Mixing properties of $a_1(1260)$ meson consisting of hadronic composite and quark composite

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A practical method to analyze the mixing structure of hadrons consisting of two components of quark and hadronic composite is discussed. As an example we investigate the properties of the axial vector meson $a_1(1260)$ and discuss its mixing properties quantitatively.

One of recent interests in the hadron structure is whether hadrons are made up of quarks described in the conventional quark model, or rather develop subcomponents of quark-clusters inside hadrons. In fact, hadronic resonant states physically observed should be mixture of more than one component like hadronic and quark-composites, an important issue is to clarify how these components are mixed in a hadron. In this work, we focus on hadron structure having two components of quark and hadronic composite. We propose a method to disentangle their mixture, by taking the $a_1(1260)$ meson as an example. The a_1 is a candidate of the chiral partner of ρ described as a $q\bar{q}$ in many models, while, in coupled-channel approaches based on the chiral effective theory, the a_1 meson has been described as a dynamically generated resonance in the $\pi\rho$ scattering [1–3]. We first solve the $\pi\rho$ scattering amplitude to find the poles corresponding to the physical a_1, and then develop a method to clarify its mixing nature. The detailed discussions are given in Ref. [4].

By solving the Bethe-Salpeter equation, we can obtain the $\pi\rho$ scattering amplitude as

$$t = \frac{v}{1 - vG}\,, \tag{1}$$

where v is the four-point Weinberg-Tomozawa interaction and G $\pi\rho$ two-body propagator. If the potential v is sufficiently attractive, the amplitude develops a pole corresponding to the $\pi\rho$-composite a_1 meson [1] without $q\bar{q}$ quark-core [5,6].

The elementary a_1 meson also contributes to the $\pi\rho$ scattering amplitude in the form of an effective $\pi\rho$ interaction: $v_{a_1} = g(s - m_{a_1}^2 + i\epsilon)^{-1}g$, where g is the coupling to $\pi\rho$ and m_{a_1}

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the bare mass of the elementary a_1 meson. The full scattering amplitude T having both interactions v and v_{a_1} is then written by,

$$T = \frac{v + v_{a_1}}{1 - (v + v_{a_1})G} \,. \tag{2}$$

This amplitude generates poles corresponding to physical resonant states of the problem. To study the mixing nature, we express equivalently the amplitude t in Eq. (1) as,

$$t \equiv g_R(s) \frac{1}{s - s_p} g_R(s), \tag{3}$$

where s_p is the pole position of the amplitude t in Eq. (1). In this form, we can interpret $(s - s_p)^{-1}$ as the one-particle propagator of the composite a_1 meson by taking an analogy with the conventional discussion of bound state problem [5]. Having the form of Eq. (3), we now rewrite the scattering amplitude T in Eq. (2) as,

$$T = (g_R, g) \frac{1}{\hat{D}_0^{-1} - \hat{\Sigma}} \begin{pmatrix} g_R \\ g \end{pmatrix} \,, \tag{4}$$

where

$$\hat{D}_0^{-1} = \begin{pmatrix} s - s_p & \\ & s - m_{a_1}^2 \end{pmatrix}, \;\; \hat{\Sigma} = \begin{pmatrix} & g_R G g \\ g G g_R & g G g \end{pmatrix} \,. \tag{5}$$

The diagonal elements of the matrix $\hat{D}_0$ are the free propagators of the two a_1's, one for the composite and the other for the elementary ones having the proper normalization, and the matrix $\hat{\Sigma}$ expresses the self-energy and interactions for these modes.

The expression of Eq. (4) makes it possible to analyze the mixing nature of the physical a_1 in terms of the original two bases. Having the amplitude in this form, $\hat{D} \equiv (\hat{D}_0^{-1} - \hat{\Sigma})^{-1}$ is identified with the propagators of the physical states represented by the bases of the elementary and composite a_1's. The residues of the diagonal elements D^{ii} defined by,

$$D^{ii} = \frac{z_a^{ii}}{s - M_a^2} + \frac{z_b^{ii}}{s - M_b^2} + (\text{regular term}) \;(i = 1, 2) \,, \tag{6}$$

have the meaning of the wave function renormalization and then carry the information on the mixing rate of the physical resonant states. The concrete forms of the potentials used in Eq. (2) are given in Ref. [4].

We find two poles at (a) $\sqrt{s} = 1033 - 107i$ MeV and at (b) $1728 - 313i$ MeV, corresponding to the physical states in the present model. These pole positions are significantly different from those of the two basis states, $\sqrt{s_p} = 1012 - 221i$ MeV and $m_{a_1} = 1189$ MeV, because of the mixing effect. In Fig. 1(a), we show the resulting pole-flow in the complex-energy plane by changing the mixing parameter x introduced by $g_{a_1 \pi\rho} \to x g_{a_1 \pi\rho}$. When the mixing is turned on, the pole starting from the composite a_1 (we refer to it as "pole-a") approaches

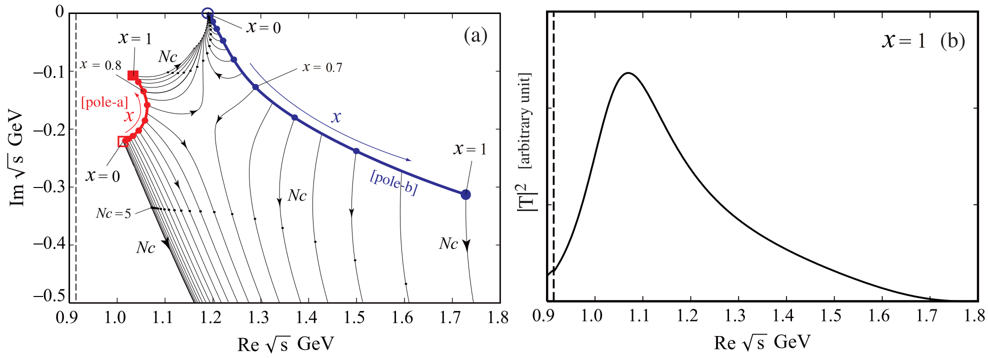

Figure 1: (a) Trajectories of the poles in the full scattering amplitude in Eq. (2) by changing the mixing parameter x (thick lines). The open square indicates the pole position of the composite a_1 and open circle indicates the elementary a_1 pole ($x = 0$). The other end points of solid circle and square correspond to the physical points ($x = 1$). Thin lines represent the pole-flows as N_c is increased from $N_c = 3$ for fixed x. (b) Squared amplitude $|T|^2$ of $\pi\rho \to \pi\rho$ process on the real energy axis for the mixing parameter $x = 1$.

the real axis, ending at $1033 - 107i$ MeV when $x = 1$ (solid square), while that from the elementary a_1 pole ("pole-b") goes far from the real axis and reaches $1728 - 313i$ MeV when $x = 1$ (solid circle). In Fig. 1(b), we show the squared amplitude $|T|^2$ in Eq. (2) (or (4)) at $x = 1$, and find that a peak structure is dominated by the pole-a. Therefore, the pole expected to be observed in experiments is the pole-a located at lower energy position that comes from the composite a_1 pole.

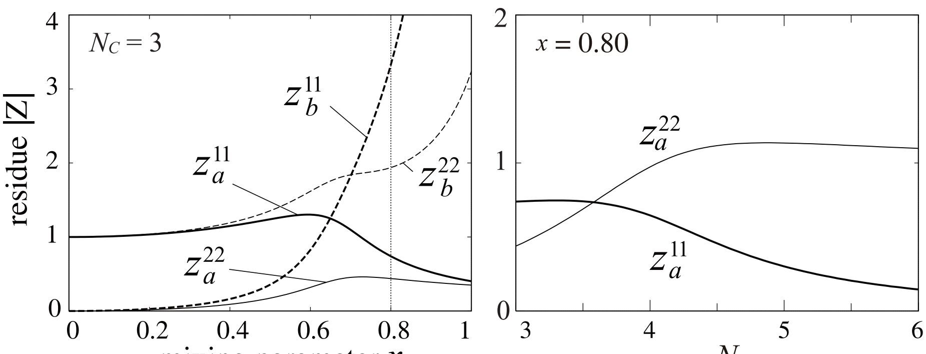

Figure 2: Absolute value of the residues defined in Eq. (6). The left panel shows the mixing parameter x dependence at $N_c = 3$ while the right panel is the N_c dependence at $x = 0.8$. The meaning of each line is indicated in the figure.

In Fig. 2(left panel) we show the absolute values of the residues as functions of the mixing parameter x. One of the most important messages can be read from the magnitude of z_a^{11} and z_a^{22} at $x = 1$, which are the residues of the possibly observed a_1 state. We can see that the pole-a at $x = 1$, although its location is close to the composite a_1 pole, has a component of the elementary a_1 meson *comparable to* that of the composite a_1. The detailed discussions are given in Ref. [4].

Next, we test the large N_c dependence of the pole positions according to the scaling law of the pion decay constant f_π as $f_\pi \to f_\pi\sqrt{N_c/3}$. In Fig. 1, we also show the trajectories of the pole positions by changing the N_c value for fixed mixing strength. The fate of the large N_c depends strongly on the mixing parameter x, although the component of the composite a_1 is always larger than that of elementary ($z_a^{11} > z_a^{22}$) at $N_c = 3$ as shown in Fig. 2 (left panel). In Fig. 2 (right panel), we show the N_c dependence of the residues of pole-a at $x = 0.8$. There we find that the magnitudes of the residues of the pole-a, z_a^{11} and z_a^{22}, interchange at $N_c \sim 3.5$. This indicates that the nature of the resonance *changes as N_c is varied*. Thus, for the mixed system of elementary and composite components, the large N_c limit does not always reflect the world at $N_c = 3$.

We have developed a general method to analyze the mixing structure of hadrons consisting of two components of quark and hadronic composites by taking the $a_1(1260)$ axial-vector meson as an example. The present analysis points out theoretically that the a_1 meson has comparable amount of the elementary a_1 component to the $\pi\rho$ composite a_1. We also have shown explicitly that the mixing nature of hadrons in the large N_c limit could differ from that at finite $N_c = 3$. Quest for evidences of the mixing nature in physical observables is an interesting future work.

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