Spin-Flavor van der Waals Forces and NN interaction

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We study the Nucleon-Nucleon interaction in the Born-Oppenheimer approximation at second order in perturbation theory including the Δ resonance as an intermediate state. The potential resembles strongly chiral potentials computed either via soliton models or chiral perturbation theory and has a van der Waals like singularity at short distances which is handled by means of renormalization techniques. Results for the deuteron are discussed.

1 Introduction

A major goal of Nuclear Physics is the derivation of the Nucleon-Nucleon (NN) interaction from Quantum Chromodynamics (QCD). In QCD the fundamental degrees of freedom are colored quarks and gluons which are *confined* to form colorless strongly interacting hadrons. Because of this the resulting nuclear forces at sufficiently large distances correspond to spin-flavor excitations, very much like the dipole excitations generating the van der Waals (vdW) forces acting between atoms (for a review see e.g. [1]). In the Born-Oppenheimer (BO) approximation and assuming no retardation and no electron cloud overlap at large distances, the atom-atom energy at a separation distance r can be calculated at second order perturbation theory as,

$$(1) \qquad V_{AA} = \langle AA|V_{\rm dip}|AA\rangle + \sum_{AA\neq A^*A^*} \frac{|\langle AA|V_{\rm dip}|A^*A^*\rangle|^2}{E_{AA}-E_{A^*A^*}} + \cdots = -\frac{C_6}{r^6} + \dots ,$$

where $|AA\rangle$ and $|A^*A^*\rangle$ is the electron wave function corresponding to a pair of well separated clusters in their atomic ground state and excited states respectively and where we assume a system with no permanent electric dipole. Driven by this compelling molecular analogy we want to analyze the NN interaction under similar dynamical assumptions.

The generalization to the NN system is straightforward, by just replacing $V_{\rm dip}$ by the One-Pion-Exchange (OPE) potential $V_{1\pi}$ and was already discussed in Ref. [2] within the context of chiral soliton models and the associated long-range spin-flavor universality. One

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considers the colorless nucleons as two quark clusters which in the chiral quark model exchange a colorless pion at large distances. The mutual (chiral) polarizability causes attraction between the nucleons, exactly in the same way as for atom-atom interactions and equivalently, using Eq. (1), one can obtain an optical potential where the effect of excited states as the Δ is included perturbatively,

$$V_{2N}(\boldsymbol{r}) = V^{1\pi}_{NN,NN}(\boldsymbol{r}) + 2\,\frac{|V^{1\pi}_{NN,N\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \frac{1}{2}\,\frac{|V^{1\pi}_{NN,\Delta\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \mathcal{O}(V^3)\,, \tag{2}$$

where $V^{1\pi}_{NN,NN}$ is the NN OPE potential and $V^{1\pi}_{NN,N\Delta}$ and $V^{1\pi}_{NN,\Delta\Delta}$ are OPE transition potentials [2]. Eq. (2) reproduces exactly the Skyrme soliton model result of Refs. [4,5]. At very short distances Eq. (2) behaves like a vdW potential $\sim -g_A^4/(\Delta f_\pi^4 r^6)$ and in fact it reduces to the Chiral Two-Pion-Exchange (ChTPE) potential at NLO-Δ [6,7] with the identification $h_A/g_A = f_{\pi N\Delta}/(2f_{\pi NN})$. Moreover, although both potentials are not completely equivalent they are very similar even at intermediate distances which explain why we achieve results for most of NN observables looking very much like those of more sophisticate chiral potentials.

2 Results

The BO-vdw potential, Eq. (2), presents a short distance singularity and to deal with it we use the method of renormalization with boundary conditions [8]. In [2] we showed satisfactory results for $^1S_0$ and $^3S_1 - ^3D_1$ phase shifts. Here and for the sake of brevity, we concentrate on deuteron properties. A more detailed study will be presented elsewhere. The deuteron is solved by fixing its binding energy $B_d = 2.224575$ MeV, the D/S ratio $\eta = 0.0256$ and the $^3S_1$ scattering length $a_{^3S_1} = 5.419$ fm from which we obtain the properties, $A_S = 0.873(8)\mathrm{fm}^{-1/2}$, $r_m = 1.945(14)$fm, $Q_d = 0.2712(1)\mathrm{fm}^2$, $P_D = 7.3(1.2)\%$, $\langle r^{-1}\rangle = 0.468(8)$, $a_{^3D_1} = 6.56(5)\mathrm{fm}^5$, $a_{E_1} = 1.549(1)\mathrm{fm}^3$ in the case in which we use the $SU(N_c)$ quark model relation with $N_c = 3$ and, $A_S = 0.886(9)\mathrm{fm}^{-1/2}$, $r_m = 1.973(18)$fm, $Q_d = 0.2789(12)\mathrm{fm}^2$, $P_D = 6.8(1.4)\%$, $\langle r^{-1}\rangle = 0.44(1)$, $a_{^3D_1} = 6.471(9)\mathrm{fm}^5$, $a_{E_1} = 1.689(3)\mathrm{fm}^3$ in the case $N_c \to \infty$. The estimate error corresponds to taking the extreme values $g_A = 1.26$ and $g_A = 1.29$. The renormalized $^1S_0$ and coupled $^3S_1$, E_1 and $^3D_1$ waves were already shown in Ref. [2]. The deuteron electromagnetic form factors in the IA using our renormalized wave functions are displayed in Fig. 1, which within uncertainties are reproduced rather well.

[2]This potential depends on the coupling constants $f_{\pi NN}$ and $f_{\pi N\Delta}$. We can use the relation $f_{\pi NN} = g_A m_\pi/(2f_\pi)$ and the $SU(N_c)$ relation [3] $f_{\pi N\Delta}/f_{\pi NN} = 3\sqrt{(N_c-1)(N_c+5)}/(\sqrt{2}(N_c+2))$ such that the only *free* parameter is actually g_A having an admissible value in between 1.26 and 1.29.

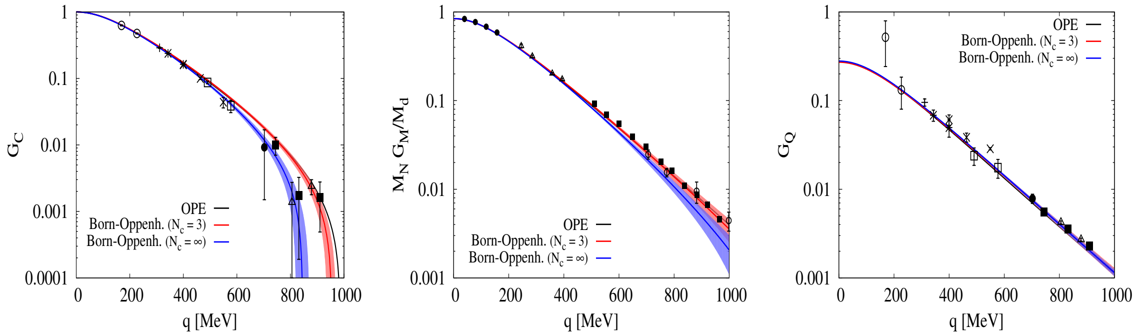

Figure 1: Deuteron charge G_C (left), magnetic G_M (middle) and quadrupole G_Q (right) form factors in the IA. The dependence with g_A is shown by light bands.

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3 Three-body force

The extension to the three nucleon force (3NF) follows from the generalization of Eq. (2) for the 3N case, at 2nd order perturbation theory, being,

$$(3) \quad V_{3N} = \langle NNN|V_{OPE}|NNN\rangle + \sum_{NNN \neq HH'H''} \frac{|\langle NNN|V_{OPE}|HH'H''\rangle|^2}{E_{NNN} - E_{HH'H''}} + \mathcal{O}(V^3)\,,$$

where H, H' and H'' represent intermediate excited states and V_{OPE} is the sum of pairwise interactions between nucleons with the exchange of a pion, *i.e.*, $V_{OPE}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = V_{OPE}(\mathbf{r}_{12}) + V_{OPE}(\mathbf{r}_{13}) + V_{OPE}(\mathbf{r}_{23})$ and $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$. Evaluating the matrix elements we obtain,

$$(4) \qquad V_{3N} = \sum_{i \neq j} V^{1\pi}_{NN,NN}(\mathbf{r}_{ij}) + \frac{1}{M_N - M_\Delta} \sum_{i \neq j} |V^{1\pi}_{NN,N\Delta}(\mathbf{r}_{ij})|^2 + V^{FM}_{ijk}\,,$$

where V^{FM}_{ijk} is the old Fujita-Miyazawa 3NF [9]. So, in the BO the 3NF decomposes into a sum of One-Pion-Exchange Two-Nucleon (1PE-2N) pair interaction, a Two-Pion-Exchange Two-Nucleon (2PE-2N) with intermediate Δ pair interaction and a genuine Two-Pion-Exchange Three-Nucleon (2PE-3N) interaction. The emergence of short distance vdW singularities in given channels is evident. Unfortunately the renormalization of singular three-body problems, even within this simplified BO approach, has not yet been achieved. We note that similar interactions have proven to be essential, after introducing cut-offs, which modify the original interaction below 2fm, to describe the binding energies of light nuclei $A \leqslant 8$ [10]. This suggests that the BO approximation may be a workable scheme for multi-nucleon forces.

4 Conclusions

We have seen how the NN interaction can be faithfully represented as a vdW force that emerges as in atomic physics where one usually uses the Born-Oppenheimer approximation. We have calculated the two- and three-body force at second order in perturbation theory although higher order may in principle be included. The two nucleon potential reproduces exactly the Skyrme model result within the same approximation and its short distances behavior is identical to ChTPE at NLO-Δ. We have shown results for the deuteron properties and EM form factors having a very good agreement with experimental data. In the 3N sector, the BO potential contains the old Fujita-Miyazawa force as well as a residual 1PE-2N and a 2PE-2N with Δ.

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