrow \gamma J / \psi$ [3] with the branching ratio: $\frac{\Gamma(X \rightarrow J / \psi \gamma)}{\Gamma\left(X \rightarrow J / \psi \pi^{+} \pi^{-}\right)}=0.14 \pm 0.05$. The QCDSR analysis in [16] gives the ratio

$$
\begin{equation*}
\frac{\Gamma(X \rightarrow J / \psi \gamma)}{\Gamma\left(X \rightarrow J / \psi \pi^{+} \pi^{-}\right)}=0.19 \pm 0.13 \tag{8}
\end{equation*}
$$

which is in complete agreement with the experimental result.
The channel of production of the $X$ from $B$ decays, $B^{ \pm} \rightarrow X(3872) K^{ \pm}$, is studied in Ref. [17]. This was done in the factorization approach, where the process can be viewed as two subprocesses with two different couplings: the parameter $\lambda_{W}$ in gives the coupling between the weak current and the $X$ state, and the form factors $f_{ \pm}\left(q^{2}\right)$ describe the weak transition $B \rightarrow K$. The branching ratio is therefore calculated was:

$$
\begin{equation*}
\mathcal{B}(B \rightarrow X(3872) K)=(1.00 \pm 0.68) \times 10^{-5}, \tag{9}
\end{equation*}
$$

The result (9) is in agreement with the experimental upper limit $\mathcal{B}\left(B^{ \pm} \rightarrow K^{ \pm} X(3872)\right)<$ $3.2 \times 10^{-4}$ [18].
In summary, we present the QCDSR analysis of the $X(3872)$ state considering a mixed charmonium-molecular current. The analysis for the state mass, decays into $J / \psi+$ pions and $J / \psi \gamma$, and the production from $B$ decays are consistent using the mixing angles in Eq. (6) and (4) with the values $\alpha=20^{\circ}$ and $5^{\circ} \leqslant \theta \leqslant 13^{\circ}$. The result presented here shows that in QCDSR is possible to explain the properties of the $X(3872)$ assuming that it is a mixture between a $c \bar{c}$ state and $D^{0} \bar{D}^{* 0}, \bar{D}^{0} D^{* 0}, D^{+} D^{*-}$ and $D^{-} D^{*+}$ molecular states.

## Acknowledgments

This work supported by FAPESP (Brazil).

## References

[1] S.-K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003).
[2] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 93, 162002 (2004); D. Acosta et al. [CDF Collaboration], Phys. Rev. Lett. 93, 072001 (2004); B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 71, 071103 (2005).
[3] K. Abe et al. [Belle Collaboration], hep-ex/0505037, hep-ex/0505038.
[4] R.D. Matheus, S. Narison, M. Nielsen and J.-M. Richard, Phys. Rev. D 75, 014005 (2007).
[5] S.H. Lee, M. Nielsen and U. Wiedner, arXiv:0803.1168.
[6] F.S. Navarra, M. Nielsen, Phys. Lett. B639, 272 (2006).
[7] T. Barnes and S. Godfrey, Phys. Rev. D 69, 054008 (2004).
[8] C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa and C. Sabelli, Phys. Rev. Lett. 103, 162001 (2009).
[9] M. Suzuki, Phys. Rev. D 72, 114013 (2005).
[10] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 102, 132001 (2009).
[11] E.S. Swanson, Phys. Lett. B 588, 189 (2004); Phys. Lett. B 598, 197 (2004)
[12] J. Sugiyama, T. Nakamura, N. Ishii, T. Nishikawa and M. Oka, Phys. Rev. D 76, 114010 (2007)
[13] M.A. Shifman, A.I. and Vainshtein and V.I. Zakharov, Nucl. Phys. B 147, 385 (1979); L.J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127, 1 (1985).
[14] R. D. Matheus, F. S. Navarra, M. Nielsen and C. M. Zanetti, Phys. Rev. D 80, 056002 (2009)
[15] L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, Phys. Rev. D 71, 014028 (2005).
[16] M. Nielsen and C. M. Zanetti, Phys. Rev. D 82, 116002 (2010)
[17] C. M. Zanetti, M. Nielsen, R. D. Matheus, Phys. Lett. B702, 359-363 (2011).
[18] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 96, 052002 (2006)


[^0]:    ${ }^{1}$ carina@if.usp.br

