Effective Quark-Quark Interaction
in Heavy Baryons

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We report results from a study of heavy-baryon spectroscopy within a relativistic constituent-quark model, whose hyperfine interaction is based on Goldstone-boson-exchange dynamics. While for light-flavor constituent quarks it is now commonly accepted that the effective quark-quark interaction is (predominantly) furnished by Goldstone-boson exchange – due to spontaneous chiral-symmetry breaking of quantum chromodynamics at low energies – there is currently still much speculation about the light-heavy and heavy-heavy quark-quark interactions. With the increasing amount of experimental data on heavy-baryon spectroscopy these issues might soon be settled. Here, we show, how the relativistic constituent-quark model with Goldstone-boson-exchange hyperfine interactions can be extended to charm and bottom baryons. It is found that the same model that has previously been successful in reproducing the light and strange baryon spectra is also in line with the existing phenomenological data on heavy-baryon spectroscopy. An analogous model with one-gluon-exchange hyperfine interactions for light-heavy flavors does not achieve a similarly good performance.

1 Framework

We view hadrons as relativistic bound states of constituent quarks Q. Baryons are are thus considered as $\{QQQ\}$ systems. Even if heavy-flavor quarks are involved, it is mandatory to work in a relativistic framework in order to prevent pathologies and/or severe shortcomings. Our relativistic constituent-quark model (RCQM) is based on a relativistically invariant mass operator $\hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}}$ that includes the Q-Q interaction according to the Bakamijian-Thomas construction [1]. In the rest frame of the baryon, $\vec{P} = \sum_i^3 \vec{k}_i^2 = 0$, the free and interacting parts of the mass operator thus read

$$(1) \qquad \hat{M}_{\text{free}} = \sum_{i=1}^{3} \sqrt{\hat{m}_i^2 + \hat{\vec{k}}_i^2} \,, \qquad \hat{M}_{\text{int}} = \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j}^{3} (\hat{V}_{ij}^{\text{conf}} + \hat{V}_{ij}^{\text{hf}}) \,,$$

where the $\vec{k}_i$ correspond to the three-momenta of the individual quarks with rest masses m_i and the mutual Q-Q potentials $\hat{V}_{ij}$ are composed of confinement and hyperfine interactions.

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By employing such a mass operator $\hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$, with baryon four-momentum $\hat{P}_\mu = (\hat{H}, \hat{P}_1, \hat{P}_2, \hat{P}_3)$, one satisfies the Poincaré algebra involving all ten generators $\{\hat{H}, \hat{P}_i, \hat{J}_i, \hat{K}_i\}$ of time and space translations, spatial rotations as well as Lorentz boosts, respectively:

$$[\hat{P}_i, \hat{P}_j] = 0 , \qquad [\hat{J}_i, \hat{H}] = 0 , \qquad [\hat{P}_i, \hat{H}] = 0 ,$$
$$[\hat{K}_i, \hat{H}] = i\hat{P}_i , \qquad [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k , \qquad [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k ,$$
$$[\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k , \qquad [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k , \qquad [\hat{K}_i, \hat{P}_j] = i\delta_{ij}\hat{H} .$$

The solution of the eigenvalue equation of the mass operator $\hat{M}$ yields the relativistically invariant mass spectra as well as the baryon eigenstates (in the standard rest frame) [2].

2 Goldstone-Boson Exchange in the $SU(3)_F$ Sector

In order to incorporate the property of spontaneous breaking of chiral symmetry of low-energy QCD, the Graz group has proposed a RCQM with Goldstone-boson-exchange (GBE) hyperfine interactions between constituent quarks [3]. In addition to a linearly rising confinement potential, as following from lattice QCD, the model comes with a spin-spin hyperfine interaction of the form

$$(2) \qquad V_{ij}^{\mathrm{hf}} = \left[V_\pi \sum_{a=1}^{3} \lambda_i^a \lambda_j^a + V_K \sum_{a=4}^{7} \lambda_i^a \lambda_j^a + V_\eta \lambda_i^8 \lambda_j^8 + V_{\eta'} \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j ,$$

where the λ_i^a are the $SU(3)_F$ Gell-Mann matrices and $\vec{\sigma}_i$ the $SU(2)_S$ Pauli matrices of quark i. The GBE is cast into the exchange of pseudoscalar mesons π, K, and η, including the singlet η' due to the $U(1)_A$ anomaly. The corresponding meson-exchange potentials as a function of the relative Q-Q distance $\vec{r}_{ij}$ read

$$(3) \qquad V_\gamma(\vec{r}_{ij}) = \frac{g_\gamma^2}{2\pi} \frac{1}{12 m_i m_j} \left[\mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - \Lambda_\gamma^2 \frac{e^{-\Lambda_\gamma r_{ij}}}{r_{ij}} \right] , \quad \gamma = \pi, K, \eta, \eta' ,$$

where g_γ is the meson-quark coupling constant, μ_γ the mass of the exchanged meson, and Λ_γ an adjusted cut-off parameter. As immediately evident, the GBE RCQM produces a spin- and flavor-dependent hyperfine interaction. It has hitherto been quite successful not only in describing the spectroscopy of all baryons with flavors u, d, and s in a unified framework but also in a number of baryon reactions [4].

3 Extension of the GBE RCQM to Heavy Baryons

We have investigated, if the same model can be extended to include also baryons with flavors c and b in a consistent manner. Thus we have generalized the hyperfine interaction
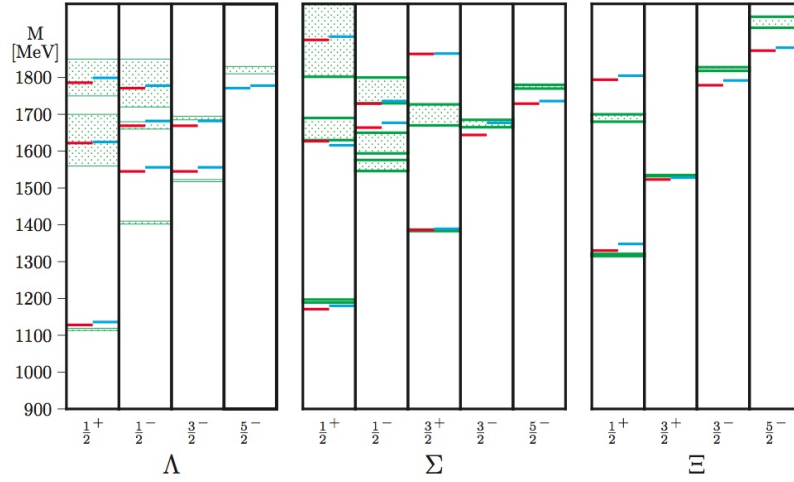
Figure 1: Comparison of $SU(5)_F$ (left/red levels) and $SU(3)_F$ (right/blue levels) hyperfine interactions in the GBE RCQM for the examples of the Λ, Σ, and Ξ baryons with definite spin and parity J^P in each column. The green boxes represent the experimental data with their uncertainties as given by the PDG [6].

of eq. (2) to $SU(5)_F$ in the following manner

$$
(4) \qquad V_{ij}^{\mathrm{hf}} = \left[V_\pi \sum_{a=1}^{3} \lambda_i^a \lambda_j^a + V_K \sum_{a=4}^{7} \lambda_i^a \lambda_j^a + V_{\eta_8} \lambda_i^8 \lambda_j^8 + V_{\eta_0} \lambda_i^0 \lambda_j^0 \right.
$$

$$
+ V_D \sum_{a=9}^{12} \lambda_i^a \lambda_j^a + V_{D_s} \sum_{a=13}^{14} \lambda_i^a \lambda_j^a + V_{\eta_{15}} \lambda_i^{15} \lambda_j^{15}
$$

$$
\left. + V_B \sum_{a=16}^{19} \lambda_i^a \lambda_j^a + V_{B_s} \sum_{a=20}^{21} \lambda_i^a \lambda_j^a + V_{B_c} \sum_{a=22}^{23} \lambda_i^a \lambda_j^a + V_{\eta_{24}} \lambda_i^{24} \lambda_j^{24} \right] \vec{\sigma}_i \cdot \vec{\sigma}_j \, ,
$$

where the various meson-exchange potentials are assumed in the same form as in eq. (3). The complete parametrization of the $SU(5)_F$ GBE RCQM can be found in ref. [5].

Obviously, this new construction also influences the Q-Q interaction in the $SU(3)_F$ sector (specifically through the diagonal elements in the flavor matrices λ_{15} as well as λ_{24}). Therefore our first concern is to maintain the good performance regarding the baryon spectroscopy with u, d, and s flavors. In Fig. 1 we give a selective comparison of the $SU(3)_F$ and $SU(5)_F$ models vis-à-vis the experimental data.

It is found that the quality of light and strange baryon spectroscopy is comparable for both the $SU(3)_F$ and $SU(5)_F$ models. In the latter, some of the levels, such as the $J^P = \frac{1}{2}^+$ Λ ground state, are even closer to experiment. A similar agreement occurs for the N, Δ, and Ω states. In particular, due to its specific flavor dependence, also the $SU(5)_F$ model produces
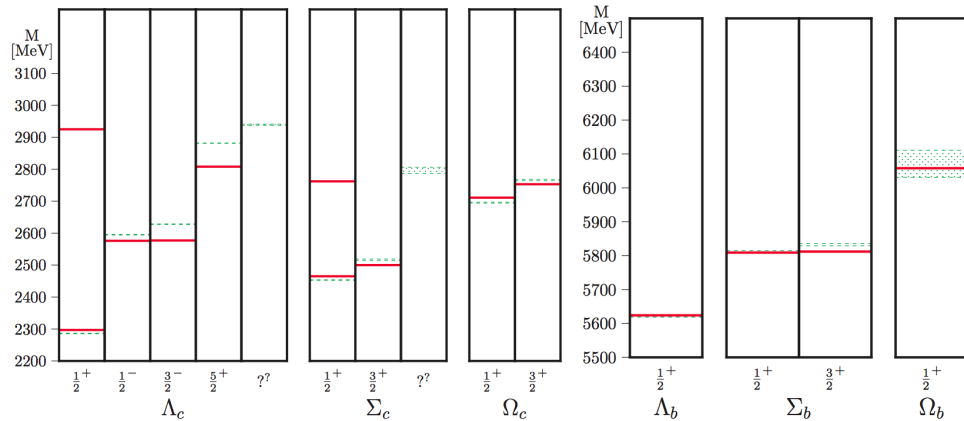
Figure 2: Heavy-baryon spectra as produced by the $SU(5)_F$ GBE RCQM (solid/red levels) in comparison to experimental data with their uncertainties, as quoted by the PDG [6] (dotted/green levels resp. boxes).

the correct orderings of positive- and negative-parity excitations simultaneously in the N and Λ spectra.

In Fig. 2 we show the results of the extended $SU(5)_F$ GBE RCQM for some heavy-baryon spectra. It appears that all levels can be reproduced satisfactorily. The same is true for the other cases not shown here because of space limitations.

At this stage we have succeeded in constructing a universal RCQM based on GBE dynamics that works in the whole baryon spectroscopy within $SU(5)_F$. For heavy baryons, additional experimental data are highly desirable in order to better determine the model parameters and put the theory to more stringent tests. Similarly, applications to describing reactions with heavy baryons (e.g., form factors and decays) will be most interesting to study.

Acknowledgments

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