Properties of light scalar mesons in the complex plane

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The flavour and glue structure of the light scalar mesons in QCD are probed by studying the couplings of the $\sigma(600)$ and $f_0(980)$ to the operators $\bar{q}q$, θ^μ_μ and to two photons.

1 Introduction

A precise identification of the exotic contents of the light scalar mesons is an old challenge for hadron physics. The idea of using the Roy dispersive representations in the complex plane [1] decisively clarified the status of the broad σ resonance and led to an accurate determination of its mass and width. The $I = 0$ scalars have the same quantum numbers as the expected lightest glueball. If all quark masses were heavy in QCD ($m_q \gtrsim 1GeV$) the glueball would be an extremely narrow meson (decaying only to two photons) with a mass slightly below two GeV [2]. Properties of the glueball in physical QCD, i.e. with three light quarks, are not yet computable in lattice simulations. A scenario suggested in [3] is that there could be two glueballs below 2 GeV, the lightest one could be the σ. Another question concerns the identification of a flavour multiplet: do the σ, κ, $a_0(980)$, $f_0(980)$ mesons belong to a nonet? This is far from obvious, e.g. several properties of the $a_0(980)$, $f_0(980)$ seem well explained by a $K\overline{K}$ molecule model [4]. I discuss below the evaluation of couplings of the σ and $f_0(980)$ to operators j_S using complex plane methods, which should provide quantitative inputs for answering some of these questions on the light scalars. Taking $j_S = \theta^\mu_\mu$ probes the gluon content, while $j_S = \bar{u}u + \bar{d}d$, $\bar{s}s$ probes the quark-antiquark content. The couplings to two photons will also be updated.

2 Poles and residues from an extended S-wave Roy solution:

Pion-pion partial-wave amplitudes obey a set of coupled integral representations as a consequence of analyticity and crossing symmetry [5]. On the real axis, at low energy, they combine with the unitarity relation into a set of non-linear equations which strongly constrain the S and the P-wave amplitudes. These equations were reconsidered recently in great detail [6]. Solutions were generated for given scattering lengths a^0_0, a^2_0 with a matching point $\sqrt{s_A} = 0.8$ GeV (i.e. using experimental inputs for $s > s_A$). The new experimental results on K_{l4} decays, analyzed with these solutions lead to very precise determinations of the scattering lengths [7]

$$a_0^0 = 0.2196 \pm 0.0028_{\text{stat}} \pm 0.0020_{\text{sys}}, \qquad (1)$$
$$a_0^2 = -0.0444 \pm 0.0007_{\text{stat}} \pm 0.0005_{\text{sys}} \pm 0.0008_{\text{ChPT}}$$

In order to improve the constraints on the $f_0(980)$ resonance, we begin by constructing a solution for the $I = 0$ S-wave over an extended region with a matching point $s_K = 4m_K^2$. Increasing the matching point from s_A to s_K leads to two differences: a) the matching point coincides with a two-particle threshold, the phase-shift has a cusp and one cannot use the continuity of the derivative argument to eliminate unphysical solutions, b) the multiplicity of the solutions (which depends on the phase-shift at the matching point [8]) increases by one unit. It turns out that unphysical solutions can still be identified provided the phase-shift is not too large ($\delta_K = \delta_0^0(s_K) \lesssim 225°$). The solutions then depend on one arbitrary parameter which can be taken as the value of the phase at one energy, e.g. $\delta_A = \delta_0^0(s_A)$. It is then determined by fitting the experimental phase-shifts in the range $[s_A, s_K]$ with the Roy solutions. One, in fact, can fit not only δ_A but also δ_K which is not very accurately determined from the data above s_K.

η_0^0	δ_A	δ_K	$\hat{\chi}^2_{\text{Hyams}}$	$\hat{\chi}^2_{\text{Kaminski}}$
shallow	$(80.9 \pm 1.4)^\circ$	$\left(190^{+5}_{-10}\right)^\circ$	2.7	1.9
deep	$(82.9 \pm 1.7)^\circ$	$\left(200^{+5}_{-10}\right)^\circ$	2.2	1.3

Table 1: Fitted values of δ_A and δ_K corresponding to two different central values (shallow-dip, deep-dip) for η_0^0.

The shape of the inelasticity, which sets in effectively at $s \geqslant s_K$ affects the phase-shift below s_K via the Roy equation. The central value of η_0^0 fitted to the determination from $\pi\pi$ production experiments displays a deep dip structure [9]. The one based on the inelastic channels displays a shallower dip (while compatible within the errors). The recent analysis of ref. [11], using constraints from a variant of the Roy equations in the region $\sqrt{s} \leqslant 1.1$ GeV, favours a deep dip structure. Our fit leads to the same conclusion, see table 1.

	$\sqrt{z_S}$ (MeV)	$\dot{S}_0^0(z_S)$ (GeV^{-2})
$\sigma(600)$	$\left(442^{+5}_{-8}\right) - i\left(274^{+6}_{-5}\right)$	$-\left(0.75^{+0.10}_{-0.15}\right) - i\left(2.20^{+0.14}_{-0.10}\right)$
$f_0(980)$	$\left(996^{+4}_{-14}\right) - i\left(24^{+11}_{-3}\right)$	$-\left(1.1^{+3.0}_{-0.4}\right) - i\left(6.6^{+0.8}_{-1.0}\right)$

Table 2: Poles and S-matrix derivatives from the extended Roy solution.

Next, one can compute the amplitude for complex energies. The position of the poles on the second Riemann sheet correspond to the zeros of the S-matrix $S_0^0(z) = 1 - 2\sigma^\pi(z)t_0^0(z)$ (see e.g. [1]) with $\sigma^\pi(z) = \sqrt{4m_\pi^2/z - 1}$. The results for the zeros and for the corresponding derivatives of S_0^0 (needed for the determination of the residues) are shown in table 2.

3 Couplings to two photons

Couplings of the light scalars to two photons can be determined from the amplitudes $\gamma\gamma \to \pi^0\pi^0, \pi^+\pi^-$. As a consequence of analyticity and unitarity, the partial-wave amplitudes $h^I_{J,\lambda\lambda'}(s)$ satisfy Omnès-type dispersive representations. A two-channel representation for $h^0_{0,++}(s)$ was reconsidered recently [12]. It should be valid in a range $\sqrt{s} \lesssim 1.1 - 1.2$ GeV where it is a good approximation to retain just two channels ($\pi\pi$, $K\overline{K}$) in the unitarity relation and has the form

$$(2)\quad \begin{pmatrix} h^0_{0,++}(s) \\ k^0_{0,++}(s) \end{pmatrix} = \begin{pmatrix} \overline{h}^{0,Born}_{0,++}(s) \\ \overline{k}^{0,Born}_{0,++}(s) \end{pmatrix} + \overline{\overline{\Omega}}(s) \times \left[\begin{pmatrix} b^{(0)}s + b'^{(0)}s^2 \\ b^{(0)}_K s + b'^{(0)}_K s^2 \end{pmatrix} + \frac{s^3}{\pi}\int_{-\infty}^{-s_0} \frac{ds'}{(s')^3(s'-s)} \times \right.$$
$$\left. \overline{\overline{\Omega}}^{-1}(s')\mathrm{Im} \begin{pmatrix} \overline{h}^{0,Res}_{0,++}(s') \\ \overline{k}^{0,Res}_{0,++}(s') \end{pmatrix} - \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{(s')^3(s'-s)} \mathrm{Im}\, \overline{\overline{\Omega}}^{-1}(s') \begin{pmatrix} \overline{h}^{0,Born}_{0,++}(s') \\ \overline{k}^{0,Born}_{0,++}(s') \end{pmatrix} \right].$$

The key ingredient in this equation is the Omnès matrix $\overline{\overline{\Omega}}$ which must be computed numerically from the T-matrix. Eq. (2) involves integrals over the left-hand cut of the amplitude which is generated by cross-channel singularities: the pion pole ($\overline{h}^{I,Born}_{0,++}$) and multi-pion cuts which are approximated by resonance poles. It also involves four polynomial parameters which appear upon writing over-subtracted dispersion relations such as to cutoff contributions from higher energy regions. They were determined in ref. [12] from a chirally constrained fit of the Belle data [13]. The representation (2) then allows one to compute the amplitude $h^0_{0,++}(s)$ for complex values of s. Extension to the second Riemann sheet is performed from standard formulas [14] involving $S^0_0(s)$. The couplings $g_{S\pi\pi}$ and $g_{S\gamma\gamma}$ are identified from the residues and one then defines the 2γ decay width [14] in terms of $|g_{S\gamma\gamma}|$. We find the following results for the σ and $f_0(980)$ (in KeV)

$$(3)\qquad \Gamma_{\sigma(600)\to 2\gamma} = \left(2.08 \pm 0.20\,^{+0.07}_{-0.04}\right),\ \Gamma_{f_0(980)\to 2\gamma} = \left(0.29 \pm 0.21\,^{+0.02}_{-0.07}\right)\ .$$

The result for the σ agrees with that presented by Hoferichter at this conference and is somewhat smaller than the one in ref. [14].

4 Couplings to gluonic and quark-antiquark operators

Let us introduce, formally at first, couplings of the scalar mesons to $\overline{q}q$ operators and to the trace of the energy-momentum tensor θ^μ_μ,

$$(4)\qquad \langle 0|\overline{u}u + \overline{d}d|S\rangle = \sqrt{2}B_0\, C^{uu}_S,\quad \langle 0|\overline{s}s|S\rangle = B_0\, C^{ss}_S,\quad \langle 0|\theta^\mu_\mu|S\rangle = m^2_S\, C^\theta_S\ .$$

In order to give precise meaning to these one considers the correlators

$$(5)\qquad \Pi_{jj}(p^2) = i\int d^4x\, e^{ipx}\langle 0|T j_S(x) j_S(0)|0\rangle$$

where $j_S(x)$ is one of the scalar operators considered above. The discontinuity along the cut in the elastic region $4m_\pi^2 \leqslant s \leqslant 16m_\pi^2$ follows from the Källen-Lehmann representation and allows one to deduce the expression for the second sheet extension of the correlator,

$$\Pi_{jj}^{II}(z) = \Pi_{jj}(z) + \frac{3}{16\pi} \frac{\sigma^\pi(z)\left(F_j(z)\right)^2}{1 - 2\sigma^\pi(z) t_0^0(z)} . \tag{6}$$

$F_j(s)$ is the pion form factor associated with j_S. $\Pi_{jj}^{II}(z)$ exhibits poles at $z = z_S$ corresponding to the scalar mesons. The coupling constants (4) are naturally identified from the residues and are thus determined in the terms of the value of the form-factor $F_j(z_S)$.

Analyticity properties of the form-factors allows them to be expressed in terms of the Omnès matrix. For this purpose, one considers the two component vector $\overline{F}(s)$ such that $\overline{F}_1(s) = F_j^\pi(s)$, $\overline{F}_2(s) = 2/\sqrt{3} F_j^K(s)$. Multiplying $\overline{F}(s)$ by the inverse of the Omnès matrix removes the right-hand cut up to a point s_2 where two-channel unitarity is no longer a good approximation. One then deduces that in the region $|s| \lesssim s_2$ the form-factors obey the following representation,

	$\sigma(600)$	$f_0(980)$
$\lvert C_S^{uu}\rvert$	$206 \pm 4_{-6}^{+4}$	$82 \pm 31_{-7}^{+12}$
$\lvert C_S^{ss}\rvert$	$17 \pm 5_{-7}^{+1}$	$146 \pm 44_{-7}^{+14}$
$\lvert C_S^{\theta}\rvert$	$197 \pm 15_{-6}^{+21}$	$114 \pm 44_{-7}^{+22}$

Table 3: *Absolute values (in MeV) of the couplings of the σ and $f_0(980)$ to operators.*

$$\begin{pmatrix} F_j^\pi(s) \\ \frac{2}{\sqrt{3}} F_j^K(s) \end{pmatrix} = \begin{pmatrix} \Omega_{11}(s) & \Omega_{12}(s) \\ \Omega_{21}(s) & \Omega_{22}(s) \end{pmatrix} \begin{pmatrix} \alpha + \alpha' s \\ \beta + \beta' s \end{pmatrix} . \tag{7}$$

The polynomial parameters in eq. (7) can be estimated from the chiral expansion at leading order [15]. The magnitudes of the resulting couplings are shown in table 3: the first and second error in the entries reflect the uncertainties in the polynomial parameters and in the Omnès matrix respectively .

In summary, one finds that both the σ and the $f_0(980)$ couple significantly to θ_μ^μ. The sum of the couplings is in qualitative agreement with the result of ref. [3]. The coupling of the σ to the $\overline{u}u + \overline{d}d$ operator can be compared to the coupling of the $a_0(980)$ to $\overline{u}d$. Converting the finite-energy sum rule result of Maltman [16] to the present normalisation gives $|C_{a_0(980)}^{ud}| = 197 \pm 37$ MeV. Thus, the σ and a_0 couplings are nearly equal, which is compatible with an assignment of both mesons into the same multiplet. We have also estimated the κ meson coupling $|C_{\kappa(800)}^{us}| \simeq 156$ MeV. The couplings of the light scalars σ, κ, $a_0(980)$, $f_0(980)$ to $\overline{q}q$ operators are therefore not unusually small, as one could expect in a naive tetraquark model, and their values are compatible with a nonet assignment.

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