Observational Signatures of a Non-Singular Bouncing Cosmology

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1 Introduction

In this contribution, we summarize the results published in Lilley *et al.* [1] and presented at the 11th workshop on nonperturbative QCD as a transverse topic. We begin with a very brief overview of standard inflationary cosmology and then motivate the study of singularity-free cosmologies. We then introduce a cosmological scenario in which inflation is preceded by a bounce. In this scenario, the primordial singularity, one of the major shortcomings of inflation, is replaced by a non-singular bounce, prior to which the universe undergoes a phase of contraction. We give a detailed study of the transfer through the bounce of cosmological perturbations sourcing today's large scale structure. We show that the bouncing phase induces oscillations superimposed on the nearly scale-invariant primordial spectra for cosmological perturbations. We discuss the effects of these oscillations in the cosmic microwave background and in the power spectrum of matter in the universe. We propose a new way to indirectly measure the spatial curvature energy density parameter in the context of this model.

2 Current Status

Measurements of the abundance of light elements (such as Helium 3 and 4, Deuterium and Lithium 7), as well as results obtained from baryonic oscillations through cosmic microwave background (CMB) measurements are consistent with our present understanding of nucleosynthesis and baryogenesis in the framework of the standard Big Bang model (an expanding universe with a radiation–dominated epoch followed by a matter–dominated epoch) and consistent with abundances of baryons in the universe at the few % level.

Structure formation, rotation curves of galaxies, the Sachs-Wolfe plateau in the (CMB), the spectrum of luminous matter in the universe, gravitational lensing effects provide evidence for the existence of dark matter in the universe and are consistent with the presence of some form of dark matter at the 20% level.

Current observations of supernovae luminosities as a function of redshift providing measurements of the Hubble parameter and the deceleration parameter of the universe indicate that the expansion of the universe is accelerating and can be explained by postulating the existence of some form of dark energy making up perhaps 75% of the energy content of the observable universe.

The COBE satellite has shown that the universe has a homogeneous and isotropic blackbody spectrum with temperature $T = 2.73$ K up to 1 part in 10^5 while the position of the first accoustic peak in the CMB is consistent with a spatially flat universe. This observed flatness, isotropy and homogeneity, as well as the absence of magnetic monopoles in the universe although unexplained in the framework of the standard Big Bang model, are easily resolved by invoking a primordial epoch of inflation [4], a framework that incorporates dark matter and also explains the origin of today's CMB anisotropies and large scale structure with compelling naturalness and ease.

The overall picture provided by inflation, dark matter, dark energy, and post–inflationary standard Big Bang cosmology nevertheless falls short at very early cosmological times. This is because the usual models of inflation are not past complete and inevitably contain a time-like singularity in the past at which the universe is of vanishing size. It is however known that theories with higher order curvature terms in the Einstein-Hilbert action, non-minimal coupling of matter fields to gravity and the low energy effective actions of some string theories do allow for *non-singular* cosmological background solutions. In these, instead of shrinking to zero size, the universe experiences a bounce: starting out large, the universe undergoes a contracting phase until it reaches a finite minimal size, after which an expanding phase occurs. The existence of bouncing solutions in these high energy effective theories warrants the study of simpler *classical* non-singular cosmological models, in which background and perturbations are completely understood and tractable. This simpler class includes models with minimally coupled scalar fields, scalar fields with non-standard kinetic terms or in geometries that depart from the Friedmann-Lemaître-Robertson-Walker (FLRW) geometry, *i.e.*Bianchi or Kantowski-Sachs spacetimes. Another possibility is to consider unconventional forms of matter, or a combination of radiation and scalar field matter.

Although bouncing cosmologies have most often been discussed as alternatives to inflation, here we shall see both the contracting and the non-singular bouncing phase as cosmological epochs that connect to inflation. In the model we describe below, the cosmological singularity is replaced by a classical bounce which can entirely be described within General Relativity (GR), and after which inflation takes place as usual. We will mostly focus, for computational simplicity, on a *symmetric* bounce, meaning that the rate of contraction (up to a minus sign) and its duration are the same as in the inflationary phase that follows. The transition to standard Big Bang cosmology after inflation through a phase of reheating remains unchanged from the standard case without bounce.

The model studied relies on a dynamical scalar field with a symmetric potential of the small field type and a positive spatial curvature term. This is the minimal possible setup in order to obtain a bounce but violate the least number of energy conditions derived from the Hawking–Penrose singularity theorems.

3 The model

We consider a homogeneous and isotropic FLRW universe with line element

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \frac{\mathrm{d}r^2}{1-\mathcal{K}r^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right], \tag{1}$$

where $a(\eta)$ is the scale factor as a function of conformal time η [where $\mathrm{d}\eta = \mathrm{d}t/a$, with t the cosmic time; below we shall use the notation $X' \equiv \mathrm{d}X/\mathrm{d}\eta$, for any function $X(\eta)$]. The constant parameter $\mathcal{K}$ describes the curvature of spatial sections and can always be rescaled such that $\mathcal{K} = 0, \pm 1$. In order to achieve a bounce with a simple scalar field and avoid breaking the null energy condition ($\rho + P \geq 0$), it is necessary that $\mathcal{K} = +1$. The simplest renormalizable potential that satisfies the conditions needed for a bounce with positive spatial curvature and which is bounded from below reads

$$V(\varphi) = M^4\left[1 - \left(\frac{\varphi}{\mu}\right)^2\right]^2, \tag{2}$$

with M and μ *a priori* free parameters.

We now study the evolution of cosmological perturbations around the homogeneous and isotropic background cosmology described by Eqs. (1) and (2), and by the Einstein equations and the conservation equation for φ . We mainly consider the scalar part of the metric perturbations (given by the gauge invariant gravitational Bardeen potential Φ) in the presence of scalar field perturbations $\delta\varphi$ but the result for tensor perturbations can be obtained with the same procedure. In longitudinal gauge, the scalar part of the perturbed metric reads

$$\mathrm{d}s^2 = a^2(\eta)\left[-(1+2\Phi)\,\mathrm{d}\eta^2 + (1-2\Phi)\,\gamma_{ij}^{(3)}\mathrm{d}x^i\mathrm{d}x^j\right], \tag{3}$$

where $\gamma_{ij}^{(3)}$ is the background metric of the spatial sections. Expressing the scalar parts of the perturbed Einstein equations in terms of Φ and the density perturbation variable $\delta\varphi$ and combining them in the appropriate way yields a second order differential equation for the modes Φ_k expressible through the following change of variables [5]

$$\Phi = \frac{\kappa}{2}(\rho+P)^{1/2}u = \frac{\kappa}{2}\frac{\varphi'}{a}u, \tag{4}$$

as

$$u_k'' + \left[k^2 - V_u(\eta)\right] u_k = 0. \tag{5}$$

The wavenumber k is the eigenvalue of the Laplace-Beltrami operator on positively curved spatial sections. It is therefore a function of an integer n, and is given by $k = \sqrt{n\,(n+2)}$ with $n = 0$ and $n = 1$ corresponding to gauge modes. The potential $V_u(\eta)$ is given by

$$V_u(\eta) = \frac{\theta''}{\theta} + 3\mathcal{K}\left(1 - c_s^2\right) \tag{6}$$

with

$$\theta = \left(\frac{3}{\kappa}\right)^{1/2} \frac{\mathcal{H}}{a\varphi'} \quad \text{and} \quad c_{\rm s}^2 = \frac{\delta P}{\delta\rho} = -\frac{1}{3}\left(1 + 2\frac{\varphi''}{\mathcal{H}\varphi'}\right). \tag{7}$$

The quantity $c_{\rm s}$ can in some regimes be interpreted as the velocity of sound.

In regions of vanishing spatial curvature (away from the high curvature bouncing region) the variable u is related to the canonical variable v in terms of which the theory can be quantized [5] by the relation (expressed in terms of the Fourier modes of u_k and v_k)

$$v_k = \left(\frac{3}{\kappa}\right)^{1/2} \theta \left(\frac{u_k}{\theta}\right)' \qquad \text{and} \qquad k^2 u_k = -z\left(\frac{v_k}{z}\right)'. \tag{8}$$

Once the theory is quantized, the $v_k(\eta)$'s and $v_k^*(\eta)$'s are the mode functions of the creation and annihilation operators, and it is in terms of those mode functions that the initial conditions for the evolution of quantum modes are given.

Contrary to u which is well-defined everywhere, v is only well-defined in regions of vanishing spatial curvature. Using the relations linking u and v, it is possible to prepare an initial quantum state by choosing the mode functions v_k of the variable v in the regions $\eta < 0$ where modes are sub-Hubble and spatial curvature is negligible but study the evolution of cosmological perturbations through the entire bouncing phase using u. The resulting form of u after the bounce can then be used to determine the form of the modes v_k at the onset of inflation and then compute the primordial spectrum of scalar perturbations.

Let us now restrict the analysis to the case of a symmetric bounce and consider the evolution of the variable u in the contracting phase, at the bounce and in the inflationary phase. The potential can very well approximated by neglecting the contribution from the spatial curvature term, neglecting the time dependence of the horizon flow functions, and joining a slow-roll exponentially contracting phase directly to a slow-roll inflationary phase[1]. The expression for the potential then simplifies to

$$V_u(\eta) = \mathcal{H}^2\left(\epsilon_1 + \frac{\epsilon_2}{2}\right) = \frac{1}{x_\pm^2}\left(\epsilon_1 + \frac{\epsilon_2}{2}\right), \tag{9}$$

where we have used the expression

$$\mathcal{H}_\pm(\eta) = -\frac{1+\epsilon_1}{\eta - \eta_\pm} = \pm\frac{1+\epsilon_1}{x_\pm}. \tag{10}$$

In the last step, we defined $x_\pm = |\eta - \eta_\pm|$, where the "$-$" or the "$+$" sign is chosen according to whether one considers times η before or after the bounce at $\eta = 0$. Up to first order in slow roll, the equation of motion for u therefore simply reads

$$u'' + \left[k^2 - \frac{1}{x_\pm^2}\left(\epsilon_1 + \frac{\epsilon_2}{2}\right)\right]u = 0\,, \tag{11}$$

where the subscript k on the mode function u is implicit. We now consider Eq. (11) separately in the two eras before and after the bounce, where the solutions for u take the form of Hankel functions of the first and second kind $H_\nu^{(1)}$ and $H_\nu^{(2)}$. We can write the solutions explicitly as

$$u^-(\eta) = \sqrt{kx_-}\left[U_1^-(k)H_\nu^{(1)}(kx_-) + U_2^-(k)H_\nu^{(2)}(kx_-)\right], \tag{12}$$
$$u^+(\eta) = \sqrt{kx_+}\left[U_1^+(k)H_\nu^{(1)}(kx_+) + U_2^+(k)H_\nu^{(2)}(kx_+)\right], \tag{13}$$

where the order ν of the Hankel functions (equal before and after the bounce) is given by

$$\nu = \frac{1}{2} + \epsilon_1 + \frac{\epsilon_2}{2}\,, \tag{14}$$

and where as before

$$x_- = |\eta - \eta_-| = \eta - \eta_-\,, \qquad x_+ = |\eta - \eta_+| = \eta_+ - \eta\,. \tag{15}$$

These two solutions (and their derivatives) can be matched at the time of the bounce $\eta = 0$ using the standard procedure. Doing so and expanding the Hankel functions for large values of $|kx_\pm|$ (that is, for $\eta \simeq 0$), one obtains

$$U_1^+ = U_2^-\left(\sigma_k + i\right)e^{-i(k\Delta\eta - \pi\nu)}, \tag{16}$$
$$U_2^+ = U_1^-\left(\sigma_k - i\right)e^{i(k\Delta\eta - \pi\nu)}, \tag{17}$$

where we retain only the leading order terms and where we have defined the parameter

$$\sigma_k = \frac{2\epsilon_1 + \epsilon_2}{k\Delta\eta} \tag{18}$$

[1]Slow roll inflation occurs when $\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2 \ll 1$ and $\epsilon_{i+1} = \mathrm{d}\ln|\epsilon_i|/\mathrm{d}N \ll 1$, with $i \geq 1$, $\mathcal{H} = a'/a$ the Hubble parameter, and $N = \ln(a/a_{\mathrm{ini}})$ the number of e-foldings of expansion.

and rewritten the conformal time difference as

$$\Delta\eta = \eta_+ - \eta_- \,. \tag{19}$$

Now the equation of motion for v in regions of vanishing spatial curvature is given by

$$v'' + \left[k^2 - \frac{1}{x_\pm^2}\left(2 + 3\epsilon_1 + \frac{3\epsilon_2}{2}\right)\right] v = 0\,, \tag{20}$$

where the subscript k on v is implicit. The solutions of Eq. (20) are given by

$$\begin{aligned} v^-(\eta) &= \sqrt{kx_-}\left[V_1^- H_\varrho^{(1)}(kx_-) + V_2^- H_\varrho^{(2)}(kx_-)\right], &(21)\\ v^+(\eta) &= \sqrt{kx_+}\left[V_1^+ H_\varrho^{(1)}(kx_+) + V_2^+ H_\varrho^{(2)}(kx_+)\right], &(22)\end{aligned}$$

where $\varrho = \nu + 1$ with ν defined in Eq. (14). From the relations in Eqs. (8) between u and v, and using Eqs. (12), (13), (21) and (22), one finds

$$U_i^\pm = \pm\left(\frac{\kappa}{3}\right)^{1/2} k^{-1} V_i^\pm, \tag{23}$$

so that the coefficients of v^- and v^+ are simply related by

$$\begin{aligned} V_1^+(k) &= V_2^-(k)\,(\sigma_k + i)\,e^{-i(k\Delta\eta - \pi\varrho)}\,, &(24)\\ V_2^+(k) &= V_1^-(k)\,(\sigma_k - i)\,e^{i(k\Delta\eta - \pi\varrho)}\,. &(25)\end{aligned}$$

Let us now quantize v in the allowed vanishing spatial curvature regions on either side of the bouncing phase. The consistency of v with the commutation relations required for quantization is ensured provided v satisfies the Wronskian condition

$$(v)'\,v^* - (v^*)'\,v = i\,. \tag{26}$$

This imposes

$$|V_1^\pm|^2 - |V_2^\pm|^2 = \mp\frac{\pi}{4}k^{-1} \tag{27}$$

on the coefficients of the Hankel functions in Eqs. (21) and (22). Let us now assume, for definiteness, that V_1^- and V_2^- can be parameterized as

$$V_1^- = \frac{\sqrt{\pi}}{2}\,\varsigma_1\,k^{-\alpha/2}e^{i\theta_1} \qquad \text{and} \qquad V_2^- = \frac{\sqrt{\pi}}{2}\,\varsigma_2\,k^{-\beta/2}e^{i\theta_2}\,, \tag{28}$$

where ς_1, $\varsigma_2 \in \mathrm{R}$ and positive, α and β are numbers while θ_1 and θ_2 are phase angles. Given Eq. (28), one finds from Eq. (27) that

$$\varsigma_2^2 k^{-\beta} = \varsigma_1^2 k^{-\alpha} - k^{-1}\,. \tag{29}$$

In order to satisfy Eq. (29), one has to choose $\alpha = \beta = 1$ and hence $\varsigma_2^2 = \varsigma_1^2 - 1$. Dropping the subscript on ς_1 so that $\varsigma_1 \to \varsigma$, one finally has

$$V_1^- = \frac{\sqrt{\pi}}{2}\,|\varsigma|\,k^{-1/2}e^{i\theta_1} \qquad \text{and} \qquad V_2^- = \frac{\sqrt{\pi}}{2}\,\left|1-\varsigma^2\right|^{1/2}\,k^{-1/2}e^{i\theta_2}\,, \tag{30}$$

where we have absorbed the choice of sign in going from $|V_1^-|^2$ and $|V_2^-|^2$ to V_1^- and V_2^- into the phases θ_1 and θ_2. Additionally, it can be checked using Eqs. (24) and (25) that for $\eta > 0$ the normalization condition is verified up to first order in slow roll,

$$|V_1^+|^2 - |V_2^+|^2 = -\frac{\pi}{4}\,k^{-1}\left(1+\sigma_k^2\right)\,. \tag{31}$$

where σ_k^2 is second order in slow roll, as can be checked from Eq. (18).

The initial conditions for v^- at times $\eta < 0$ have a simple interpretation. They represent, through the phases θ_1 and θ_2 and through the amplitude parameter ς, a deviation from the vacuum state, which can be recovered provided $\varsigma = 1$ and $\theta_1 = (\pi/2)\varrho + \pi/4$. Furthermore, provided one *does* choose to start with vacuum initial conditions, it can be seen from Eqs (24) and (25) that v^+ is given by a single mode function. Consequently, in this case and at first order in slow roll, we can expect to recover the standard form of the primordial spectrum of scalar fluctuations.

Let us now use the late time (super-Hubble) behaviour of v^+ to compute the primordial spectrum of scalar perturbations in the expanding phase, $\mathcal{P}_\zeta$, where, in the subscript, the variable $\zeta = v/z$ is the curvature perturbation in the comoving gauge. The spectrum $\mathcal{P}_\zeta$ is given by the usual expression [6]

$$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2}|\zeta|^2 = \frac{k^3}{2\pi^2}\left|\frac{v^+}{z}\right|^2. \tag{32}$$

computed on super-Hubble scales, *i.e.*in the limit $kx_+ \ll 1$. It is convenient to split the explicit expression for $\mathcal{P}_\zeta$ into a first standard (tilt) part $\mathcal{P}_\zeta^{\rm std}$ and a second part $\mathcal{P}_\zeta^{\rm osc}$, which will be seen to be oscillatory in k. Doing so, one finds

$$\mathcal{P}_\zeta^{\rm std} \simeq \frac{H_p^2}{m_{\rm Pl}^2\pi\epsilon_{1\,\rm p}}\left\{1-\left[2(1+C)\epsilon_{1\,\rm p}+C\epsilon_{2\,\rm p}+(2\epsilon_{1\,\rm p}+\epsilon_{2\,\rm p})\ln\left(\frac{k}{k_{\rm p}}\right)\right]\right\}, \tag{33}$$

and

$$\begin{aligned}\mathcal{P}_\zeta^{\rm osc} \;\simeq\;& \varsigma^2+\left|1-\varsigma^2\right|-2\varsigma\left|1-\varsigma^2\right|^{1/2}\times \\ & \left[\cos\left(2k\Delta\eta+\theta_1-\theta_2\right)+\pi\left(2\epsilon_{1\,\rm p}+\epsilon_{2\,\rm p}\right)\sin\left(2k\Delta\eta+\theta_1-\theta_2\right)\right].\end{aligned} \tag{34}$$

where $C = \gamma_{\rm E} + \ln 2 - 2$ and $\gamma_{\rm E}$ is Euler's constant, and where the $\epsilon_{i\,\rm p}$ and $H_{\rm p}$ are evaluated at the pivot scale $k_{\rm p}$, defined as the logarithmic mean of the observable range of spatial scales on the CMB sky. The quantity $\Delta\eta = |\eta_+ - \eta_-|$ is the characteristic conformal timescale of the bounce and can be seen to set the frequency of oscillations, while ς set their amplitude. A similar can be obtained for tensor perturbations using the same procedure as outlined above.

4 Observables

The scalar multipole moments $\mathcal{C}_\ell^{\mathrm{s}}$ are related to the two–point function of CMB temperature fluctuations on the sky. For a spatial curvature contribution to the energy content of the universe $|\Omega_\mathcal{K}| \ll 1$ and for $\ell \lesssim 10$ but for $\mathcal{K} = +1$, the scalar multipole moments $\mathcal{C}_\ell^{\mathrm{s}}$ can be related to the primordial scalar curvature perturbation spectrum $\mathcal{P}_\zeta$ by

$$\mathcal{C}_\ell = \frac{2\pi^2}{25} \sum_{n=2}^{\infty} \frac{\mathcal{P}_\zeta M_n \left| P_{-1/2+n}^{-1/2-\ell} \left[\cos\left(\chi\right)\right]\right|^2}{n\left(n+1\right)\left(n+2\right)\sin\left(\chi\right)}, \tag{35}$$

with

$$M_n = \begin{cases} \displaystyle\prod_{i=0}^{\ell} \left(i+1\right)^2 - n^2 & \text{for } \ell \leq n \\ 0 & \text{for } \ell > n \end{cases}$$

and where $P_{-1/2+n}^{-1/2-\ell}$ are the associated Legendre polynomials while $\chi = \eta_0 - \eta_{\mathrm{lss}} \simeq \ell_H/a_0 = \sqrt{|\Omega_\mathcal{K}|}$ is the (conformal) radial distance to the last scattering surface, with η_0 and η_{lss}, the conformal times today and at the time of last scattering, respectively. With the exception that the integral over k should be replaced by a sum over n, the textbook expression used in the case $\mathcal{K} = 0$, is a very good approximation to Eq. (4) for $|\Omega_\mathcal{K}| \ll 1$. The expression for the scalar $\mathcal{C}_\ell$'s can also be carried over to the case of the $\mathcal{C}_\ell$'s for tensor perturbations.

The presence of the slowly oscillating sinusoidal functions in $\mathcal{P}_\zeta$ introduces an oscillation in the kernel of the sum in Eq. (4). In particular, one can in general expect constructive or destructive interference between the oscillations of the $P_{-1/2+n}^{-1/2-\ell}$'s having as a characteristic scale χ with the oscillations of the cosine and sine terms in $\mathcal{P}_\zeta$ of characteristic frequency $2\Delta\eta$. Summed over n, this induces slow oscillations in the $\mathcal{C}_\ell$'s at small ℓ values. The precise shape and locations of the deviations from the standard $\mathcal{C}_\ell$'s induced by the oscillations depend on $\Omega_\mathcal{K}$ via χ and on $\Delta\eta$ while their amplitude depends on ς.

In Figure 1, we present the results of a numerical simulation using the publically `CAMB` code. The quantities shown in the dashed red and full blue lines are defined by

$$\Delta\mathcal{C}_\ell = \mathcal{C}_\ell^{\mathrm{THEORY}} - \mathcal{C}_\ell^{\mathrm{WMAP}} \tag{36}$$

where the theoretical $\mathcal{C}_\ell$'s are the $\mathcal{C}_\ell$'s obtained, respectively, either from an inflationary cosmology or from the bouncing cosmology described in this paper. The error bars are the WMAP error bars for the measured $\mathcal{C}_\ell$'s. The $\Delta\mathcal{C}_\ell$'s were computed for $\varsigma = 1.01$, 1.1 and 1.2 and for increasing $\Delta\eta$. These results were obtained for $\mu = 3$, a number of e-foldings between Hubble exit of the largest observable modes and the end of inflation $N_\star = 50$, $\Upsilon \simeq 2.055$ where Υ parameterizes the scale factor at the bounce, $a_{\rm b}$, in units of the Planck length. For this choice of parameters, $\epsilon_1 = 0.0025$, $\epsilon_2 = 0.045$, $N_{\rm inf} = 65$, and for a period of radiation and matter domination (neglecting reheating) lasting 65 e-folds, such that $\Omega_{\mathcal{K}} = -0.002$. The figure mainly illustrates that at the frequency $\delta\eta = 0.01$, and for $\varsigma = 1.01$, the oscillatory features in the primordial spectrum appear to improve the fit to the WMAP data in comparison with what is obtained in the absence of oscillations in $\mathcal{P}_\zeta$. For larger values of ς, such as 1.1 and 1.2, the effect of the oscillations worsens the fit near the first accoustic peak at $\ell \simeq 200$ over a widening range of ℓ values as ς is increased. At higher frequency, for $\delta\eta = 0.01$, the resulting $\mathcal{C}_\ell$'s have large fluctuations that appear to be phase-shifted with respect to the result obtained in the inflationary case. At low ℓ, oscillations are not visible but there remains a change in amplitude with respect to the $\Delta\mathcal{C}_\ell$'s obtained from inflation. This could be expected from Eq. (4) for the values of ς and $\delta\eta$ chosen. The matter power spectrum $\mathcal{P}_\delta$ is related to the two–point correlation function of visible matter on the sky. $\mathcal{P}_\delta$ can be related to the primordial power spectrum $P_\zeta(k)$ by a k-dependent transfer function $T(k)$, *i.e.*one can write $P_\delta(k) = T(k)\,P_\zeta(k)$. Roughly speaking, $T(k) \simeq 1$ for small k, and $T(k) \propto k^{-2}$ at large wavenumbers. The oscillating behaviour observed in the primordial perturbation spectrum is therefore expected to be transmitted to the matter power spectrum. Again, using the `CAMB` code with standard cosmological parameter values, we have computed the theoretical matter power spectra for the parameters specified above. The results are shown in blue in Fig. 2. The theoretical spectra were then convolved with the observational window functions of the Sloan Digital Sky Survey (SDSS) and are shown by the red dots in Fig. 2. These theoretical results are compared with the SDSS data, shown in black in the figure. The convolution evidently smoothes out the oscillations, and it is clear from Fig. 2 that there exist values of ς and $\Delta\eta$ such that the resulting matter spectra are fully degenerate with those obtained from a standard slow roll power spectrum.

5 Conclusions

This paper refines and completes the analysis begun in Ref. [2] and continued in Ref. [3]. In Ref. [2], the authors focused on the immediate vicinity of the bouncing phase and conducted a detailed analysis of the bounce-inducing background cosmology and on the transfer of fluctuations through the bounce by modifying the exact solution obtained when considering a de Sitter universe with closed spatial sections. In Ref. [3], the framework used in Ref. [2] was used to show that for symmetric or quasi-symmetric general relativistic bouncing cosmologies with $\mathcal{K} = +1$ the peak in the potential for the variable u could never be large so that the metric perturbation is always sub-Hubble in the vicinity of the bounce. A cosmology smoothly connecting a contracting phase and a bouncing phase to an inflationary phase was then proposed

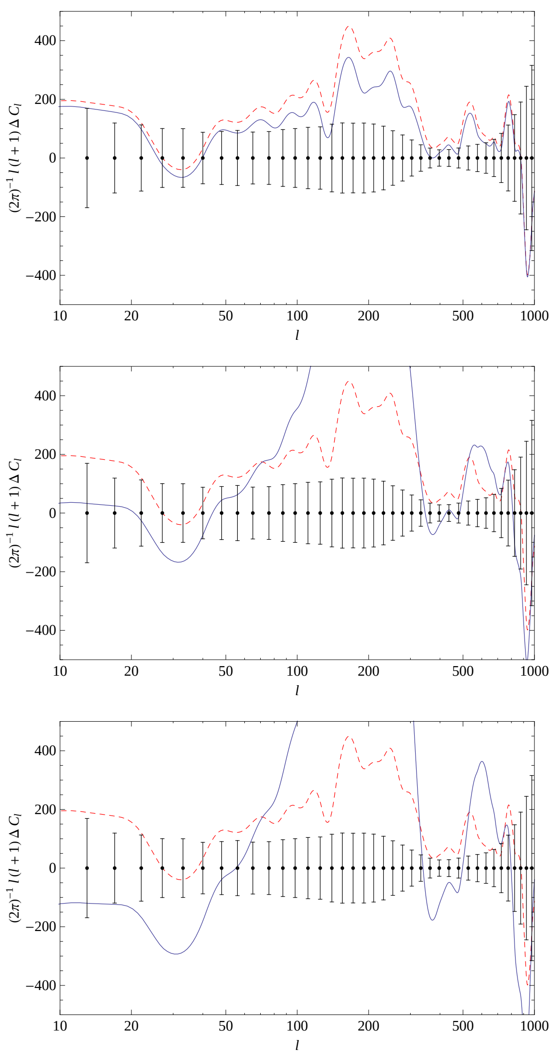

Figure 1: $\Delta\mathcal{C}_\ell$ spectra for $\varsigma = 1.01$, 1.1 and 1.2, from top to bottom respectively and for increasing values of $\Delta\eta$ from left to right, generated using `CAMB`. The error bars are the WMAP error bars on the measured values of the $\mathcal{C}_\ell$'s, the red dashed lines are the $\Delta\mathcal{C}_\ell$'s for an inflationary cosmology and the blue line in full represents the $\Delta\mathcal{C}_\ell$'s for the bouncing cosmology presented in this work.

and analyzed at both the background and linear perturbations level. In the present work, we have provided a detailed summary of the background cosmology exploited in Ref. [3] and have performed a much more detailed calculation of the transfer of perturbations through the contracting, bouncing and inflationary phases of the cosmological background discussed in Ref. [3]. We have parameterized the initial state of

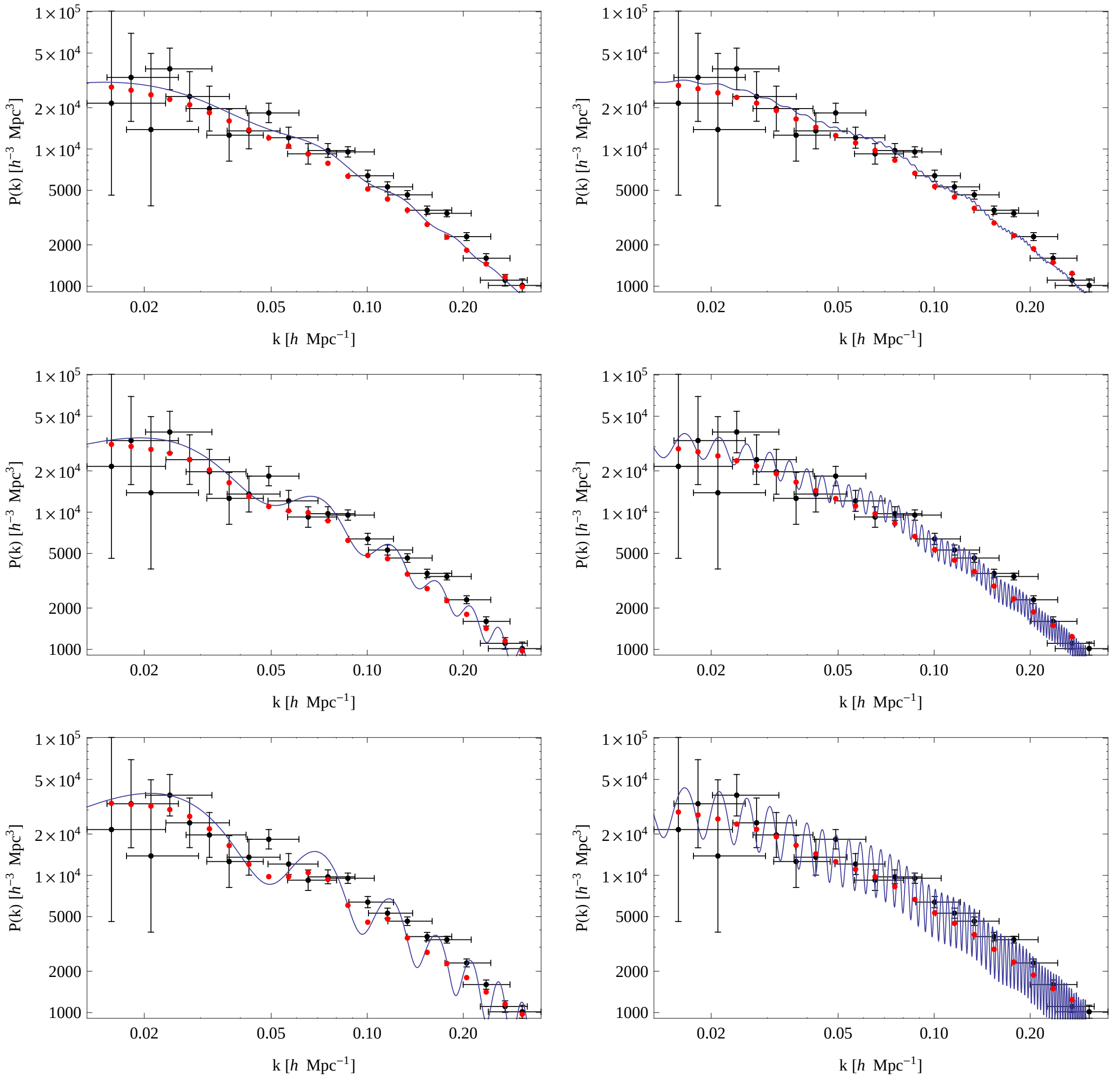

Figure 2: Matter power spectrum for $\varsigma = 1.01$, 1.1 and 1.2, from top to bottom respectively and for increasing $\Delta\eta$ from left to right. The black dots with error bars are the data points from the Sloan Digital Sky Survey (SDSS), the blue line is the theoretical prediction of the bouncing cosmological model obtained using `CAMB`, and the red dots are the simulated data points obtained by convolving the blue line with the SDSS window functions.

perturbations prior to the bounce in terms of the vacuum state and used the solution for the canonical variable v (far from the high spatial curvature region) in terms of Hankel functions, to reduce the number of unknown parameters needed to define the initial state of first order perturbations. We then computed the effects of the bounce and the choice of initial conditions on the scalar and tensor mode primordial spectra $\mathcal{P}_\zeta$ and $\mathcal{P}_h$. We discussed the modified low multipoles of the CMB angular power spectrum, the modified COBE normalization and also provided numerical evidence, using `CAMB`, that the $\mathcal{C}_\ell$'s and matter power spectrum $\mathcal{P}_\delta$ are affected by the combined effect of the choice of initial conditions and the bouncing cosmology.

The crucial point is the appearance of oscillations in the power spectra. These oscillations mainly depend on the initial state of the perturbations through the free parameter ς and on a new cosmological scale $\Delta\eta$. The former sets the amplitude of the oscillations while the latter sets the oscillatory frequency. As exhibited in the figures of the $\mathcal{C}_\ell$'s and $\mathcal{P}_\delta$, there exists values of ς and $\Delta\eta$ for which the oscillations induced by the bouncing phase and by the choice of initial conditions do not conflict with the WMAP and SDSS data. In fact, there appears to be parameter ranges in which the $\mathcal{C}_\ell$'s and the $\mathcal{P}_\delta$ derived from $\mathcal{P}_\zeta$ are fully degenerate with those obtained from a standard slow-roll primordial spectrum. The tensor-to-scalar ratio is also modified. It departs from the standard result in two ways, first by a modification of the overall amplitude and secondly by the existence of a scale-depedent oscillation.

We can also identify a new way of indirectly measuring the spatial curvature of the universe, assuming the oscillations can be attributed to a bouncing cosmology of the kind described in this paper and can indeed be measured. The scale factor at the bounce as well as the model parameter μ determine both $\Omega_\mathcal{K}$ and $\Delta\eta$, thereby establishing a one-to-one relationship between the two. If the frequency of oscillations could be measured together with the spectral index of $\mathcal{P}_\zeta$, then $\Omega_\mathcal{K}$ is determined, as well as a_b.

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