

Large- N_c universality of phases in QCD and QCD-like theories

Masanori Hanada
Department of Physics
University of Washington
Seattle, WA 98195-1560, USA

1 Introduction

QCD with a finite baryon chemical potential, despite its importance, is not well understood because the standard lattice QCD simulation is not applicable due to the sign problem. Although sign-free QCD-like theories have been studied intensively, relation to QCD with a finite baryon chemical potential was not clear until recently [1, 2]. In this talk we explain the large- N_c equivalences between QCD and various QCD-like theories, which lead us to a unified viewpoint for QCD with baryon and isospin chemical potentials, $SO(2N_c)$ and $Sp(2N_c)$ gauge theories. In particular two-flavor QCD with the baryon chemical potential is equivalent to its phase quenched version in a certain parameter region, which is relevant for heavy ion collision experiments.

2 Basic Idea

Consider QCD at a finite baryon chemical potential,

$$\mathcal{L} = \frac{1}{4g^2} \text{tr}(F_{\mu\nu})^2 + \sum_{f=1}^{N_f} \bar{\psi}_f (\gamma^\mu D_\mu + m_f + \mu\gamma^4) \psi_f, \quad (1)$$

where the gauge group is $SU(3)$, N_f is the number of flavors, ψ_f are quarks of mass m_f in the fundamental representation, and μ is the quark chemical potential which is related to the baryon chemical potential μ_B as $\mu_B = 3\mu$. This system suffers from the *fermion sign problem* – the fermion determinant $\prod_{f=1}^{N_f} \det(\gamma^\mu D_\mu + m_f + \mu\gamma^4)$ becomes complex, rendering importance sampling impossible in practice.

In order to circumvent this difficulty, gauge theories which do not suffer from the sign problem at finite density have been studied. Consider QCD and QCD-like

theories¹ of the form

$$\mathcal{L}_G = \frac{1}{4g_G^2} \text{tr}(F_{\mu\nu}^G)^2 + \sum_{f=1}^{N_f} \bar{\psi}_f^G (\gamma^\mu D_\mu^G + m_f + \mu_f \gamma^4) \psi_f^G, \quad (2)$$

where G is the gauge group e.g. $SU(N_c)$, μ_f is a generic quark chemical potential, and fermions ψ^G are not necessarily in the fundamental representation. The main examples are QCD with an isospin chemical potential μ_I (i.e. $N_f = 2$, $\mu_1 = -\mu_2 = \mu_I/2$) and degenerate mass $m_1 = m_2$, two-color QCD of even number of flavors and degenerate mass, $SU(N_c)$ Yang-Mills with adjoint fermions, and $SO(2N_c)$ and $Sp(2N_c)$ Yang-Mills theories. However, these theories look quite different from $N_c = 3$ QCD; for example the flavor symmetry is explicitly broken in the first case. Therefore it is important to understand *what we can learn from these theories*, or in other words, *in what sense they are similar to real QCD with the baryon chemical potential*.

In [1, 2, 3], an answer to this question has been given. The statements are

- $SO(2N_c)$ YM with μ_B , $Sp(2N_c)$ YM with μ_B and $SU(N_c)$ QCD with μ_I are large- N_c equivalent both in the 't Hooft limit (N_f fixed) and the Veneziano limit (N_f/N_c fixed), everywhere in the T - μ plane. (Fermions are in the fundamental (vector) representations.)
- $SO(2N_c)$ YM with μ_B , $Sp(2N_c)$ YM with μ_B , $SU(N_c)$ QCD with μ_I and $SU(N_c)$ QCD with μ_B are large- N_c equivalent in the 't Hooft limit, outside the BEC/BCS crossover region of the former three theories. (Fermions are in the fundamental representations.)
- More generally, $SO(2N_c)$, $Sp(2N_c)$ and $SU(N_c)$ theories with fermion mass m_1, \dots, m_{N_f} and chemical potential μ_1, \dots, μ_{N_f} are equivalent. The signs of the chemical potential can be flipped without spoiling the equivalence. (Fig. 2)
- $SO(2N_c)$ YM with the N_f complex adjoint fermions and μ_B , $SU(N_c)$ YM with the N_f complex adjoint fermions and μ_B , and $SU(N_c)$ YM with the $2N_f$ complex anti-symmetric fermions and μ_I are large- N_c equivalent everywhere in the T - μ plane.

These statements have been derived by using a string-inspired large- N_c technique, which is called the *orbifold equivalence* [6, 7, 8, 9]. As shown in [1, 2, 3], there are orbifold projections relating $SO(2N_c)$ and $Sp(2N_c)$ theories with μ_B , QCD with μ_B and QCD with μ_I (Fig. 1). At large- N_c , the orbifold equivalence guarantees these theories are equivalent in the sense a class of correlation functions (e.g. the expectation value

¹In this paper we call $SU(N_c)$ Yang-Mills with N_f fundamental fermions ‘QCD’. $SU(N_c)$ Yang-Mills with fermions in other representations and $SO(2N_c)/Sp(2N_c)$ theories are referred to as ‘QCD-like theories’.

of the chiral condensate and π^0 correlation functions) and the phase diagrams determined by such quantities coincide, as long as the projection symmetry is not broken spontaneously [9]. A similar argument shows QCD with adjoint fermions and μ_B is equivalent to QCD with fermions of two-index antisymmetric representation, which is the so-called the Corrigan-Ramond large- N_c limit, with μ_I (Fig. 3). In order for these equivalence to hold, orbifolding symmetries must not be broken spontaneously. This requirement is always satisfied for the equivalences between $SO(2N_c)$ YM with μ_B , $Sp(2N_c)$ YM with μ_B and QCD with μ_I . For the equivalences between these three theories and QCD with the baryon chemical potential, ‘outside the BEC/BCS crossover region’ is required for the symmetry realization. This region is relevant for the search for the QCD critical point, which attract intense interest over the decade. Our answer to the problem is strikingly simple – one can study it by using the sign-free theories. In the case of the two-flavor theory, QCD with μ_I is nothing but the phase-quenched version of QCD with μ_B . Therefore, the sign problem is merely an illusion, up to the $1/N_c$ correction. Furthermore, for gluonic observables, the leading $1/N_c$ corrections to the large- N_c limit which carries the information of the chemical potential are the same in these theories.

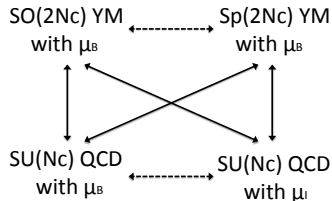


Figure 1: A web of equivalences. Arrows with solid lines represent equivalences through orbifold projections. Arrows with dashed lines are the ‘parent-parent’ and ‘daughter-daughter’ equivalences which arise as combinations of two orbifold equivalences.

3 Orbifold equivalence

3.1 Pure Yang-Mills theory

The notion of the orbifold equivalence came from the string theory [6, 7]. Soon it was proven by using only field theory techniques [8, 9], without referring to the string theory. As a simple example, let us consider the equivalence between $SO(2N_c)$ and $SU(N_c)$ pure Yang-Mills theories. (The projection from $Sp(2N_c)$ to $SU(N_c)$

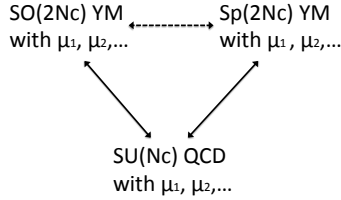


Figure 2: More general version of the equivalences. Values of the quark chemical potentials can be different.

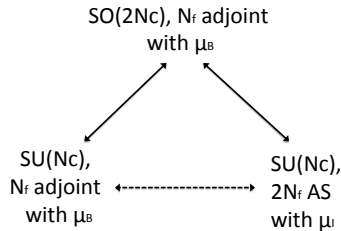


Figure 3: Equivalences in the Corrigan-Ramond limit. $SU(N_c)$ YM with anti-symmetric fermions can be regarded as a special kind of large- N_c limit of three-color QCD (the Corrigan-Ramond limit), because anti-symmetric and fundamental representations are equivalent at $N_c = 3$. Unfortunately, $SU(N_c)$ YM with anti-symmetric fermions and μ_B cannot be incorporated in these equivalences.

is obtained in a similar manner.) To perform an orbifold projection, one identifies a discrete subgroup of the symmetry group of the ‘parent’ theory, which is the $SO(2N_c)$ theory in this case, and requires the fields to be invariant under the discrete symmetry. This gives a ‘daughter’ theory, which is the $SU(N_c)$ YM.

The details are as follows. Let us take $J_c \in SO(2N_c)$ to be $J_c = i\sigma_2 \otimes 1_{N_c}$, which generates a \mathbb{Z}_4 subgroup of $SO(2N_c)$. Here 1_N is an $N \times N$ identity matrix. We require the gauge field A_μ to be invariant under

$$A_\mu \rightarrow J_c A_\mu J_c^{-1}, \quad (3)$$

which generates a \mathbb{Z}_2 subgroup of $SO(2N_c)$. A generic $SO(2N_c)$ gauge field A_μ can be written in $N_c \times N_c$ blocks as

$$A_\mu = i \begin{pmatrix} A_\mu^A + B_\mu^A & C_\mu^A - D_\mu^S \\ C_\mu^A + D_\mu^S & A_\mu^A - B_\mu^A \end{pmatrix}, \quad (4)$$

where fields with an ‘ A ’ (‘ S ’) superscript are anti-symmetric (symmetric) matrices. Under the \mathbb{Z}_2 symmetry, A_μ^A and D_μ^S are even while B_μ^A and C_μ^A are odd, and hence the orbifold projection sets $B_\mu^A = C_\mu^A = 0$; the ‘daughter’ field is

$$A_\mu^{\text{proj}} = i \begin{pmatrix} A_\mu^A & -D_\mu^S \\ D_\mu^S & A_\mu^A \end{pmatrix}. \quad (5)$$

By using a unitary matrix

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1_{N_c} & i1_{N_c} \\ 1_{N_c} & -i1_{N_c} \end{pmatrix}, \quad (6)$$

it can be rewritten as

$$PA_\mu^{\text{proj}}P^{-1} = \begin{pmatrix} -\mathcal{A}_\mu^T & 0 \\ 0 & \mathcal{A}_\mu \end{pmatrix}, \quad (7)$$

where $\mathcal{A}_\mu \equiv D_\mu^S + iA_\mu^A$ is a $U(N_c)$ gauge field. However, the difference between $U(N_c)$ and $SU(N_c)$ is a $1/N_c^2$ correction and can be neglected at large- N_c .² The gauge part of the action after the orbifold projection is thus simply

$$\mathcal{L}^{\text{gauge,proj}} = \frac{2}{4g_{SO}^2} \text{Tr} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu}, \quad (8)$$

where $\mathcal{F}_{\mu\nu}$ is the $SU(N_c)$ field strength. Let us identify it with the Lagrangian of the daughter theory times two,

$$\mathcal{L}_{SO} \rightarrow 2\mathcal{L}_{SU}, \quad (9)$$

or equivalently let us take $g_{SU}^2 = g_{SO}^2$, where g_{SU} is the gauge coupling constant of the $SU(N_c)$ theory. This factor two is necessary in order for the ground state energies, which are proportional to the degrees of freedom, to match. Given this projection, expectation values of the gauge-invariant operators in parent theory, $\mathcal{O}^{(p)}[A_\mu]$, agree with the expectation values of counterparts in the daughter theory, which are obtained by replacing A_μ with $\mathcal{A}_\mu^{\text{proj}}$, $\mathcal{O}^{(d)}[\mathcal{A}_\mu] \equiv \mathcal{O}^{(p)}[A_\mu^{\text{proj}}]$ in the large- N_c limit [8, 9], as long as the projection symmetry is not broken spontaneously.

3.2 Introducing fundamental fermions

²When one studies $U(N_c)$ theory, it is difficult to control the $U(1)$ part and the effect of the chemical potential can be washed away by a lattice artifact. In order to avoid it one should simulate $SU(N_c)$ theory on the lattice.

3.2.1 Orbifold projection of fundamental fermions

In this section, we introduce the orbifold projection for fundamental fermions [1, 2]. By using $\omega = e^{i\pi/2} \in U(1)_B$, we define the projection by

$$\psi_f = \omega J_c \psi_f. \quad (10)$$

By using

$$\begin{pmatrix} \psi_f^+ \\ \psi_f^- \end{pmatrix} \equiv P \psi_f, \quad (11)$$

the action of the \mathbb{Z}_2 symmetry is just $(\psi_f^+, \psi_f^-) \rightarrow (-\psi_f^+, \psi_f^-)$. The projection consists of setting $\psi_f^+ = 0$.

The action of the daughter theory is

$$\mathcal{L} = \frac{1}{4g_{SU}^2} \text{Tr} \mathcal{F}_{\mu\nu}^2 + \sum_{f=1}^{N_f} \bar{\psi}_f^{\text{SU}} (\gamma^\mu \mathcal{D}_\mu + m_q + \mu \gamma^4) \psi_f^{\text{SU}}, \quad (12)$$

where $\mathcal{F}_{\mu\nu}$ is the field strength of the $SU(N_c)$ gauge field $\mathcal{A}_\mu = D_\mu^S + iA_\mu^A$, $\psi_f^{\text{SU}} = \psi_f^-$, and $\mathcal{D}_\mu = \partial_\mu + i\mathcal{A}_\mu$. This is an $SU(N_c)$ gauge theory with N_f flavors of fundamental Dirac fermions and the baryon chemical potential $\mu_B = \mu N_c$. So the orbifold projection relates $SO(2N_c)$ gauge theory to large N_c QCD.

On the other hand, in order to obtain fermions at finite μ_I for even N_f , we use $J_c \in SO(2N_c)$ [or $J_c \in Sp(2N_c)$] and $J_i \in SU(2)_{\text{isospin}} [C SU(N_f)]$ defined by

$$J_i = -i\sigma_2 \otimes 1_{N_f/2}. \quad (13)$$

We choose the projection condition to be

$$(J_c)_{aa'} \psi_{a'f'} (J_i^{-1})_{f'f} = \psi_{af}. \quad (14)$$

The flavor N_f -component fundamental fermion is decomposed into two $(N_f/2)$ -component fields,

$$\psi = (\psi_i \ \psi_j), \quad (15)$$

with i and j being the isospin indices. If we define $\varphi_\pm = (\psi_\pm^i \mp i\psi_\pm^j)/\sqrt{2}$ and $\xi_\pm = (\psi_\pm^i \pm i\psi_\pm^j)/\sqrt{2}$, φ_\pm survive but ξ_\pm disappear after the projection (14). Because φ_\pm couple to $(A_\mu^{\text{SU}})^C$ and A_μ^{SU} respectively, the action of the daughter theory is expressed as

$$\mathcal{L}_{\text{SU}} = \frac{1}{4g_{\text{SU}}^2} \text{tr}(F_{\mu\nu}^{\text{SU}})^2 + \sum_{f=1}^{N_f/2} \sum_{\pm} \bar{\psi}_{f\pm}^{\text{SU}} (\gamma^\mu D_\mu + m \pm \mu \gamma^4) \psi_{f\pm}^{\text{SU}}, \quad (16)$$

where $\psi_+^{\text{SU}} \equiv \sqrt{2}\varphi_-$ and $\psi_-^{\text{SU}} \equiv \sqrt{2}\varphi_+$. This theory has the isospin chemical potential $\mu_I = 2\mu$.

3.2.2 The 't Hooft limit vs the Veneziano limit

The proof of the orbifold equivalence of the pure Yang-Mills theories in [8] can be applied even with the fundamental fermion, when the chemical potential is zero. Note that two projections (10) and (14) are equivalent when the chemical potential is absent. Both are a \mathbb{Z}_4 subgroup of the flavor symmetry which mixes two Majorana flavors. Once the chemical potential is turned on, they are not equivalent. The flavor symmetry J_i used in (14) satisfies the assumption used in [8], and the proof can be repeated straightforwardly. On the other hand, $\mathbb{Z}_4 \in U(1)_B$ used in (10) does not satisfy that assumption; however it is still possible to show that all planar diagrams with at most one fermion loop coincide. Because the fermion loops are suppressed by the factor N_f/N_c , the equivalence through (10) holds in the 't Hooft large- N_c limit (N_f fixed) while the one through (14) holds also in the Veneziano limit (N_f/N_c fixed).

The above argument has an implication for the $1/N_c$ correction. Let us consider QCD with μ_B and that with μ_I . In the 't Hooft large- N_c limit, gluonic operators trivially agree because the fermions are not dynamical. Let us consider finite- N_c , say $N_c = 3$ and $N_f = 2$. Then the largest correction to the 't Hooft limit comes from one-fermion-loop planar diagrams, which, as we have seen, do not distinguish μ_B and μ_I . Therefore gluonic operators should behave similarly even quantitatively; the difference is at most $(N_f/N_c)^2$ (two-fermion-loop planar diagrams) or $(1/N_c^2) \cdot (N_f/N_c)$ (one-fermion-loop nonplanar diagrams). In particular, the deconfinement temperatures, which are determined from the Polyakov loop, should be close.

3.2.3 Symmetry realization and validity of the equivalence

As we have seen so far, $SU(N_c)$ QCD with μ_B/μ_I , $SO(2N_c)$ YM and $Sp(2N_c)$ YM should be equivalent in the large- N_c limit as long as the projection symmetries are not broken spontaneously. In this section we discuss the phase structures of these theories and clarify when the symmetries are broken. It turns out that $SU(N_c)$ QCD with μ_I , $SO(2N_c)$ YM and $Sp(2N_c)$ YM with μ_B should be equivalent everywhere in T - μ parameter space. The equivalence to $SU(N_c)$ QCD with μ_B is not valid outside the BEC/BCS crossover region of other three theories. (In [1, 10] a possible cure to this is discussed.)

Let us start with $SO(2N_c)$ YM with μ_B . A crucial difference from QCD is that there is no distinction between ‘matter’ and ‘antimatter’ because the gauge group is real. In other words, ‘fundamental’ and ‘anti-fundamental’ representations are equivalent. For this reason, mesons in this theory are not necessarily neutral under $U(1)_B$; one can construct ‘baryonic mesons’ and ‘anti-baryonic mesons’ out of two ‘quarks’ and ‘antiquarks’, respectively. Because they couple to μ_B , as we increase the value of μ_B the lightest ‘baryonic meson’ condenses at some point. Then the $U(1)_B$ symmetry is broken to \mathbb{Z}_2 and the equivalence to QCD with μ_B fails. (Note that we have used \mathbb{Z}_4 subgroup of $U(1)_B$ for the projection.)

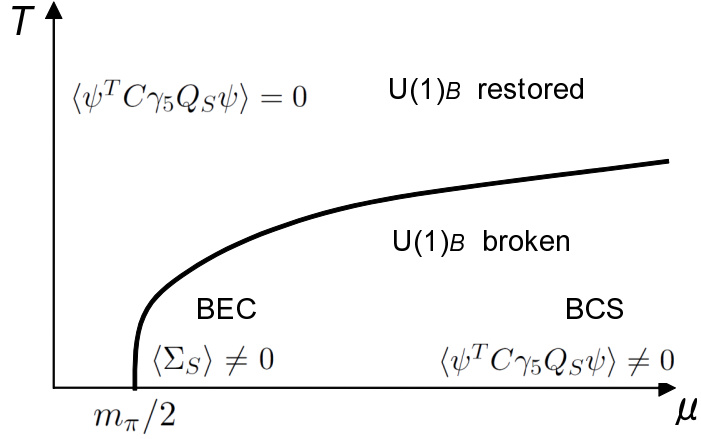


Figure 4: Phase diagram of $SO(2N_c)$ gauge theory at finite μ_B . (Figure taken from [2].)

In order to identify the lightest baryonic meson, let us consider the chiral symmetry breaking in this theory. When $m = \mu_B = 0$, the Lagrangian (2) has the $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_A$ symmetry at the classical level at first sight. However, chiral symmetry of the theory is known to be enhanced to $SU(2N_f)$. Here $U(1)_A$ is explicitly broken by the axial anomaly. One can actually rewrite the fermionic part of the Lagrangian (2) manifestly invariant under $SU(2N_f)$ using the new variable $\Psi = (\psi_L, \sigma_2 \psi_R^*)^T$:

$$\mathcal{L}_f = i\Psi^\dagger \sigma_\mu D_\mu \Psi, \quad (17)$$

where $\sigma_\mu = (\sigma_k, -i)$ with the Pauli matrices σ_k ($k = 1, 2, 3$). The chiral symmetry $SU(2N_f)$ is spontaneously broken down to $SO(2N_f)$ by the formation of the chiral condensate $\langle \bar{\psi}\psi \rangle$, leading to the $2N_f^2 + N_f - 1$ Nambu-Goldstone bosons living on the coset space $SU(2N_f)/SO(2N_f)$: neutral pions $\Pi_a = \bar{\psi}\gamma_5 P_a \psi$, ‘baryonic pions’ (or simply ‘diquark’) $\Sigma_S = \psi^T C \gamma_5 Q_S \psi$ and ‘anti-baryonic pions’ $\Sigma_S^\dagger = \psi^\dagger C \gamma_5 Q_S \psi^*$. It is easy to see the fate of these bosons under the orbifold projection. The projection to QCD with μ_B maps neutral pions to pions in QCD, and baryonic and anti-baryonic pions are projected away. On the other hand, the projection to QCD with μ_I sends neutral/baryonic/anti-baryonic pions to π^0 , π^+ and π^- , respectively. Therefore the (baryonic) pions in $SO(2N_c)$ YM and those in QCD have the same mass m_π . In the same way as the π^+ condensation in QCD with μ_I at $\mu = m_\pi/2$, baryonic pions condense at $\mu = m_\pi/2$ (Fig. 4 and Fig. 5).

At sufficiently large μ , the one-gluon exchange interaction in the $\psi\psi$ -channel is attractive in the color symmetric channel, leading to the condensation of the diquark pairing $\langle \psi^T C \gamma_5 Q_S \psi \rangle$. This diquark condensate does not break $SO(2N_c)$ symmetry.

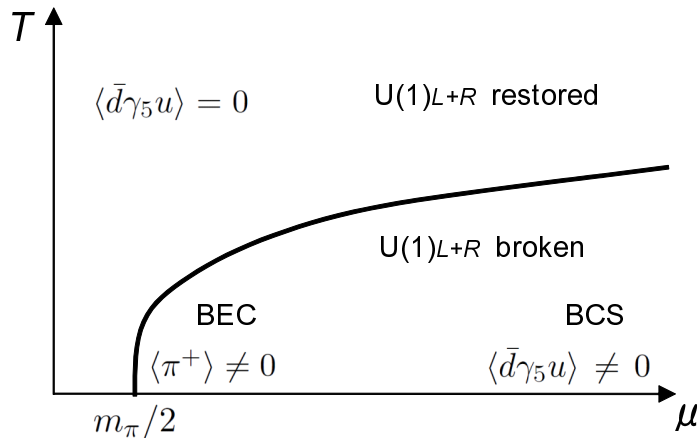


Figure 5: Phase diagram of QCD at finite $\mu_I = 2\mu$. (Figure taken from [2].)

This BCS pairing has the same quantum numbers and breaks the same $U(1)_B$ symmetry as the BEC $\langle \Sigma_S \rangle$ at small μ_B , and there should be no phase transition for $\mu > m_\pi/2$ along μ axis. The phase diagram of this theory is similar to that of QCD at finite μ_I , as shown in Fig. 4 and Fig. 5. This is because the condensates in two theories are related each other through the orbifold projection, and furthermore, the condensation does not break the flavor symmetry used for the projection.

In the same manner, $Sp(2N_c)$ YM and QCD with μ_I are equivalent everywhere in T - μ plane; see Fig. 6. (For further details, see [2].)

QCD with μ_B behaves rather differently, because μ_B does not couple to mesons. This does not lead to a contradiction, however. Because baryons are much heavier than pions, phenomena characteristic to QCD with μ_B (e.g. formation of hadronic matter) takes place after the equivalence is gone due to the $U(1)_B$ breakdowns in $SO(2N_c)$ and $Sp(2N_c)$ Yang-Mills.

At high temperature and small μ_B , the symmetry is not broken and hence the equivalence works. This region is relevant for the heavy ion collision experiments. Note that this region, where the symmetry is intact, is exactly where the reweighting method works in principle (but of course difficult in practice). In such a region, one should not spend too much computational resource for the reweighting; just by ignoring the phase one can obtain reasonable results.

4 Conclusion and outlook

We have pointed out that QCD and various QCD-like theories with chemical potentials are equivalent at large- N_c through the orbifold equivalence, at least to all order in perturbation theory. QCD with the isospin chemical potential and

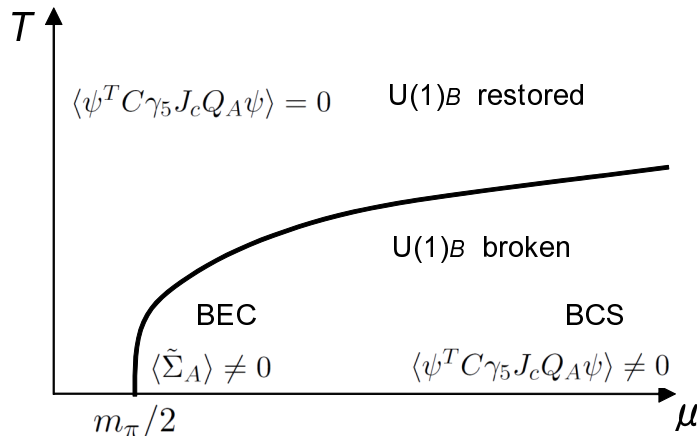


Figure 6: Phase diagram of $Sp(2N_c)$ gauge theory at finite μ_B . $\tilde{\Sigma}_A = \psi^T C \gamma_5 J_c Q_A \psi$, where Q_A ($A = 1, 2, \dots, N_f(N_f - 1)/2$) are antisymmetric $N_f \times N_f$ matrices in the flavor space. (Figure taken from [2].)

$SO(2N_c)/Sp(2N_c)$ Yang-Mills with the baryon chemical potential are equivalent everywhere in the T - μ plane, and furthermore, they are equivalent to QCD with the baryon chemical potential outside the BEC-BCS crossover region.

Our result has immediate implication for the study of the chiral and deconfinement transitions in high- T , small- μ region. In this region it is reasonable to assume the $1/N_c$ correction is not very large (for example, as we have seen in § 3.2.2, the leading corrections to the large- N_c limit of the gluonic operators agree), and hence we can expect that the Monte-Carlo results of the QCD with isospin chemical potential describe the QCD with the baryon chemical potential with rather good accuracy. Furthermore, by using the $SO(2N_c)$ theory, one can study three-flavor theory without suffering from the sign problem. Similar study e.g. phase quenched simulation of $SU(3)$ 3-flavor QCD has been performed [4] and the results suggest that the QCD critical point does not exist. It is very important to study these sign-free theories numerically, further in detail, in order to find (or exclude) the QCD critical point.

I would like to thank Aleksey Cherman, Carlos Hoyos, Andreas Karch, Daniel Robles-Llana, Laurence Yaffe and Naoki Yamamoto for fruitful collaborations which this talk is based on. I also thank Brian Tiburzi for stimulating discussions and comments. My work is supported by Japan Society for the Promotion of Science Postdoctoral Fellowships for Research Abroad.

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