Quasi-particle degrees of freedom in finite temperature SU(N) gauge theories

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1 Introduction

Many interesting indications on the gluonic sector of Quantum Chromodynamics can be drawn from lattice studies on the high temperature transition of pure non-abelian gauge theories $SU(N_c)$, where all difficulties related to the presence of fermions and to the details of the chiral simmetry breaking are absent. In addition, recent lattice simulations on $SU(N_c)$ gauge theories at finite temperature T and for large number of colors N_c [1, 2] are available, which allows for a specific analysis of the role of N_c at the transition.

We shall revise the $SU(N_c)$, $N_c = 3, 4, 6$, lattice data on pure gauge theories at finite temperature by means of a quasi-particle approach, with particular attention to the dimensionless interaction measure Δ, defined in terms of the energy density ϵ, the pressure p and the temperature T as $\Delta(T) = (\epsilon - 3p)/T^4$, and its scaling with N_c, and also with particular attention to the role of the quasi-particle effective mass, the screening mass and the gluon condensate [3, 4].

We shall focus on the temperature range which goes from the critical deconfinement temperature T_c up to about 4 T_c. In fact, the region below T_c is dominated by the gluon condensate and glueball degrees of freedom, while only for extremely large values of T, $T >> T_c$, perturbation theory starts to give a reliable description of the lattice data. The region here considered, between T_c and 4 T_c, is very interesting because, due to the transition, new degrees of freedom emerge. This is particularly evident in the interaction measure Δ, which shows a steep increase just above T_c, followed by a smooth decrease toward zero, which is reached only at very large temperatures where the perturbative, almost non-interacting, massless gluon degrees of freedom are relevant. Then, we shall mainly concentrate on the interaction measure Δ and on a possible description of this temperature region in terms of non-pertubative massive quasi-particle degrees of freedom, with a temperature dependent mass which is supposed to include all the interaction effects.

[1]Speaker.

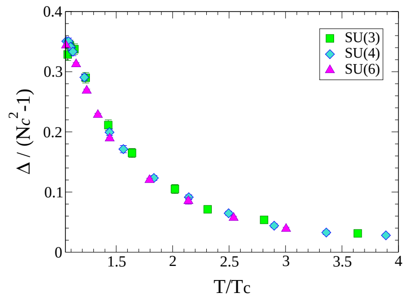

Figure 1: Interaction measure lattice data above 1.05 T_c.

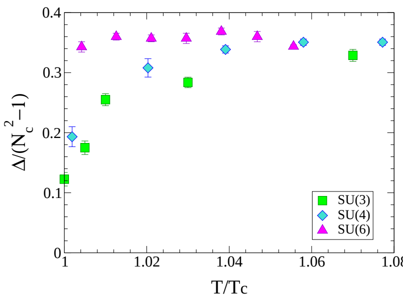

Figure 2: Interaction measure lattice data between T_c and 1.05 T_c.

2 The interaction measure $\Delta(T)$

Before considering the specific quasi-particle model, it is interesting to point out some scaling properties of the interaction mesure $\Delta(T)$ which is directly related to the trace of energy-momentum tensor

$$\Delta(T) = \frac{(\epsilon - 3p)}{T^4} = \frac{\Theta^\mu_\mu(T)}{T^4} \tag{1}$$

where the zero temperature part ($\overline{\Theta}^\mu_\nu$) of the full energy-momentum tensor $\mathrm{T}^\mu_\nu(T)$ has been removed :

$$\Theta^\mu_\nu(T) = \mathrm{T}^\mu_\nu(T) - \overline{\Theta}^\mu_\nu \tag{2}$$

and, accordingly to the normalization in Eq. (2), the energy density ϵ and the pressure p in Eq. (1) vanish at $T = 0$ [5, 6].

As already noticed, the lattice data for Δ show a very large peak around 1.05 T_c. The data above the peak are plotted in Figure 1 for $N_c = 3, 4, 6$, with each curve rescaled by $N_c^2 - 1$, showing that the scaling of Δ with $N_c^2 - 1$ in this temperature region is almost exact [1, 2]. For lower temperatures, namely between T_c and 1.05 T_c, such a scaling is slightly violated, as observed in Figure 2. This clearly indicates a rearrangement of the degrees of freedom because at large temperatures each different type of gluon contributes to the energy density and pressure, while below T_c color is confined and proportionality with the gluon degrees of freedom, $N_c^2 - 1$, is lost. Figure 2 shows the transition between these two regimes.

It must be remarked at this point that the interaction measure shows another approximate scaling [7], namely $T^2 \cdot \Delta$ is almost T independent in the region between T_c and 4 T_c, which, even if accidental *i.e.* not corresponding to a specific physical mechanism, is nevertheless interesting because not only simple perturbative approximations, but also hard thermal loop resummations [8] fail to reproduce it [3]. This, again, indicates that a non-perturbative description in the temperature region just above T_c is required.

Before considering any specific model, it is important to recall the role of the gluon condensate $\langle G^2 \rangle$, which is known to provide a contribution to $\Theta^\mu_\mu(T)$ and therefore to $\Delta(T)$ [5]. It has been evaluated by lattice simulations at finite temperature, in quenched and unquenched QCD [9] and it turns out that for $T < T_c$ the gluon condensate is almost T independent and is the sum of two equal contributions, namely the chromo-electric and chromo-magnetic part. For $T > T_c$, in correspondence of the deconfinement transition, the chromo-electric part quickly decreases to zero while the chromo-magnetic part remains constant in the temperature interval considered ($T_c < T < 4\ T_c$).

Therefore, between T_c and 4 T_c the gluon condensate is essentially T independent and it is about one half of its value at $T = 0$. The corresponding contribution to the interaction measure is $\langle G^2 \rangle / T^4$ which is largely suppressed at large temperature and, in addition, the T^{-4} behavior is not in agreement with the mentioned temperature dependence of the interaction measure ($T^2 \cdot \Delta \sim$ constant). This indicates that the gluon condensate alone cannot explain the observed properties of Δ.

About the dependence of the gluon condensate on N_c, we recall that

$$G^2 \equiv -\frac{\beta(g)}{2g}\ G^a_{\mu\nu} G_{\mu\nu a} = \frac{11 N_c g^2}{96\pi^2}\ G^a_{\mu\nu} G_{\mu\nu a} \tag{3}$$

where, in the right hand side, we have inserted the β-function at leading order in the coupling constant g. In the t'Hooft scaling regime, which corresponds to the limit of large N_c and small coupling g with the product $g^2 N_c$ fixed, we expect the right hand side of Eq. (3) to be proportional to $N_c^2 - 1$ due to the sum over the $SU(N_c)$ index a. However, as noticed before in Figure 2, $\Delta(T_c)$ shows a deviation from the simple $N_c^2 - 1$ scaling.

Therefore the gluon condensate does not show the same scaling properties of the interaction measure, so that the dominant component of Δ must have a different origin. Moreover, the quantitative value of the gluon condensate is estimated on the basis of the zero temperature phenomenological input [10]

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G_{\mu\nu a} \right\rangle \simeq 0.012 \pm 0.006 \; GeV^4 \tag{4}$$

and this, as it will be clear below, only gives a very small contribution , when compared to the numerical values of Δ computed by lattice simulations.

3 Quasi-particle approach

The observed approximate scaling of Δ with $N_c^2 - 1$ for $N_c \geq 3$, naturally suggests a quasi-particle behavior of the effective degrees of freedom, with the typical degeneracy, $N_c^2 - 1$, of the gluons and with an effective, temperature dependent mass that turns out to be divergent or, at least, very large at T_c [11, 12, 13, 14].

The partition function for the very simple case of free quasi-particles in a volume V, at temperature T and with temperature dependent mass $m(T)$ is

$$\ln \mathcal{Z}(T,V) = 2V(N_c^2 - 1) \int \frac{d^3k}{(2\pi)^3} \ln \left[f_T(k) \exp \left(\sqrt{\vec{k}^2 + m^2(T)}/T \right) \right] \tag{5}$$

where $f_T(k)$ is the distribution

$$f_T(k) = \left[\exp \left(\sqrt{\vec{k}^2 + m^2(T)}/T \right) - 1 \right]^{-1} \tag{6}$$

and all thermodynamical quantities are obtained by deriving Eq. (5). For our purpose, the energy density ϵ, the pressure p, the entropy density s are respectively, $\epsilon = (T^2/V) \, \partial \ln \mathcal{Z}/\partial T$, $p = T \; \partial \ln \mathcal{Z}/\partial V$ and $s = (\epsilon + p)/T$. The interaction measure Δ is directly obtained from ϵ and p as $\Delta = (\epsilon - 3p)/T^4$.

Obviously the temperature dependence of the mass must be taken into account when differentiating with respect to T. Note also that the additional effect of a temperature independent bag pressure (or gluon condensate) B corresponds to the changes $p \to p - B$, and $\epsilon \to \epsilon + B$, with no change in the entropy density s.

The factor 2 in front of the color multiplicity factor $N_c^2 - 1$ in Eq. (5) corresponds to the number of polarization degrees of freedom. In general, the representations of the Poincaré group of massive and massless particles in this case would suggest that our massive physical constituents carry three, rather than two, spin degrees of freedom. However, this is valid for free particles and, in fact, the comparison of all predicted thermodynamic quantities with the observed high temperature lattice results clearly shows much better agreement when only two polarization states are considered, as already noticed in [11].

In particular all lattice QCD results hint at an asymptotic limit of the ϵ/T^4 consistent with $2\,(N_c^2 - 1)$ degrees of freedom. In other words, a simple shift to massive gluons with three spin degrees of freedom cannot satisfactorily explain the effects of the interaction apparently still present in the gluon gas above the critical temperature.

It is well known that gauge symmetry forbids a mass term in the lagrangian for the elementary gluons and, in order to preserve the symmetry, one can expect to observe the generation of mass through a dynamical mechanism, such as the Schwinger mechanism [15] in which the mass comes from the appearance of a pole in the self-energy. In fact this effect has been explicitly pointed out and it has been argued that the longitudinal polarization component could be cancelled by the scalar massless pole [16, 17, 18].

On the other hand, in the modified Hard Thermal Loop perturbation theory approach, where each order already includes some aspects of gluon dressing and which leads to a rather rapid convergence of the expansion, the contribution of longitudinal gluons vanishes in the limit $g \to 0$, and, in particular, one also obtains the right number of degrees of freedom for the Stefan-Boltzmann form [19].

Moreover, from a comparison of the lattice glueball spectrum with the predictions of constituent models it has recently been argued [20] that massive gluons should in fact be transversely polarized, since two massless gluons cannot combine to form a longitudinally polarized massive gluon [21]. According to these indications we limit ourselves to consider just two polarization degrees of freedom for the effective quasi-particle in Eq. (5).

Let us now turn to the most important ingredient in our approach, that is the effective temperature dependent mass $m(T)$ which contains the non-perturbative dynamics. Previous analyses [11, 12, 13] show that $m(T)$ strongly increases near T_c and a qualitative explanation of this aspect, which we shall recall below, has been suggested in [3, 4]. To illustrate this point, one describes the mass of the quasi-gluon in the strongly coupled region as the energy contained in a region of volume V_{cor} whose characteristic size is given by the correlation range ξ, so that in three spatial dimensions one gets (η is the anomalous dimension and t is the reduced temperature $t = T/T_c$):

$$m(t) \simeq \epsilon(t) V_{cor} = \epsilon(t) \int dr\; r^2 \frac{\exp[-r/\xi(t)]}{r^{1-\eta}} \tag{7}$$

where it is understood that all dimensionful quantities are expressed in units of the critical temperature. In the case of a second order phase transition, the correlation length shows the power law divergence $\xi(t) = (t-1)^{-\nu}$ at $t = 1$, which indicates that the associated fluctuations have an infinite range at criticality, and the corresponding component of the energy density vanishes as $\epsilon(t) \simeq (t-1)^{1-\alpha}$ where α is the specific heat critical exponent. In this case Eq. (7) predicts a power law divergence of the mass $m(1)$.

For the gauge groups $SU(N_c)$, with $N_c = 3, 4, 6$ here considered, a first order phase transition and consequently a finite correlation length, is expected at T_c and power law behavior at criticality is modified by the finite scale ξ. However, in the case of weak first order transitions one should expect a behavior of the thermodynamical quantities at T_c not totally different from that observed in second order transitions, and therefore a finite but large correlation length and a correspondingly large $m(1)$. In particular, as discussed in [22], the thermodynamical quantities approach T_c (from larger values of the temperature $t > 1$) as in a second order phase transition with the critical temperature shifted to a lower value: $1 \rightarrow \delta$ with $0 \sim (1-\delta) << 1$.

According to this suggestion, and recalling that for $t >> 1$ the mass is expected to grow linearly with the temperature, which is the only dimensionful scale available, we parametrize the mass as[4] :

$$m(t) = \frac{a}{(t-\delta)^c} + bt \tag{8}$$

where a, b, c, δ are constant parameters.

The quasi-particle mass $m(t)$ should not be confused with the screening mass $m_D(t)$. The relation between $m(t)$ and $m_D(t)$, has been clarified in [11] where it is shown that

$$m_D^2 = \frac{g^2 N_c}{\pi^2 T} \int_0^\infty dk\ k^2 f_T^2(k) \exp \frac{\sqrt{\vec{k}^2 + m^2(t)}}{T} \tag{9}$$

and, following [11], the leading order QCD coupling g^2 is evaluated at the average, M^2, over the squared quasi-particle momenta, i.e.

$$M^2(T) = \frac{4}{3} \frac{\int dk\ k^4\ f_T(k)}{\int dk\ k^2\ f_T(k)} \tag{10}$$

In the next Section we display $m(t)$ and $m_D(t)$ which show totally different behaviors when approaching T_c.

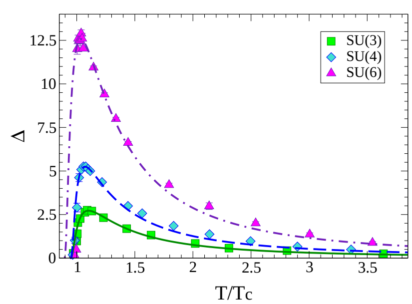

Figure 3: The interaction measure as obtained from a fit to the $SU(3,4,6)$ lattice data with $c = 0.5$ fixed.

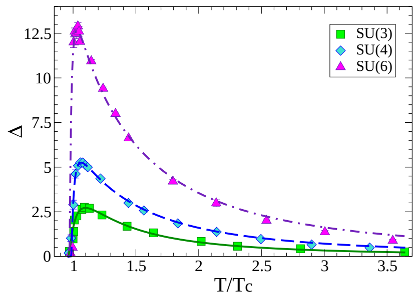

Figure 4: The same fit as in Figure 3 but with c treated as a free parameter.

4 Analysis of the lattice data

Now we check the simple model outlined in the previous Section against the lattice data for $SU(3,4,6)$ [2] . These theories undergo a weakly first order transition and we shall resort to the large but finite mass in Eq. (8) for computing the various thermodynamical quantities.

As a first check that our choice of describing the weakly first order phase transition by means of a quasi-particle model is a reasonable assumption, we verify that the mean field behavior produces a good fit to the data by fixing the exponent c in Eq. (8) to its mean field value: $c = 0.5$. The results are plotted in Figure 3 and we get $\delta = 0.94,\ 0.94,\ 0.89$ and $\chi^2/dof = 2.6,\ 7.7,\ 6.5$ respectively for $N_c = 3,\ 4,\ 6$. As expected, δ turns out to be very close to 1.

The full analysis is then performed by releasing the constraint on the parameter c which now turns out to slightly decrease from the mean field value: $c = 0.46,\ 0.35,\ 0.33$ respectively for $N_c = 3,\ 4,\ 6$ and the other parameters are collected in Table 1 (see values in brackets). As expected the χ^2/dof is smaller than the case with $c = 0.5$ and the parameter δ is very close to one for all values of N_c.

The corresponding curves are shown in Figure 4 where the details at large t are well visible. In order to have a closer look at the region just above the transition, we

N_c	$m(1)$	δ	χ^2/dof
3	(6.6) 7.7	(0.95) 0.95	(2.0) 2.1
4	(7.4) 7.5	(0.98) 0.96	(0.8) 1.1
6	(4.4) 2.6	(0.97) 0.73	(3.1) 4.9

Table 1: Some parameters from the fit to the lattice data. The values on the left within brackets correspond to the simple quasi-particle model, those on the right are obtained by including the gluon condensate.

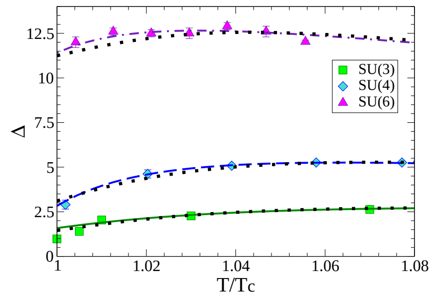

Figure 5: Enlargement of Figure 4 in the region $t \simeq 1$, with the additional insertion of three black dotted curves obtained by including in the fit the gluon condensate contribution to Δ.

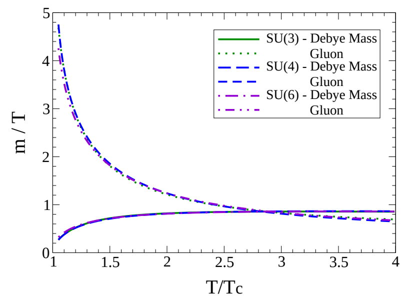

Figure 6: The screening mass m_D compared with $m(t)$ as obtained from the fit to the lattice data.

have enlarged the figure around $t \simeq 1$ and the plot is displayed in Figure 5, where we have also included three black dotted curves which will be discussed afterward. Even very close to $t = 1$ the agreement with the data is evident.

The mass as a function of the temperature is plotted in Figure 6. We note that even if we have not forced any specific dependence of the mass on the coupling g and on N_c, the resulting masses manifest an independence of $m(t)$ on the the $SU(N_c)$ gauge group for temperature above the peak in the interaction measure. This could be expected if one considers the parametric dependence of the mass in a perturbative approach, $m_g^2 \simeq g^2 N_c T^2$, along with the t'Hooft scaling of the coupling, $g^2 \simeq 1/N_c$. Moreover, the comparison between the gluon effective mass $m(t)$ and the Debye screeening mass $m_D(t)$ according to Eqs. (9,10), is also displayed in Figure 6.

As a further check of our model on other thermodynamical quantities, respectively in Figures 7 and 8 we plot the pressure p and the speed of sound, defined as $c_s^2 = \partial p/\partial \epsilon$, which correctly tends to the Stefan-Boltzmann limit $c_s^2 = 1/3$ at large temperatures.

Now we are able to see how, in accordance to the previous Section, the inclusion of

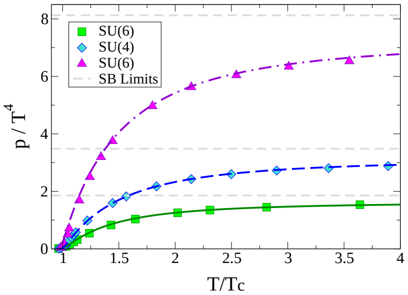

Figure 7: Full fit to the $SU(3,4,6)$ pressure lattice data.

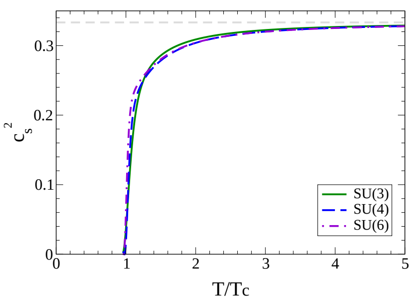

Figure 8: The speed of sound as obtained from the fit to the lattice data.

the gluon condensate can affect our results. As already anticipated the gluon condensate does not have the correct dependence on the temperature T and the number of colors N_c and, in addition, its global contribution to Δ, due to the phenomenological value in Eq. (4) is expected to be small.

In fact, it is straightforward to add this contribution and observe the induced changes in the parameters of our quasi-particle model. Results are again shown in Table 1 (values on the right - no brackets) and the corresponding plots are reported in Figure 5 as black dotted curves. We do not show the full curves at larger t as they become identical to the old ones because the gluon condensate is suppressed at larger t by the factor t^{-4} and therefore it becomes irrelevant. The small changes displayed in Figure 5 seem to worsen the agreement close to $t = 1$ and this is confirmed by the increase of the χ^2/dof in Table 1. The parameters δ and $m(1)$ show a sizeable change only in the case $N_c = 6$.

Finally we note that, when the gluon condensate is included and going from $N_c = 6$ to $N_c = 3$, Table 1 shows a slight increasing trend of the masses $m(1)$. This is, at least, in line with the possible occurrence of a power law diverging mass at T_c in the case of the $SU(2)$ second order phase transition, as suggested after Eq. (7).

5 Conclusions

Our results show that a quasi-particle approach, where the effective mass is related to the features of the deconfinement transition, gives a very good description of the interaction measure and of the thermodynamical quantities for the first order phase transition occurring in $SU(3,4,6)$.

We find that the scaling of the interaction measure with $(N_c^2 - 1)$ is observed for $T > 1.1\ T_c$ but is violated near T_c. Accordingly, above 1.1 T_c the mass behavior is independent of N_c in agreement with a perturbative parametric dependence on $g^2\ N_c$ and the $1/\sqrt{N_c}$ t'Hooft scaling of the coupling which, on the other hand, is broken very close to T_c. In all cases $N_c = 3, 4, 6$, the mass at the critical temperature is large but finite and δ is slightly smaller than one. This is clearly in agreement with the first order nature of the deconfinement transition for $N_c \geq 3$.

In addition the gluon condensate, which is essential to understand lattice data below T_c where the simple glueball gas contribution is highly insufficient [23], in the region $T_c \leq T \leq 1.1\ T_c$ gives only a very small correction and is practically irrelevant at higher temperatures.

An important check to our model would come from the extension of the analysis to the $SU(2)$ theory because in this case a second order phase transition is expected, with a corresponding divergent effective mass at the critical point and $\delta = 1$. Unfortunately, to our knowledge, no recent lattice analysis of the $SU(2)$ case with an accurate extrapolation to the continuum, thus eliminating finite lattice size effects [24], is presently available.

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