Does $I = 1$ Isospin State Exist in $c\bar{c}$ Meson?

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We analyze for X series $c\bar{c}$ mesons with the Diquark Cluster Model except for X(3872) meson. The mass formula for the $q^k\bar{q}^h$ system, which consistens of u,d,s and c quark, is given by

$$M = m_q\, n_T + M(1p_{1/2})\, n(1p_{1/2}) + M(1p_{3/2})\, n(1p_{3/2}) + M(1d_{3/2})\, n(1d_{3/2})$$
$$+ M(1d_{5/2})\, n(1d_{5/2}) + M(2s_{1/2})\, n(2s_{1/2})$$
$$+ \Delta_0\, (n_{\phi_0} + n_{\overline{\phi}_0}) + \Delta_1\, (n_{\phi_1} + n_{\overline{\phi}_1}) + \Sigma\Delta_{TS}.$$

The M is the meson mass and m_q is the quark mass and the total quark number n_T = k + h where k is a number of quark and h is a number of antiquark. The M(nlj) is the single excitation energy from the $1s_{1/2}$ shell with the principal quantum number n and orbital and total angular momentum l and j.
The n(nlj) is the total number of excited quarks and antiquarks in the shell. The u or d quark quark-quark interaction in the s state diquark cluster is the following.
The interaction energy of the diquark cluster in the s state are

$$\Delta_0 = a - 3/4b \text{ for the spin 0, } \Delta_1 = a + 1/4b \text{ for the spin 1,}$$
$$a = 187 \text{ MeV, } b = 195 \text{ MeV.}$$

When one s quark includes for the s state insted of u or d quark, the interaction parameter a is the same but $b_s = (m/m_s)b$ where m is the u or d quark mass and m_s is the mass of s quark. Relating to the u,d and s quark parameters were already published [1]. The charm quark mass m_c = 1646 MeV and the interacion parameter a_c = 38.4 MeV are oblained using the mass of Λ_c and X(3845) meson. The n-c quark interaction, b is $b_c = (m/m_c)$ b. The s-c quark intraction case, a_{cs} = - 34.7MeV is obtained by the X(4140) meson mass.

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The interaction energy of the diquark cluster in the p state are

$$\Delta_{00} = \Delta_{11} = \Delta_{01} = 0, \Delta_{10} = -60 \text{ MeV}.$$

for u or d quark.
And Δ^{s}_{60} = - 19 MeV, Δ^{s}_{3*0} = 0 in the p state if s quark includes one. Δ^{c}_{60} = - 5.5 MeV, Δ^{c}_{3*0} = 0 in the p state if c quark includes one. One of the c or s quark in the p shell and the other one is in the s shell, Δ^{cs}_{60} = - 3.4 MeV, Δ^{c}_{3*0} = 0. The first suffix of Δ_{ts} presents an isospin or flavour and second one is a spin,respectivly.

There are no any free parameter remains in our calculations. We will discusses the calculated results in three part. The first part is an ordinary cn $\overline{\text{cn}}$ quark system except X(3872).

In our calculation, X(3945) is the ground state of cn $\overline{\text{cn}}$ system.
We fixed the interaction parameter a_c by this meson. The mass, isospin, and spin for X(3940), X(4160), X(4260), X(4360) and X(4660) mesons are calculated and results are shown in Table 1.

Meson	Exp.	Configuration/[TS][TS] : $I^g(J^{pc})$	Cal.
X(3945)	3916	$[(1s_{1/2})_c(1s_{1/2})]^2$	Input
	$0^+(?^{?+})$	$[1/2\ 0][1/2\ 0] : 0^+(0^{++})$	
X(3940)	3942	$[(1s_{1/2})_c(1s_{1/2})]^2$	3951
	$?^?(?^{??})$	$[1/2\ 0][1/2\ 1] : 0^+(0^{++}),0^-(1^{+-})$	
$X(4050)^{\pm}$	4051	$[(1p_{1/2})_c(1s_{1/2})][(1s_{1/2})_c(1s_{1/2})]$	4056
	$?^?(?^?)$	$[1/2\ 0][1/2\ 1] : 1^-(1^-)$	
X(4160)	4156	$[(1p_{1/2})_c(1s_{1/2})][(1s_{1/2})_c(1p_{1/2})]$	4161
	$?^?(?^{??})$	$[1/2\ 0][1/2\ 1] : 0^+(1^{++})$	
$X(4250)^{\pm}$	4248	$[(1s_{1/2})_c(1p_{1/2})][(2s_{1/2})_c(1s_{1/2})]$	4279
	$?^?(?^?)$	$[1/2\ 0][1/2\ 1] : 1^-(1^-)$	
X(4260)	4263	$[(2s_{1/2})_c(1s_{1/2})][(1s_{1/2})_c(1p_{1/2})]$	4264
	$?^?(1^{--})$	$[1/2\ 1][1/2\ 0] : 0^-(1^{--})$	
X(4360)	4361	$[(1p_{1/2})_c(1s_{1/2})][(1s_{1/2})_c(1d_{3/2})]$	4383
	$?^?(1^{--})$	$[1/2\ 1][1/2\ 1] : 0^-(1^{--})$	
$X(4430)^{\pm}$	4443	$[(1p_{1/2})_c(1s_{1/2})][(1s_{1/2})_c(1p_{3/2})]$	4413
	$?^?(?^?)$	$[1/2\ 1][1/2\ 1] : 1^+(1^+)$	
X(4660)	4664	$[(1p_{1/2})_c(1s_{1/2})][(1s_{1/2})_c(2s_{1/2})]$	4637
	$?^?(1^{--})$	$[1/2\ 0][1/2\ 1] : 0^-(1^{--})$	

Table 1 : cn $\overline{\text{cn}}$ meson (unit MeV)

The mass, spin and parity of X(3940), X(4160), X(4260) are very good reproduced the experimental results. According to the Belle experiment [2], X(4050)$^{\pm}$, X(4250)$^{\pm}$ are decaying to $\pi^+\chi_{c1}(1p)$ and the BaBar and the Belle [3] experiment shown X(4430)$^{\pm}$ goes to $\pi^+\psi(2s)$ decay instated of two π mesons decay for these mesons.

In the strong decay the isospin is conserved. These three X mesons must have the isospin 1 because their decay products. This is the strong evidence that these mesons are consisting of the four quark meson. Our calculated results are naturally explaining for this situation. X(4050)$^{\pm}$, X(4250)$^{\pm}$ mass are reproduced quite well. All chaged three mesons are also in Table 1.

X(4260) is very interesting meson. If cn $\overline{\text{cn}}$ configuratin is used, we obtained very good agreement with the experimental mass. But using the Maiani et al configuration [4], the result is the same as Maiani's even it is completely different calculation method but it is poor agreement with the experimental mass.

X(4140) and X(4350) mesons are the four quark charmed strange meson because of the decay mode. X(4140) is the ground state of cs $\overline{\text{cs}}$ system. We use this meson to determine the parameter a_{cs} . We calculate the mass, isospin and spin with no free parameters remaining. The result is shown in Table 2. The mass of X(4350) is excellent reproduced and also it is suggest that this is the exotic meson $J^{PC} = 1^{-+}$.

Meson	Exp.	Configuration/[TS][TS] : $I^g(J^{pc})$	Cal.
X(4140)	4143	$[(1s_{1/2})_c(1s_{1/2})_s]^2$	Input
	$0^+(?^{?+})$	$[0\ 0][0\ 0] : 0^+(0^{++})$	
X(4260)	4263	$[(1p_{1/2})_c(1s_{1/2})_s][(1s_{1/2})_c(1p_{1/2})_s]$	4336
	$?^?(1^{--})$	$[0\ 1][1\ 0] : 0^-(1^{--})$	
X(4350)	4351	$[(1p_{3/2})_c(1s_{1/2})_s][(1s_{1/2})_c(1s_{1/2})_s]$	4347
	$0^?(?^{?+})$	$[0\ 0][0\ 1] : 0^+(1^{-+})$	
?	?	$[(1d_{3/2})_c(1s_{1/2})_s][(1s_{1/2})_c(1s_{1/2})_s]$	4443
	?	$[0\ 1][0\ 0] : 0^+(1^{++})$	

Table 2 : cs $\overline{\text{cs}}$ meson (unit MeV)

Concluding remarks:

The interesting results are I = 1 state exists for $c\bar{c}$ meson. These states cannot be explained by the simple $c\bar{c}$ meson. This is the strong evidence of the existence of the four quark state.

X(4140) and X(4350) are cs $\overline{cs}$ the charmed strange four quark meson. Especially X(4350) is possible as the exotic meson $J^{PC} = 1^{-+}$ from our calculation.

References

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