Rapid Thermalization by Baryon Injection in Gauge/Gravity Duality

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1 Condensed matter physics and heavy ion collisions

The gauge/gravity correspondence is now a mature subject in high energy physics, particularly because of its success in the application to non-perturbative QCD. The subject is called holographic QCD, and we got familiar with the concept of the gauge/gravity duality, or the AdS/CFT correspondence, in the context of generalizations of QCD. The applications of the holographic principle is now going beyond the QCD, as it is expected since the concept of the AdS/CFT correspondence is not limited to QCD-like theories. What is beyond the expectation about the development in the application of the holographic principle is that the holography seems to work not only for gauge theories. This point is still under discussion, but in fact we now know that some problems and questions in condensed matter physics may be answered by using the holography. In particular, questions related to strongly correlated electrons in condensed matter physics, such as mechanisms for high-temperature superconductors and non-Fermi liquids, can be interestingly modeled by a gravity model in higher dimensions.

The goal of the condensed matter physics and that of QCD, heavy ion collisions, look very different. However, there are some questions which are closely related. For example, the heavy ion collisions probe high density phase of QCD, and also a time-dependent process for a formation of the quark-gluon plasma. In condensed matter physics, one is interested in such kind of time-dependent process and thermalization, from a different motivation: it is essential for device and material modeling as a thermalization generically destroys important electric transportability. With different motivations, we can find a good and shared problem: the thermalization via changing the external parameters.

What is the external parameters for the heavy-ion collisions? The answer is not simple, since it involves various quantum effects of QCD at the incident of the collision of nuclei. So far, proposed methods to describe the initial condition of the heavy ion collisions include color-glass condensate etc., but in this talk I would like to propose a different approach: *a time-dependent quark number density.*

Once two heavy ions collide, one can view it as a sudden change of the quark number at the collision point. The quarks (baryons) pass by, and the quark number goes back to zero after the collision. In this approximation, we ignore the effect of the kinetic energy of the colliding baryons (which is normally a trigger for the formation of the quark gluon plasma). Since the difficult issue concerning the understanding of the formation process is indeed the question of how the collision of baryons itself may be converted to the energy of the thermal plasma, here we take a radical assumption that one may ignore the kinetic energy of the incident baryons. Instead, as an external manipulation, we change the quark number density by hand, in a time-dependent manner.

Interestingly, this manipulation is quite common in condensed matter physics. One can change the electron density in media by hand, by introducing some electric potentials to the material put close to the media. This manipulation is generally called “quantum quench”, and our trial here is to apply this quantum quench to the QCD baryon density to see the thermalization, and to see whether we can actually approximate the heavy ion collisions. We use the holography to calculate the response of this quantum quench, the thermalization time scale. As we shall see, the time-scale computed in this way is consistent with the known requirement of the rapid thermalization for RHIC experiments. We also have a conjecture for LHC heavy-ion program, for the thermalization time scale.

This talk is based on a collaboration with N. Iizuka and T. Oka [1].

2 Problem, cause and solution

The problem which we are going to address, under the approximation mentioned above, is a possible derivation of the rapid thermalization at heavy ion collisions. It is expected [2] that the thermalization time scale is

$$t_{\rm th} < 2[{\rm fm/c}] \tag{1}$$

which means a quite rapid process. This constraint comes from hydrodynamic simulation of the quark-gluon plasma expansion.

Why is it difficult to derive this thermalization time-scale? There are two reasons: first, our QCD is strongly coupled and it goes through a phase transition from the confined phase to a deconfined phase, in the heavy ion collisions. Second, any thermalization is a non-equilibrium and time-dependent process which is quite difficult to analyze, and it is indeed difficult to even define the concept of the thermalization. These are obviously two hard causes which make the analysis difficult.

Now, a way to solve is in the AdS/CFT correspondence. For both the causes, indeed the AdS/CFT can provide a strategy. In AdS/CFT, a deconfined phase of gluons at a finite temperature is provided by a black hole geometry in the bulk. Therefore, formation of a black hole horizon is equivalent, in the AdS/CFT dictionary, to a thermalization and a deconfinement. Once one is able to describe the formation process of the black hole horizon in the bulk, it can be interpreted as a thermalization.

In order to create the black hole horizon in the gravity dual, one needs to start with a certain initial condition which is strong enough to make the geometry curved and create the horizon. The previous trials include a forced Bjorken-like expansion of space, which indeed creates a black hole horizon [3]. Our approach is different: we do not make the space expand to mimic the heavy ion collisions. Instead, we change the baryon number density by hand in a time-dependent manner, to create the horizon. In reality, both of these should be responsible to the thermalization of the heavy ion collision. Our claim here is that not only the Bjorken expansion of space but also the sudden change of the baryon number (quark number) can give an energetic source for the thermalization.

Once we accept this idea, then the issue is which process is more relevant for the thermalization. The answer to this question depends on how rapid the thermalization is for each of these two approaches. We shall see below that our approach, the quantum quench, can give a reasonable value for the thermalization time scale, as compared to (1).

3 Solving the behavior of the time-dependent baryon number density

There is a well-known procedure for the finite baryon number density in AdS/CFT, when the baryon number is not time-dependent. The baryon number is translated to the terminology in the gravity dual as an electric charge on the flavor D-brane in the AdS bulk geometry. So, if we like to change the baryon number density in a time-dependent way, we need to throw-in the electric charge from the boundary of the flavor D-brane in the bulk. The thrown-in electric charge changes the electromagnetic fields on the flavor D-brane, and it is dictated by an effective action of the flavor D-brane (Dirac-Born-Infeld action). See the schematic picture, Fig. 1.

More specifically, the D-brane action on the flavor D-brane is

$$S = -\mu_7 \int d^8\xi \sqrt{-\det(G_{ab} + 2\pi\alpha' F_{ab})} \tag{2}$$

and we add the time-dependent source term as

$$\delta S = \mu_7 V_3 \mathrm{Vol}(S^3) \int dt dz (A_t j^t + A_z j^z) \tag{3}$$

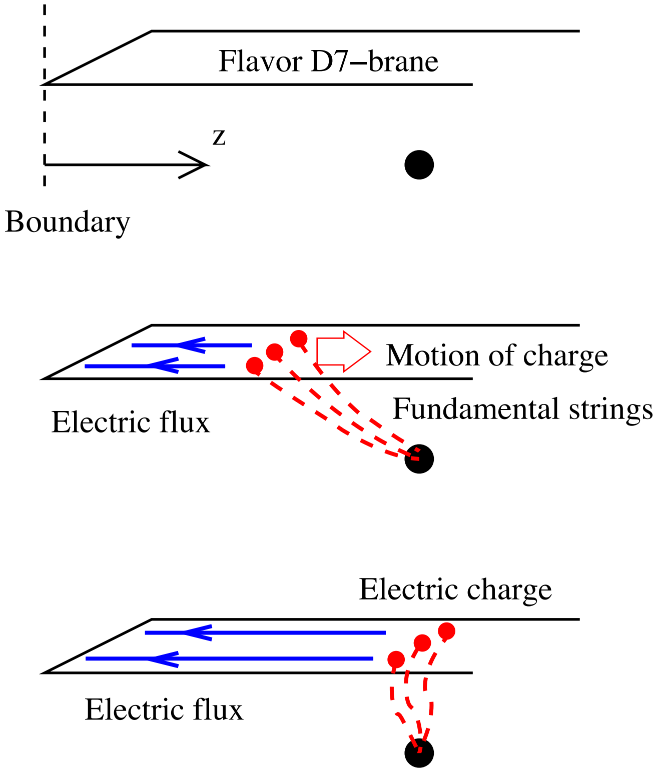

Figure 1: Electric charges on the flavor D-brane are thrown in, which induces the electric flux on the flavor D-brane.

where the source is

$$j^t = j^z = g'(t-z) \tag{4}$$

with the time-dependent baryon number density $n_B(t)$,

$$g(t) = \frac{2(2\pi\alpha')^4}{\pi}\lambda n_B(t). \tag{5}$$

Here λ is the 'tHooft coupling constant of QCD. The stringy parameter α' disappears in the final result of AdS/CFT. This is a generalization of the "holographic quantum quench" which was studied by Das, Nishioka and Takayanagi [4]. One can work with a specific example of the AdS/CFT setup: for example a flavor D7-brane in $AdS_5 \times S^5$ geometry, then for a massless quark, we find a solution for the field strength on the D7-brane as

$$(2\pi\alpha')F_{tz} = \frac{R^2 z g(t-z)}{\sqrt{(2\pi\alpha')^2 R^{12} + z^6 (g(t-z))^2}}. \tag{6}$$

Here R is the radius of the AdS_5 background geometry.

4 Emergent horizons and thermalization time scale

Since the effective action on the flavor D-brane is non-linear, the time-dependent solution on the flavor D-brane creates a nontrivial background effective metric for small fluctuations. The effective metric is a functional of the time-dependent baryon number density, so for a typical time-dependence of the baryon number density, indeed the effective metric provides a horizon, which is interpreted as a thermalization.

The fluctuation of the scalar field η on the D7-brane is written as

$$S = -\int dt dz d^3 x^i d^3 \theta^I \frac{\sqrt{-\tilde{g}}}{2} \tilde{g}^{MN} \partial_M \delta\eta \partial_N \delta\eta \tag{7}$$

where

$$-\tilde{g}_{tt} = \tilde{g}_{zz} = \mu_7^{1/3} R^{4/3} z^{-4/3} (1 - z^4 R^{-4} (2\pi\alpha'^2) F_{tz}^2)^{5/6} \tag{8}$$

and a similar expressions for the other components of the effective metric $\tilde{g}_{MN}$. From this, one can identify the location of the apparent horizon as

$$(\partial_z - \partial_t) \left[z^2 (1 - z^4 R^{-4} (2\pi\alpha'^2) F_{tz}^2) \right] = 0. \tag{9}$$

This equation determines a curve at which the apparent horizon is located.

If two nuclei collide and pass by, the baryon number density suddenly goes up, and then comes back to zero. This can be typically approximated with

$$\rho(t) = n_B \exp\left[-(2w)^2 (t - 1/w)^2 \right] \tag{10}$$

where w is a typical frequency of the change of the baryon number density. One can take a linear profile instead, or some other profiles, for example, a step function which is nonzero only while two nuclei are overlapping.

The information of the created horizon is conveyed to the AdS boundary via a geodesic path. The time at which this information is reached to the boundary is denoted by t_{th} which is nothing but the thermalization time scale as we interpret.

For any profile of the time-dependent baryon density, which suddenly goes up and again vanishes with the time scale $1/w$, we find

$$t_{\rm th} \sim \min{}_{\{k=0,1,2,3\}} \left\{ \left(\frac{\lambda}{n_B^2 w^k} \right)^{1/(6+k)} \right\}. \tag{11}$$

This is our formula for the thermalization time scale.

5 Physics parameters and discussions

Now, with the formula (11) at hand, we can substitute the values for the parameters of the heavy ion collisions at RHIC experiments. We have

$$n_B \sim 2\gamma n_N, \quad 1/w \sim 2A^{1/3}/\gamma[\mathrm{fm/c}], \quad \gamma = E/m_{Au} \sim 100. \tag{12}$$

Then the formula (11) gives

$$t_{th} < 1 \ \ [\mathrm{fm/c}]\,. \tag{13}$$

So we obtain a rapid thermalization, whose time scale is consistent with hydrodynamic simulations of the expansion of the quark-gluon plasma.

We can employ parameters for LHC collisions, which gives a value of the thermalization time scale of order 0.1 [fm/c]. This is quite a rapid thermalization.

Let us discuss issues concerning our calculations and setup. First of all, our horizon is on the flavor D-brane, so what is thermalized is quarks and mesons, and not gluons. This may look strange since for the formation of the quark gluon plasma one needs to have the thermalization of gluons. However, note that the AdS/CFT works only for a large number of colors. Even though we change the quark sector in a time-dependent manner, the gluon sector is not affected easily, as there is a difference in numbers of degrees of freedom. So this feature is a deficit of the holographic approach triggered by the quantum quench in the quark sector.

Then how universal is our result? At least we can claim the following universality.

- No effect of a small quark mass. It is possible to argue that small mass does not change the story which we have given here. In particular, a rough estimate gives a constraint on the quark mass as $m_q \ll (\sqrt{2}\lambda n_B/\pi)^{1/3}$ so that it does not affect the thermalization time scale.

- Can be applied to confining gauge theories. In this talk I have concentrated on the AdS_5 geometry which corresponds to a deconfinement phase of the large N_c QCD. One can instead employ a confining geometry which differs only at the IR part of the geometry. So it is obvious that for large enough baryon density the effect of the confinement does not change the story on the flavor D-brane.

- No relevance to supersymmetries. Since we have not used any fermions in the gravity side of the AdS/CFT, the effect of the supersymmetries is irrelevant. However, if one breaks the supersymmetry in such a way that it changes the asymptotic geometry (which is the AdS_5 in the present case), it would significantly alter the result of the thermalization time scale.

Application to more realistic gauge theories closer to the real QCD is left for a future work.

I am grateful to my collaborators, and also to the organizers of the workshop for a pleasant workshop for discussions.

References

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