Phase structure of YM theory on a small torus

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1 Introduction

One of the outstanding problem in the quantum field theory is the understanding of the thermodynamical phase structure of the non-abelian gauge theory. Although it is difficult to solve it in general situation, if Yang-Mills theory is on a sphere, perturbative analysis is possible [1, 2, 3, 4]. In this case, since the spatial components of the gauge fields are massive, we can perturbatively integrate out them and obtain an effective action for the temporal component A_0. Owing to the gauge invariance, this effective action is described by the Polyakov loop operator.

$$W_0 \equiv \frac{1}{N} \mathrm{Tr} P \left(\exp \left[i \int_0^\beta A_0 dx^0 \right] \right). \tag{1}$$

Interestingly this effective action exhibits the confinement/deconfinement transition in the large N limit. Thus this model is important as a toy model for Yang-Mills theory on R^n.

In this talk, I show that we can evaluate the phase structure of large N $SU(N)$ Yang-Mills theory on torus if the volume of the torus is sufficiently 'small' [5, **?**, 6]. (I will explain the definition of 'small' soon.) Although the gauge field are classically massless in this case contrary to the sphere case, they obtain masses dynamically and the low energy effective action would be described by some Polyakov loop operators. As a result, we are able to evaluate the phase structure in this case too. Since this model has been studied in lattice gauge theories (see [7] for review) and holography [8, 9, 10, 11, 12], our results would be valuable to verify the validity of these methods. Indeed we found some problems of the holography in our model. These problems are related to a more serious issue in holographic QCD [9], which was pointed out in [13]. The resolution of these problems was proposed in [14]. The understanding of our model had played an important role to lead this resolution.

2 Set up

Let us consider a Euclidean $d+D$-dimensional gauge theory on a $d+D$-dimensional torus with radii L_μ [1]

$$S=\int_0^\beta dt\left(\prod_{M=1}^{D+d-1}\int_0^{L_M}dx^M\right)\frac{1}{4g_{d+D}^2}\mathrm{Tr}F_{\mu\nu}^2. \tag{2}$$

Here the length of the temporal circle is denoted as $L_0=\beta$ and the rest are denoted as $L_M, M=1,...,d+D-1$. The phases of (2) are characterized by Wilson lines around the $d+D$ noncontractible cycles of the torus:

$$W_\mu=\mathrm{Tr}U_\mu\equiv\frac{1}{N}\mathrm{Tr}P\left(\exp\left[i\int_0^{L_\mu}A_\mu dx^\mu\right]\right), \tag{3}$$

where no sum over μ is intended. These Wilson loops transform nontrivially under the centre symmetry. For sufficiently large radii L_μ, all W_μ vanish, signifying unbroken centre symmetry. Since W_0 can be interpreted as $\exp[-S_q]$, where S_q is the action for a static quark, the phase with $\langle W_0\rangle=0$ exhibits confinement. As is well-known, as β is reduced (i.e. the temperature is increased), below a certain critical value β_c, $\langle W_0\rangle$ becomes non-zero, signaling a deconfinement transition together with a breaking of the centre symmetry $Z_N^{d+D}\to Z_N^{d+D-1}$. It has been argued from lattice studies (see [7]) that as the other radii are successively reduced, one has a cascade of analogous symmetry breaking transitions $Z_N^{d+D-1}\to Z_N^{d+D-2}\to...\to 1$.

While it would be fascinating to study all the above phases analytically, in the current study we will be able to study the phases of a $D+2$ dimensional pure Yang Mills theory on T^{D+2} (i.e. (2) with $d=2$) in which a D-dimensional torus (with radii $L^I/(2\pi), I=1,2,...,D$) is taken as small (ensuring broken Z_N symmetries in those directions), leaving the remaining $d=2$ directions (including time) of variable size. Such a theory is given by a Kaluza-Klein reduction of (2) on the small T^D, and is described by the following action:

$$S=\int_0^\beta dt\int_0^L dx\,\mathrm{Tr}\left(\frac{1}{2g^2}F_{01}^2+\sum_{I=1}^D\frac{1}{2}\left(D_\mu Y^I\right)^2+\frac{m^2}{2}(Y^I)^2-\sum_{I,J}\frac{g'^2}{4}[Y^I,Y^J][Y^I,Y^J]\right). \tag{4}$$

Here Y^I comes from the gauge field components A_{I+1} and the covariant derivative is defined as $D_\mu=\partial_\mu-i[A_\mu,\ \ ]$. A naive KK reduction leads to massless Y^I's and $g=g'$; however, a mass m for the adjoint scalars as well as radiative splitting between g and g' is induced from loops of KK modes.

We will analyse this model and show the phase structure in the following sections.

[1] Our notation for spacetime coordinates is: $\{x^0\equiv t, x^M\}, M=1,...,d+D-1$. We will further split the $d+D-1$ coordinates into d 'large' dimensions $\{x_0,x^i\}, i=1,...,d-1$ and and D 'small' dimensions $x^I, I=1,2,...D$ (the meaning of 'large' and 'small' is explained below).

3 Outline of the derivation of the effective action

In this section, we will show the outline for the derivation of the low energy effective theory from (4). We consider two limits: $L \to \infty$ and $L \to 0$, and evaluate the phase structure by changing β. Since the system has a Z_2 symmetry $\beta \leftrightarrow L$, we can obtain the phase structure for large β and small β regime as well. (However we have not succeeded in the evaluation in the intermediate regime.)

3.1 Effective action in $L \to \infty$

We consider the derivation of the low energy effective action in the large L case. We are integrating out the adjoint scalars Y^I in a $1/D$ expansion [5, ?, 6, 15]. Through the comparison with numerical studies [16, 17], this expansion is expected to be valid if $D \geq 2$.

Let us take the following limit,

$$g, g' \to 0, \quad N, D \to \infty \quad s.t. \quad \tilde{\lambda} \equiv g^2 DN, \quad \tilde{\lambda}' \equiv g'^2 DN \quad \text{fixed.} \tag{5}$$

Then we can find a non-trivial saddle point characterized by

$$\left\langle \mathrm{Tr} Y^I Y^I \right\rangle = \frac{DN^2}{\tilde{\lambda}'} \Delta_0^2. \tag{6}$$

Here Δ_0 is given by

$$\Delta_0 = \sqrt{\frac{\tilde{\lambda}'}{2\pi} \log\left(\frac{2\pi\Lambda^2}{\tilde{\lambda}'}\right)} + \cdots, \tag{7}$$

for a low temperature case. (This expression is valid if temperature is not so higher than the critical temperature for the confinement/deconfinement transition.) Here Λ is a cut off scale.

Around this saddle point, the adjoint scalar Y^I obtain a dynamical mass Δ_0 and interactions $(Y^I)^{2n}$ $(n \geq 2)$ are suppressed by a factor of $1/D$ [2]. Thus if the theory is weak coupling ($\tilde{\lambda} \ll \Delta_0^2$), we can integrate out Y^I perturbatively and obtain an effective action for a matrix

$$U(x) = P \exp[i \int_0^\beta dt A_0(x,t)] = \exp[i\beta A_0(x)], \tag{8}$$

[2] In the small L case, the next order of the $1/D$ expansion has been evaluated [5]. There, such $1/D$ corrections do not change the nature of the phase structure. We can expect that the same thing will happen in our two dimensional gauge theory also. Hence we do not evaluate the $1/D$ corrections in large L case.

as

$$S/DN^2 = C(\tilde{\lambda}', \Delta_0) + \int_{-\infty}^{\infty} dx \left[\frac{1}{2N} \mathrm{Tr}\left(|\partial_x U|^2 \right) - \frac{\xi}{N^2} |\mathrm{Tr} U|^2 \right], \tag{9}$$

for a large L and low temperature [6]. Here

$$\xi = \sqrt{\frac{\Delta_0}{2\pi \tilde{\lambda}^2 \beta^3}} e^{-\Delta_0 \beta}, \quad C(\tilde{\lambda}', \Delta_0) = \frac{\beta L \Delta_0^2}{8\pi} \left(1 + \frac{\pi \Delta_0^2}{\tilde{\lambda}'} \right). \tag{10}$$

Note that ξ is a monotonically decreasing function of β. In the next section, we evaluate the phase structure of this effective potential.

3.2 Effective action in $L \to 0$

Through a similar procedure to the $L \to \infty$ case, we can derive an effective action[3] for a small L by using the large D expansion [5] as

$$S_{eff}(\{u_n\})/DN^2 = \frac{3}{8} \beta \tilde{\lambda}_1^{1/3} + a|u_1|^2 + b|u_1|^4 + \cdots, \tag{11}$$

where $\tilde{\lambda}_1$ is a one dimensional 'tHooft like coupling $g'^2 N(D+1)/L$. Here

$$a = \left(\frac{1}{D} - e^{-\beta \tilde{\lambda}_1^{1/3}} \right), \quad b = \frac{1}{3} \beta \tilde{\lambda}_1^{1/3} e^{-2\beta \tilde{\lambda}_1^{1/3}}, \tag{12}$$

and $u_n = \mathrm{Tr} U^n$, where U for small L is defined by (8) with the zero mode of A_0 along the L-direction.

4 Phase structure of the Yang-Mills theory

Now we evaluate the phase structure of the effective action (9) and (11). In each case, the phases are characterized by the distribution of the eigenvalue density

$$\rho(\theta, x) = \frac{1}{N} \sum_{i=1}^{N} \delta(\theta - \theta_i(x)), \tag{13}$$

where $\exp(i\theta_i(x))$ are the eigenvalues of (8). Depending on β and L, the distribution changes three configurations: uniform, non-uniform and gapped. See figure 1. Note that, since the Z_N symmetry along the temporal circle is related to the translation symmetry of the eigenvalue density along the θ coordinate, the uniform distribution corresponds to the confinement phase (in which the Z_N symmetry is preserved) and the other two distributions correspond to the deconfinement phase (in which the Z_N symmetry is broken).

[3] The effective action is calculated for $m = 0$. Note that the mass from the KK modes for small L is proportional to $\sqrt{\lambda_1 L}$. Thus the mass correction is small and we ignore it.

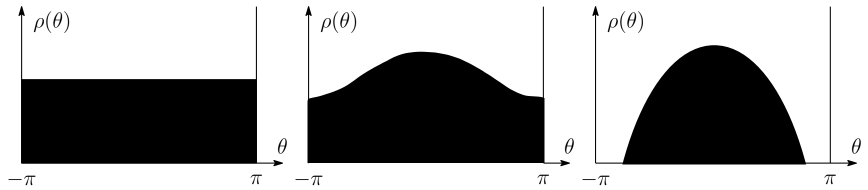

Figure 1: Configurations of eigenvalue density $\rho(\theta)$ in the unitary matrix model. The left plot is the uniform distribution , the middle one is the non-uniform distribution and the right one is the gapped distribution.

4.1 The phase transition at Large L

Phase transitions in the system (9) have been discussed in [18, 19] in different contexts. We will adopt their result to infer about phase transitions in our two-dimensional gauge theory (4) at large L. By comparing the free energies, we can evaluate the stabilities of the solutions. They can be summarised as follows:

- Independent of the value of ξ, the uniform solution always exists. We call this phase as I.
- For $\xi < \xi_0 = 0.227$, only one solution (phase I) exists and is stable.
- At $\xi = \xi_0$, there is nucleation of two gapped solutions. One is unstable (phase II) and another is meta-stable (phase III).
- At $\xi = \xi_1 = 0.23125$, a GWW type phase transition [20, 21] occurs in phase II and the gapped solution becomes a solution with non-uniform distribution (Phase IV).
- At $\xi = \xi_2 = 0.237$, there is a first order phase transition between the phases I and III. Above ξ_2, the phase III is stable and the phase I is meta-stable.
- At $\xi = \xi_3 = 1/4$, phase IV merges into phase I, and the uniform solution becomes unstable beyond ξ_3.

These are summarised in figure 2.

Using (10), we can read off the critical temperatures corresponding to these transition points:

$$\begin{aligned}\beta_m \equiv \beta(\xi_m) &= \frac{3}{2\Delta_0} W\left[\frac{2}{3(2\pi\xi_m^2)^{1/3}}\left(\frac{\Delta_0^2}{\tilde{\lambda}}\right)^{2/3}\right] \\ &\approx \frac{1}{\Delta_0}\left(\log\left(\frac{\Delta_0^2}{\tilde{\lambda}}\right) - 1.53 - \log\xi_m\right), \quad m = 0, 1, 2, 3. \qquad (14)\end{aligned}$$

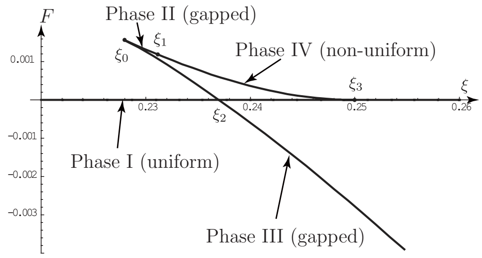

Figure 2: Free energy vs ξ in the four phases. The gapped and non-uniform solutions here are numerically evaluated. Since ξ is a monotonically increasing function of temperature (see (10)), the uniform distribution (Phase I) is stable at low temperatures and the gapped distribution (Phase III) is stable at higher temperature. A first order phase transition between these two phases happens at ξ_2.

Here Lambert's W function is employed. In the second step we have assumed $\Delta_0^2/\tilde{\lambda} \gg 1$ and $\xi_m = O(1)$.

As we come down from $\beta = \infty$ (go up in temperature), there is a first order phase transition at β_2 from the centre symmetric phase ($\mathrm{Tr}U = 0$) to the broken symmetry phase ($\mathrm{Tr}U \neq 0$) at an inverse temperature

$$\beta_{cr} \equiv \beta_2 \approx \frac{1}{\Delta_0} \log\left(\frac{\Delta_0^2}{\tilde{\lambda}}\right). \tag{15}$$

The Euclidean model (4) is symmetric under the interchange of $(t, \beta) \leftrightarrow (x, L)$. Hence, similarly to (15), we can deduce a phase transition in L from the $\mathrm{Tr}V = 0$ phase[4] to $\mathrm{Tr}V \neq 0$ (at large enough β) at a critical length $L_{cr} = \beta_{cr}$. The existence of a finite L_{cr} above confirms that the transition which we found at $L \to \infty$ and $\beta = \beta_{cr}$ between $\mathrm{Tr}U = 0$ and $\mathrm{Tr}U \neq 0$ indeed happens in the $\mathrm{Tr}V = 0$ phase. Therefore the expression for β_{cr} is valid even at finite L as long as $\mathrm{Tr}V = 0$, since large N volume independence [22, 23] ensures that gauge invariant quantities like the free energy and vev of Wilson loop operators do not depend on L in the $\mathrm{Tr}V = 0$ phase. Thus, the correct definition of 'large L' in this section is

$$L \gg L_{cr} = \beta_{cr}, \tag{16}$$

which ensures that we are in the $\mathrm{Tr}V = 0$ phase. β_{cr} is defined in (15).

4.2 The phase transition at small L

In this section, we discuss the phase structure of the effective action (11) for small L ($L \ll L_{cr}$). Phase transitions in the system (11) have been discussed in [2] in a

[4] We define $V = \exp(iLA_1)$.

different context. To see the phase structure, it is convenient to define the moments of the eigenvalue density

$$\rho(\theta) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} u_n e^{-in\theta}. \tag{17}$$

We summarise the phase structures [5],

- $\beta > \beta_{c1}$: The stable solution is given by $u_n = 0$ $(n \geq 1)$. The eigenvalues of A_0 are distributed uniformly.
- $\beta_{c1} > \beta > \beta_{c2}$: The stable solution is given by $u_1 \neq 0$, $u_n = 0$ $(n \geq 2)$. The eigenvalue distribution is non-uniform and gapless.
- $\beta_{c2} > \beta$: The stable solution is given by $u_n \neq 0$ $(n \geq 1)$. The eigenvalue distribution is gapped.
- The phase transition at $\beta = \beta_{c1}$ is of second order and the transition at $\beta = \beta_{c2}$ is a third order (GWW type) transition.

Contrary to the case of large L, an intermediate non-uniform phase exists at small L. The critical temperatures are calculated up to $O(1/D)$ in [5] as

$$\beta_{c1}\tilde{\lambda}_1^{1/3} = \log \tilde{D} \left(1 + \frac{1}{\tilde{D}} \left(\frac{203}{160} - \frac{\sqrt{5}}{3}\right)\right), \tag{18}$$

$$\begin{aligned} &\beta_{c2}\tilde{\lambda}_1^{1/3} - \beta_{c1}\tilde{\lambda}_1^{1/3} \\ &\quad = \frac{\log \tilde{D}}{\tilde{D}} \left[-\frac{1}{6} + \frac{1}{\tilde{D}} \left(\left(-\frac{499073}{460800} + \frac{203\sqrt{5}}{480}\right) \log \tilde{D} - \frac{1127\sqrt{5}}{1800} + \frac{85051}{76800}\right)\right], \end{aligned} \tag{19}$$

where $\tilde{D} = D + 1$ and $\tilde{\lambda}_1 = (g')^2 N(D+1)/L$. These critical temperatures have been evaluated in numerical studies [10, 11, 16, 17] and our large D results agree with them. See table 1.

In the β-L plane the transition lines appear as curves $\beta \propto L^{1/3}$ passing through the origin. Since our analysis is valid only for $L \ll L_{cr}$, we should trust these transition lines only in that region, as we have depicted in figure 3. By using the $\beta \leftrightarrow L$ reflection symmetry, we can also infer phase transition lines for $\beta \ll \beta_{cr}$ described by $L \propto \beta^{1/3}$, as shown in figure 3.

We should mention that considerations in this section are valid up to $\tilde{\lambda}' \lesssim \lambda_{max}$ where $\lambda_{max} = L/\beta^3$ for $\beta \ll \beta_{cr}$, and $\lambda_{max} = \beta/L^3$ for $L \ll L_{cr}$ [5], which can be large close to the origin.

4.3 Phases of 2d gauge theory on T^2

In the last two subsections, we have studied confinement/deconfinement type transitions in the model (4) for large and small values of the spatial size L.

We have found that the nature of the transition depends on L. We can summarise these results in figure 3, where we supplemented our calculations with the reflection symmetry $\beta \leftrightarrow L$ of the model.

Note that the gravity analysis [6, 12] and lattice study [7] indicate that the intermediate phase structure is given by the middle joining pattern in figure 3.

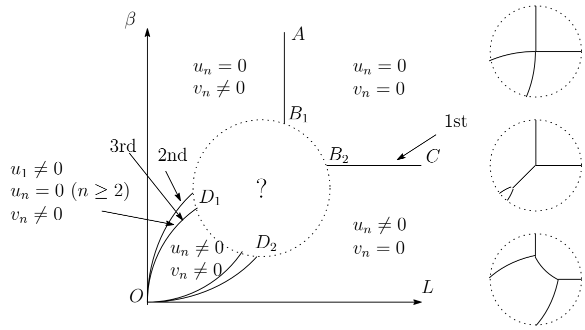

Figure 3: Phase structure of the 2d gauge theory at weak coupling. There are essentially 4 phases characterized by non-zero values of various Wilson lines. The inner region, with both Wilson lines non-zero, includes 2 additional phases in which the eigenvalue distribution is gapless but non-uniform. The orders of the phase transitions (1st, 2nd, 3rd) are indicated. Our analysis does not apply to the intermediate region enclosed by the dotted lines. Possible connections between the phase boundaries across this region are suggested in the inset (where boundaries of the intermediate phases are omitted for simplicity). The gravity analysis and lattice studies conform to the second pattern.

D=9	T_{c1}	T_{c2}	R^2	F_0
Numerical result	0.8761	0.905	2.291	6.695
$1/D$ expansion	0.895	0.917	2.28	6.72
error	2%	1%	0.5%	0.3%

Table 1: Numerical analysis [16] vs. $1/D$ expansion [5] in $D = 9$ case. We have evaluated the $1/D$ expansion up to $O(1/D)$ and the errors are expected as $O(1/D^2) \sim 1/9^2 \sim 1\%$. Indeed the $1/D$ expansion reproduces the numerical analysis in this order. R^2 and F_0 in this table denote $\langle Y^{I2} \rangle$ and free energy in the confinement phase.

5 Conclusions

In this talk, we have shown the recent progress of the understanding of the thermodynamics of the Yang-Mills theory on a torus. It may be valuable to compare our results to other studies.

Comparison to dual gravity The construction of the gravity dual of the non-supersymmetric gauge theory was proposed in [8]. Its finite temperature extension was introduced in [9]. However, it turned out that a naive application of the construction in [9] did not work in our model (2) because of an ambiguity of the fermion boundary condition along the each cycle of the torus [6]. Furthermore a more serious problem of the holography in the finite temperature case [9] exists even in a Yang-Mills theory on a flat space [13, 14] and the resolution of this problem was proposed in [14].

According to this resolution, we can compare the gravity and our results without any ambiguity and confirmed that these two are consistent [6].

Comparison to lattice As mentioned in the Introduction, our model (4) can be regarded as a dimensional reduction of a $D+2$ dimensional pure Yang-Mills theory compactified on a small T^D. Since T^D is small, $W_I = \mathrm{Tr}U_I$ $(I = d, \cdots, d+D-1)$ must be non-zero. Therefore we can regard the phase structure in figure 3 as a part of the $D+2$ dimensional pure Yang-Mills theory in the $W_I \neq 0$ phase. Such a Yang-Mills theory on T^3 and T^4 have been studied in lattice gauge theory [7] and our results are consistent with them.

Yang-Mills theory on sphere We showed that the order of the phase transition of the Yang-Mills theory on a torus does depend on the size of the cycle. On the hand, if we consider a weak coupling Yang-Mills theory on a sphere, the order of the phase transition is 1st order in S^3 case [3] and is 2nd+3rd order in S^2 case similar to the small L case [4]. These results show that the phase structure of Yang-Mills theory is very rich. The understanding of the origin of these differences of the transitions must be valuable.

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Discussion

M. Hanada: Is it possible to apply the $1/D$ expansion to $d \geq 3$ case?

Morita: It may be possible to derive the effective action for the gauge field by integrating out the D adjoint scalars through the $1/D$ expansion. However, the analysis of this effective action must be very difficult.