Calculation of the pion electromagnetic form factor from lattice QCD

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We present a lattice calculation of the vector form factor of the pion for two flavours of non-perturbatively $O(a)$ improved Wilson fermions. For the measurements we utilise the CLS ensembles which include various lattice spacings and pion masses down to about 250 MeV. To obtain a fine momentum resolution near zero momentum transfer (q^2) partially twisted boundary conditions are employed using several twist angles. Due to the fine resolution around $q^2 = 0$ we are able to determine the slope of the form factor and, in turn, extract the charge radius of the pion without any model dependence. The results for the form factor and the charge radius are then compared to chiral perturbation theory and phenomenological models which are used to extrapolate the results to the physical point.

Over the last years Monte-Carlo simulations of lattice QCD started to produce accurate and reliable results for a number of quantities of phenomenological interest, such as e.g. the spectrum of the low lying hadrons and light quark masses (see e.g. [1] and [2]). Despite good agreement between theory and experiment for these quantities there are others where lattice QCD does not coincide with experiment. The origin of these discrepancies is not clear since a number of systematic effects have to be controlled both in experiment and simulations. A particular example where experiment and theory are not in satisfactory agreement are observables connected with structural properties of the nucleon, such as electric and magnetic form factors as well as the nucleon axial charge (see e.g. [3–6]). Another technically simpler observable where similar systematic effects enter is the pion electromagnetic form factor $f_{\pi\pi}(q^2)$, where q is the momentum transfer. The fact that $f_{\pi\pi}$ receives no contribution from quark-disconnected diagrams for two degenerate flavours makes it the ideal observable to perform a precision calculation in lattice QCD. Nevertheless, in the region of small momentum transfers the extraction of $f_{\pi\pi}(q^2)$ usually suffers from an intrinsic model dependence, since a direct calculation in that region has not been possible so far for both experiment and theory. Related to the pion form factor in that kinematical regime is the pion charge radius $\langle r_\pi^2 \rangle$ defined by the linear behavior of $f_{\pi\pi}(q^2)$ at $q^2 = 0$,

$$(1) \qquad f_{\pi\pi}(q^2) = 1 + \frac{\langle r_\pi^2 \rangle}{6} q^2 + \dots \quad \Rightarrow \quad \langle r_\pi^2 \rangle = 6 \frac{d f_\pi(q^2)}{dq^2}\bigg|_{q^2=0} .$$

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β	a[fm]	lattice	# masses	$m_\pi L$	Labels	Statistic
5.20	0.08	64×32^3	3	$6.0 - 4.0$	A3 – A5	$\mathcal{O}(100)$
5.30	0.07	64×32^3	2	6.2, 4.7	E4, E5	$\mathcal{O}(100)$
5.30	0.07	96×48^3	1	5.0	F6	233
5.50	0.05	96×48^3	3	$7.7 - 5.3$	N3 – N5	$\mathcal{O}(100)$

Table 1: Compilation of simulation parameters.

The extraction of $\langle r_\pi^2 \rangle$ is thus mostly governed by the modeling of the q^2-dependence of $f_{\pi\pi}$ unless results are available around $q^2 \approx 0$. In lattice QCD the accessible momenta are usually obtained by Fourier transformation and thus constrained by finite lattice volume. This has changed recently by the introduction of partially twisted boundary conditions [8,9] that in principle allow arbitrary small momentum transfers in lattice simulations [10].

Our calculation of $f_{\pi\pi}$ employs ensembles generated in the context of the CLS project[2] and include three different lattice spacings with three different pion masses each. The main parameters of the ensembles used in the analysis are shown in table 1. To reduce the statistical noise we use stochastic $Z(2) \times Z(2)$ wall sources for the computation of the quark propagators, see e.g. [7]. The momenta are generated by five twist angles tuned so as to obtain as many as 30 values of q^2 below the lowest accessible q^2 from Fourier-momentum. We express our result in units of the Sommer scale r_0 [11] in the chiral limit, which has been measured for these ensembles in [12]. To compare our data to experimental results and results from other collaborations we use $r_0 = 0.471$ fm as obtained in [13]. To make optimal use of the generated data we extract $f_{\pi\pi}$ using a combination of the three different ratios of two- and three-point functions defined in [10]. The error bars are estimated with the bootstrap method using 1000 samples. For more details of the simulations see [14] and [6] as well as our upcoming publication.

The results for the pion form factor on our lightest ensemble for each lattice spacing are shown in figure 1, together with the results from [15–17] and the experimental results from [18]. Our points reach down to $(r_0 q)^2 \lesssim -10^{-4}$ with small statistical uncertainties. This enables us to use a linear fit to $f_{\pi\pi}(q^2)$ in the region $(r_0 q)^2 \leqslant -0.15$ to extract $\langle r_\pi^2 \rangle$ without any model dependence. The results from the linear fit are shown in figure 2 (left) for all ensembles, together with the other results quoted above. We see consistency with the results from other collaborations. Note that our results might show some residual cut-off dependence, investigated in our forthcoming publication.

Since our quark masses are bigger than the physical ones, we have to perform a chiral extrapolation to the physical point, guided by chiral perturbation theory (χPT). The χPT expressions for $f_{\pi\pi}$, and thus also for $\langle r_\pi^2 \rangle$, have been worked out to next-to-next-to leading order (NNLO) in [19]. As an intermediate step to compare to the NNLO formulae we start with the comparison to NLO, which was derived in [20]. In figure 2 (right) we show the results for the only free parameter $\bar{\ell}_6$ from the fit with $(r_0 q)^2 \leqslant -0.15$ against $(m_\pi r_0)^2$.

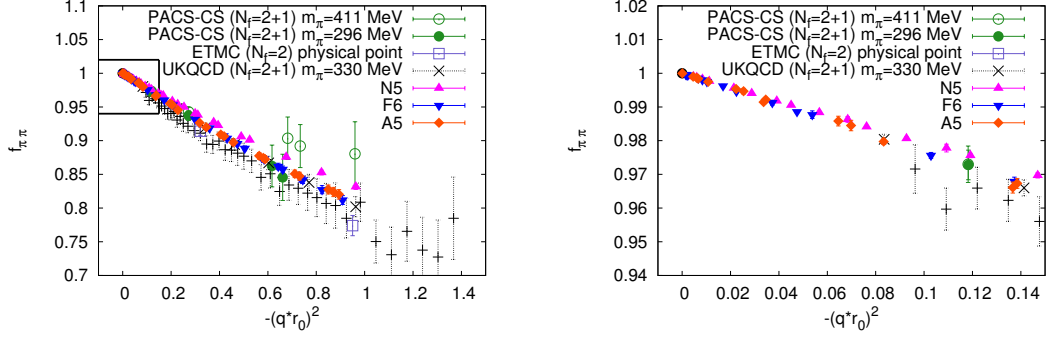[2]https://twiki.cern.ch/twiki/bin/view/CLS/WebHome

Figure 1: Results for the pion form factor for the lightest quark mass for each lattice spacing compared with the results from PACS-CS [15], ETMC [16] and UKQCD [17], as well as the experimental results from [18]. The right figure is the inset in the top left-hand corner.
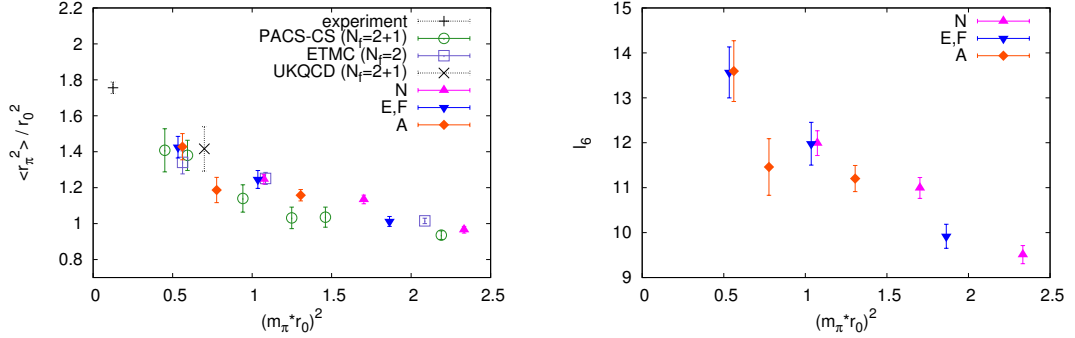

Figure 2: Left: Results for the pion charge radius extracted from linear fits to $f_{\pi\pi}(q^2)$ in the region $(r_0 q)^2 \leqslant -0.15$ against $(m_\pi r_0)^2$ together with results from other collaborations. **Right:** Results for $\bar{\ell}_6$ against $(m_\pi r_0)^2$.

If χPT to NLO were a good description for the mass range of our simulations we would expect $\bar{\ell}_6$ to be constant, which is apparently not the case. This observation is consistent with the findings of earlier studies as e.g. in [15, 16]. To make a statement on $\langle r_\pi^2 \rangle$ at the physical point we thus envisage to use χPT to NNLO. In addition, finite volume effects as well as lattice artefacts, which might still be present in our analysis, are not taken into account so far. The discussion of the corresponding analysis of both, χPT to NNLO and finite volume effects and lattice artefacts, is postponed to a later publication.

Conclusions: In this proceedings article we have given an overview on our ongoing determination of the electromagnetic form factor of the pion in lattice QCD. We use twisted boundary conditions to attain a high density of measurements around $q^2 = 0$ which allows us to extract the charge radius without residual model dependence. We have compared our measurements to χPT at NLO and conclude that NLO is insufficient to describe the data in this mass range consistently. In the final analysis we are going to compare our measurements to χPT at NNLO and perform detailed studies on cut-off effects, finite volume effects and contributions to the form factor from excited states.

3

Acknowledgments

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