Confinement by dual superconductivity: an update.

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1 Introduction

An appealing mechanism for color confinement is dual superconductivity of QCD vacuum[1][2][3]. In the confined phase magnetic charges (Monopoles) condense in the vacuum, in the same way as Cooper pairs condense in an ordinary superconductor. The chromo-electric field acting between a $\overline{q}q$ pair is then constrained into Abrikosov flux tubes, and the energy is proportional to the distance, which means confinement.

Dual superconductivity of QCD vacuum has been explored by numerical simulations on the lattice, with two different strategies:

1) Observe monopoles in field configurations produced by numerical simulations on the lattice. Assuming that monopoles are the dominant degrees of freedom (monopole dominance[4][5]) try to extract from the observations an effective action for them[6][7], and read condensation from it.

$$L_{eff} = \frac{m^2}{2}\Phi^2 - \frac{\lambda}{4}\Phi^4 + ... \tag{1}$$

Positive m^2 signals dual superconductivity.

2)Define an order parameter $\langle\mu\rangle$, the vacuum expectation value of a gauge invariant operator μ carrying non zero magnetic charge. $\langle\mu\rangle \neq 0$ signals dual superconductivity, expected in the confined phase. In the deconfined phase $\langle\mu\rangle = 0$[8]

We shall review the status and recent progress of both approaches.

[1]Presented the talk

2 Approach 1).

Monopoles are U(1) objects[9]: they belong to a $U(1)$ subgroup of the gauge group, usually known as abelian projection. For the 'tHooft Polyakov monopole this $U(1)$ is identified as the invariance group of the $v.e.v.$ $\langle\Phi\rangle$ of the Higgs field[10]. In QCD there is no Higgs field . The simplifying assumption of Ref.[3] that physics is independent on the choice of the abelian projection is openly contradicted by the fact that the number and the locations of monopoles in lattice configurations strongly depend on it. In Ref.s[11][12] it has been shown that the existence of a monopole in a site of a configuration is a gauge invariant concept. It is not true that all abelian projections are equivalent, since each monopole identifies a privileged direction in color space, that of the magnetic field at large distances, and the corresponding generator is diagonal in the Maximal Abelian Gauge (MAG). Detection of monopoles on the lattice by the usual method is gauge dependent. In the MAG all monopoles are detected.

This legitimates the studies performed in the MAG, e.g. in Ref.s[6][7]. The fact that the results were not convincing, in the sense that many terms were needed in the expansion Eq.(1) and the sign of m^2 was not stable, may be due to the assumption that monopole dominance is valid at all values of the coupling. Monopoles are instead expected to dominate only at the deconfining transition, and there the analysis should maybe repeated and improved.

3 Approach 2)

The prototype definition of the order parameter $\langle\mu\rangle$ refers to the simplest case of continuum $U(1)$ gauge theory[13].

$$\mu(\vec{x},t) = \exp\left(i\int d^3y \vec{E}_\perp(\vec{y},t)\cdot\vec{A}^0_\perp(\vec{y}-\vec{x})\right) \tag{2}$$

Here the sub-fix $_\perp$ means transverse, and $\vec{A}^0_\perp(\vec{y}-\vec{x})$ is the vector potential generated at the site $\vec{y}$ by a monopole sitting at $\vec{x}$, in the transverse gauge $\vec{\nabla}\vec{A}^0_\perp = 0$. $\vec{E}_\perp(\vec{x},t)$ is the conjugate momentum to the transverse field $\vec{A}_\perp(\vec{x},t)$ independent on how the theory is quantized,and therefore

$$\mu(\vec{x},t)|\vec{A}_\perp(\vec{z},t)\rangle = |\vec{A}_\perp(\vec{z},t)+\vec{A}^0_\perp(\vec{z}-\vec{x})\rangle \tag{3}$$

Eq.(3) is the field theory analog of the elementary translation

$$\exp(ipa)|x\rangle = |x+a\rangle \tag{4}$$

In computing $\langle\mu\rangle$ by the path-integral formalism, one gets

$$\langle\mu(\beta)\rangle = \frac{Z(\beta(S+\Delta S))}{Z(\beta S)} \tag{5}$$

$$\Delta S = \int d^3y \vec{E}_\perp(\vec{y},t) \cdot g\vec{A}^0_\perp(\vec{y}-\vec{x}) \tag{6}$$

$\beta \equiv \frac{1}{g^2}$. The fields in the path integral are the canonical fields times g: in the monopole operator there is a $\frac{1}{g}$ factor coming from the magnetic charge of the monopole, which obeys Dirac quantization condition and another one coming from the fact that the electric field in Eq.(2) is the canonical one.

It proves convenient to make use of the quantity

$$\rho(\beta) \equiv \frac{\partial \ln\langle\mu(\beta)\rangle}{\partial\beta} = \langle S\rangle_S - \langle S+\Delta S\rangle_{S+\Delta S} \tag{7}$$

Since $\langle\mu\rangle_{\beta=0} = 1$, $\langle\mu\rangle$ is computed in terms of ρ as

$$\langle\mu(\beta)\rangle = \exp\left(\int_0^\beta \rho(\beta')d\beta'\right) \tag{8}$$

The construction can be translated to the lattice formulation as follows [13]

$$S = \Sigma_{n,\mu<\nu} Re(1-\Pi_{\mu\nu}(n)) \tag{9}$$

$$S+\Delta S = \Sigma_{n,\mu<\nu} Re(1-\Pi'_{\mu\nu}(n)) \tag{10}$$

$\Pi_{\mu\nu}(n)$ is the parallel transport along the elementary square of the lattice (plaquette) at the site n in the plane $\mu\nu$,

$\Pi'_{0,n}(t,\vec{y}) = \Pi_{0,n}(t,\vec{y})\exp\left(-iA^0_n(\vec{x}-\vec{y})\right)$ $n_0 = t$ $\vec{n} = \vec{y}$
$\Pi'_{\mu\nu}(n) = \Pi_{\mu\nu}(n))$ otherwise.

In terms of states

$$Z(\beta(S+\Delta S)) = \langle\mu_{out}|0_{in}\rangle$$
$$Z(\beta S) = \langle 0_{out}|0_{in}\rangle \tag{11}$$

As a consequence of commutativity one can prove that [See Section 4]

$$\langle\mu_{out}|\mu_{in}\rangle = \langle 0_{out}|0_{in}\rangle \tag{12}$$

Up to irrelevant phase factors $\langle\mu\rangle$ is a probability amplitude, as should be any good order parameter:

$$\langle\mu\rangle = \frac{\langle\mu_{out}|0_{in}\rangle}{(\langle 0_{out}|0_{in}\rangle)^{\frac{1}{2}}(\langle\mu_{out}|\mu_{in}\rangle)^{\frac{1}{2}}} \tag{13}$$

An order parameter for the same system had been defined in Ref.[14] for $U(1)$ lattice theory with Villain action, via its correlation functions, and proved to describe the

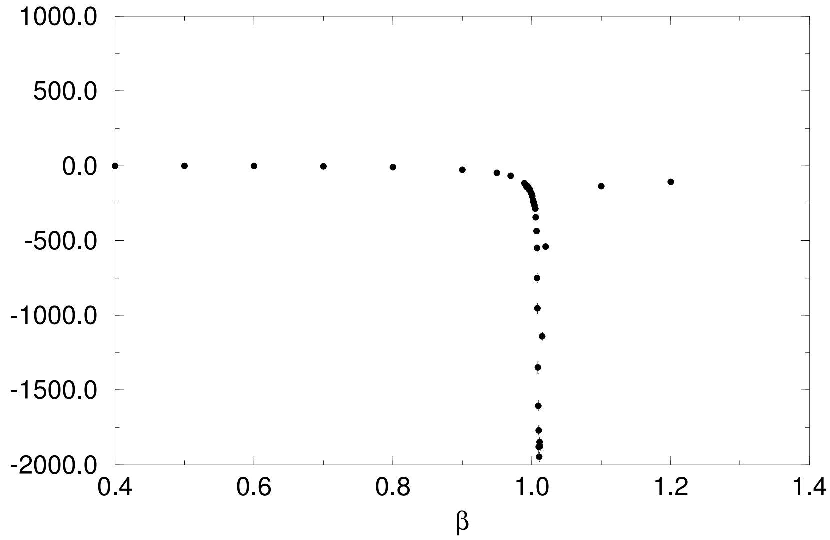

Figure 1: ρ for $U(1)$ lattice gauge theory.

observed deconfining transition as a change from superconductor to normal. It was also proved that it was the VEV of a magnetically charged, Dirac like gauge invariant operator. The result was then extended to the usual Wilson form of the action in Ref.[15]. In Ref.[13] it was proved that the lattice version of the order parameter defined by Eq.s(5)(10) is equal to that of Ref.[14], is an alternative form of it. It has the advantage of being more manageable and easily extendable to other models.

Indeed a numerical test was made [13] by measuring ρ on the lattice. From Eq.(8) one expects the following behaviors

i) for $\beta < \beta_c$ (the transition coupling), ρ has a finite limit as the volume $L^3 \to \infty$, meaning, by Eq.(8) that $\langle\mu\rangle \neq 0$

ii) for $\beta > \beta_c$ $\rho \propto c_1 - |c_2|L$, meaning $\langle\mu\rangle \to 0$ as $L^3 \to \infty$

iii) for $\beta \approx \beta_c$ ρ has a sharp negative peak, corresponding to the rapid decrease to zero of μ.

Near the critical point the following scaling law is expected to hold

$$\rho \approx L_s^{\frac{1}{\nu}} \Phi(\tau L_s^{\frac{1}{\nu}}) \qquad (14)$$

$\tau \equiv (1 - \frac{T}{T_c})$ and ν is the critical index of the correlation length.

All the expectations were met by the numerical simulations, the transition is compatible with first order, and the quality of data is shown in Fig.1[13].

The construction was heuristically extended to non abelian gauge theories, specifically $SU(2)$ in Ref. [16] and $SU(3)$ without quarks [17] and with quarks [18]. The idea was to fix an abelian projection and try to add an abelian monopole to the corresponding $U(1)$ component of the field, in a similar way as was done in the abelian case. Again expressions like Eq.'s (9)(5) and Eq.(10) result, where now for gauge group $SU(N)$ $\beta = \frac{2N}{g^2}$ and

$$\begin{aligned} \Pi_{\mu\nu}(n) &= Tr\left[U_\mu(n)U_\nu(n+\hat{\mu})U^\dagger_\mu(n+\hat{\nu})U^\dagger(n)\right] \quad (15)\\ \Pi'_{i0}(t,\vec{n}) &= Tr[U_i(t,\vec{n})U_0(t,\vec{n}+\hat{i})U^\dagger_i(t+1,\vec{n})M_i(\vec{n}+\hat{i}) \\ &\quad U^\dagger(t,\vec{n})] \\ M_i(\vec{n}+\hat{i}) &= \exp\left(iA^0_i(\vec{x}-\vec{n})\hat{\Phi}\right) \quad (16) \end{aligned}$$

Here $\hat{\Phi}$ is the generator identifying the $U(1)$ of the abelian projection.

On the lattice the monopole is defined in terms of the component of the gauge field along the $U(1)$ generator corresponding to the abelian projection, and due to non commutativity these components are defined up to terms $O(a^2)$ coming from the Baker-Haussdorf composition of the product of parallel transports. a is the lattice spacing. The above construction creates a monopole modulo terms of $O(a^2)$ [16].

The numerical determinations of ρ of Ref.s [16] ,[17], [18] for the lattices and within the statistics available at the time, gave results consistent with the expectations, namely

i) A volume independent ρ at $\beta < \beta_c$, meaning by Eq.(8) $\mu \neq 0$

ii) A negative divergence with the spatial size L of the lattice of the form $\rho \propto c_1 - |c_2|L$, implying $\langle\mu\rangle \to_{L\to\infty} 0$.

iii) Sharp negative peak at β_c. Scaling in the vicinity of the transition compatible with $3d$ Ising for $SU(2)$ and with first order for $SU(3)$.

Confinement in G_2 gauge theory is a specially interesting case. The group has no center, and therefore no central vortices. If central vortices were the relevant excitations for confinement the system should not confine[19]. Proving that there is confinement by use of the order parameter $\langle\mu\rangle$ would then be a strong indication in favor of dual superconductivity as a mechanism for confinement[23]. On this argument see also Ref.s[21] [22]. In this case the procedure described above gave a qualitatively positive result for ρ , but superimposed to a negative background diverging at large sizes L of the lattice also in the low β region. This means that $\langle\mu\rangle$ is zero everywhere and is not an order parameter. We then went back to the analysis of $SU(2)$ [16] and $SU(3)$ [17], redoing it with larger lattices and statistics [See Section 4 below] and we found the same kind of disease [23] . Similar results were published in Ref.[24].

After many attempts we now understand the origin of the problem, which comes from a tacit assumption made in the construction of the order parameter which proves to be not valid in the non abelian case, and we have solved it by defining an improved order parameter which is free of it, and still is the VEV of a magnetically charged operator.

4 The improved order parameter

As explained in the previous sections the order parameter $\langle\mu\rangle$ is given by Eq.(5) or, in terms of states by the ratio

$$\langle\mu\rangle = \frac{\langle\mu_{out}|0_{in}\rangle}{\langle 0_{out}|0_{in}\rangle} \tag{17}$$

The correct definition should be that of Eq.(13). In terms of Feynman integrals

$$\langle\mu_{out}|\mu_{in}\rangle = Z(\beta(S+\overline{\Delta}S)) \tag{18}$$

where [25]

$$S+\overline{\Delta}S = \Sigma_{n,\mu<\nu} Re(1-\overline{\Pi}_{\mu\nu}(n)) \tag{19}$$

and $\overline{\Pi}_{\mu\nu}(n) = \Pi_{\mu\nu}(n)$ for all n, μ, ν except fot the $0i$ plaquettes at the time t when the monopole is created, which are replaced by

$$\begin{aligned}\overline{\Pi}_{0i}(t,\vec{n}) &= \frac{1}{\mathrm{Tr}[I]}\mathrm{Tr}\big[U_i(t,\vec{n})M_i^\dagger(\vec{n}+\hat{\imath}) \times \\ &\quad U_0(t,\vec{n}+\hat{\imath})M_i(\vec{n}+\hat{\imath})U_i^\dagger(t+1,\vec{n})U_0^\dagger(t,\vec{n})\big]\end{aligned} \tag{20}$$

with $M_i(\vec{n})$ defined by Eq.(16). Eq.(20) is valid for a generic gauge group. In the case of $U(1)$ everything commutes and $M_i^\dagger(\vec{n}+\hat{\imath})$ which is unitary, cancels with $M_i(\vec{n}+\hat{\imath})$, so that $S+\overline{\Delta}S = S$ and $\langle 0_{out}|0_{in}\rangle = \langle\mu_{out}|\mu_{in}\rangle$ as already anticipated in Eq.(12). For $U(1)$ the discretization preserves the unitarity of the continuum operator of Eq.(2).

In the general case the correct order parameter is,up to phase factors,

$$\langle\overline{\mu}\rangle = \frac{Z(\beta(S+\Delta S))}{Z(\beta S)^{\frac{1}{2}} Z(\beta(S+\overline{\Delta}S))^{\frac{1}{2}}} \tag{21}$$

As long as $\langle\mu_{out}|\mu_{in}\rangle = Z(\beta(S+\overline{\Delta}S))$ is different from zero in the thermodynamical limit the two versions of the order parameter $\langle\overline{\mu}\rangle$ of Eq.(4) and $\langle\mu\rangle$ of Eq.(17) are equivalent, since each of them is zero or non-zero at the same time. This was tacitly assumed to be the case in all the previous works starting from Ref.[16]. What happens in fact is that the lattice implementation of the operator Eq.(2) is not unitary, as was in the abelian case, not only, but the amplitude $\langle\mu_{out}|\mu_{in}\rangle$ tends to zero in the thermodynamical limit, and so does the amplitude $\langle\mu_{out}|0_{in}\rangle$ whilst of course $\langle 0_{out}|0_{in}\rangle$ stays finite. This is the reason why $\langle\mu\rangle$ of Eq.(17) vanishes as $L\to\infty$ and is not a good order parameter.

However what happens is that the ratio $\frac{Z(\beta(S+\Delta S))}{Z(\beta(S+\overline{\Delta}S))^{\frac{1}{2}}}$ stays finite: the state with a monopole added to the vacuum is well defined as a direction in Hilbert space in the thermodynamical limit.

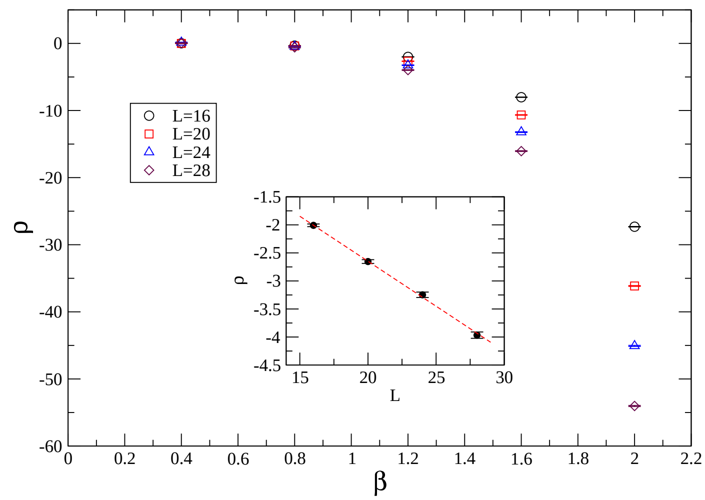

Figure 2: ρ for the $SU(2)$ lattice gauge theory at low β calculated for the Wu-Yang monopole of charge 4. The inset shows the linear divergence in the lattice size for $\beta = 1.2$.

In analogy with Eq(7) we define a quantity $\overline{\rho}$ and make use of the definition Eq.(4)

$$\overline{\rho}(\beta) \equiv \frac{\partial \ln\langle\overline{\mu}(\beta)\rangle}{\partial\beta} = \frac{1}{2}\langle S\rangle_S + \frac{1}{2}\langle S + \overline{\Delta}S\rangle_{S+\overline{\Delta}S} - \langle S + \Delta S\rangle_{S+\Delta S} \tag{22}$$

$\langle\overline{\mu}(\beta)\rangle$ is the probability amplitude to find in the vacuum a state with one monopole added, and is therefore a legitimate candidate for an order parameter. The analog of Eq.(8) reads

$$\langle\overline{\mu}(\beta)\rangle = \exp\left(\int_0^\beta \overline{\rho}(\beta')d\beta'\right) \tag{23}$$

and again dual superconductivity corresponds to finite $\overline{\rho}$ at low β, negative divergent at large β $\overline{\rho} \propto c_1 - |c_2|L$ as the spatial size L goes large and scaling of the form $\rho \approx L_s^{\frac{1}{\nu}}\Phi(\tau L_s^{\frac{1}{\nu}})$ near the transition with the appropriate critical index ν.

A strong coupling expansion of ρ corresponding to the old order parameter shows that it is negatively divergent with L at large volumes, even at low β's consistently with the numerical determinations, as discussed above. The same analysis on $\overline{\rho}$ instead gives a convergent result at large L. We shall not report the details of this

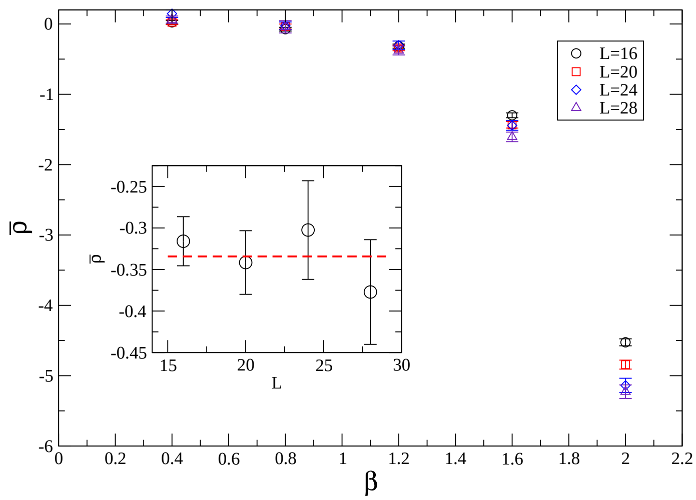

Figure 3: $\overline{\rho}$ for the $SU(2)$ lattice gauge theory at low β calculated for the Wu-Yang monopole of charge 4. The inset shows the dependence on the lattice size for $\beta = 1.2$ together with the best fit to a constant.

computation which are contained, together with a comprehensive analysis of the new order parameter in Ref.[25].

Here we shall instead concentrate on the numerical results for $SU(2)$ gauge group. Figure 2 shows the volume dependence of ρ, with lattice sizes and statistics much larger than it was in Ref.[16]: ρ is proportional to L with a negative coefficient, which makes $\langle\mu\rangle$ zero in the thermodynamical limit. Notice that, with larger error bars, this behavior was missed.

Figure 3 shows the low β behavior of $\overline{\rho}$: here there is no sign of divergence. Notice that the vertical scale in the two figures differs by an order of magnitude. The size of the errors is about the same in the two cases.

The overall behavior of $\overline{\rho}$ is shown in Figure 4, the scaling in the vicinity of the transition in Figure 5.

The behavior of $\overline{\rho}$ at large β (deconfined phase) is displayed in Figure 6, showing that, in the thermodynamical limit $\overline{\rho} = 0$

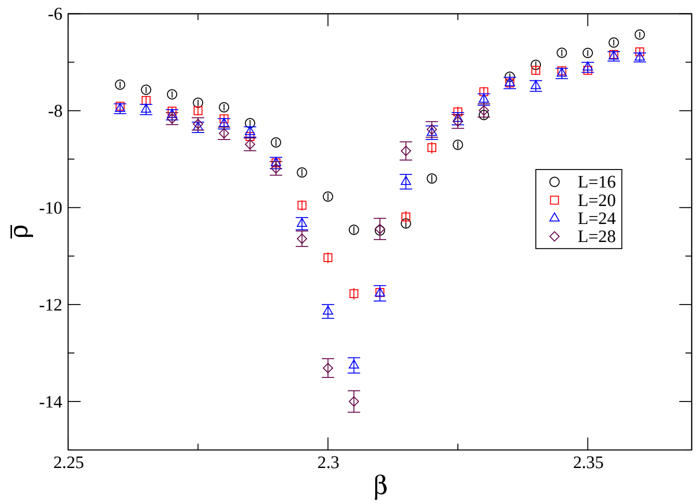

Figure 4: $\overline{\rho}$ for the $SU(2)$ lattice gauge theory near the deconfinement transition ($N_t = 4$) calculated for the Wu-Yang monopole of charge 4. When the error bars are not explicitly shown they are smaller than the symbols.

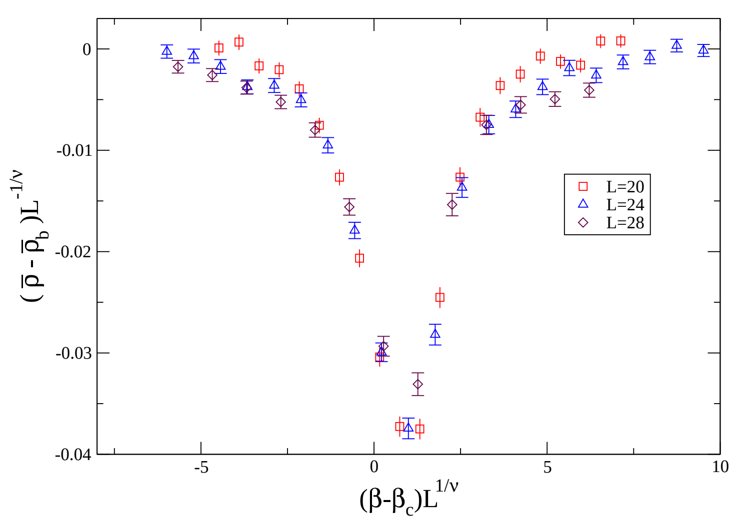

Figure 5: Scaling of the critical part of $\overline{\rho}$ near the $N_t = 4$ deconfinement transition.

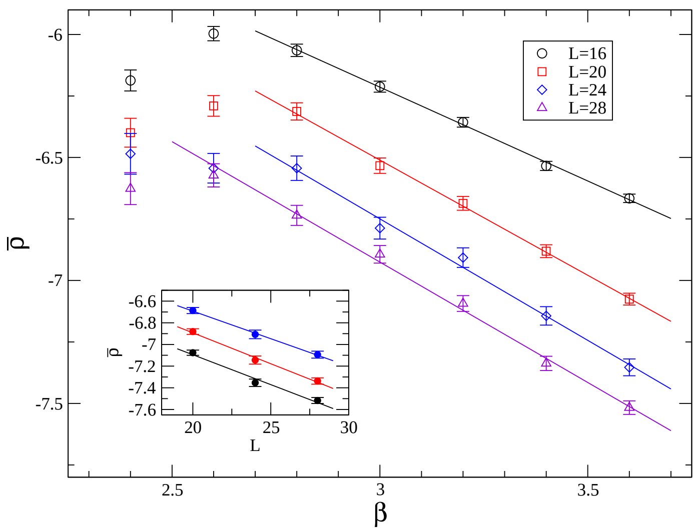

Figure 6: $\overline{\rho}$ for the $SU(2)$ lattice gauge theory for $\beta > \beta_c$ ($N_t = 4$) calculated for the Wu-Yang monopole of charge 4. In the inset the scaling with the lattice size at $\beta = 3.2, 3.4, 3.6$ is shown together with linear fits.

Finally Figure 7 shows $\overline{\rho}$ on a lattice with $N_t = 6$. The peak is properly shifted to the new location of the transition, as identified e.g. by the susceptibility of the Polyakov loop. The peak is not a lattice artifact but is physical.

5 Conclusions and outlook.

We have cured a drawback of the order parameter for dual superconductivity in QCD, which was due to a tacit assumption on the normalization of the state obtained by applying to the vacuum the operator which creates a monopole. The new order parameter is the correct probability amplitude for the transition from the vacuum to the state with an additional monopole. Numerical results give evidence that dual superconductivity is the mechanism of confinement.

We will now play the same game for other gauge systems, in particular G_2 gauge theory. From the strong coupling analysis it appears that also the order parameter of non-gauge systems like $3d$ XY model in which a phase transition exists related

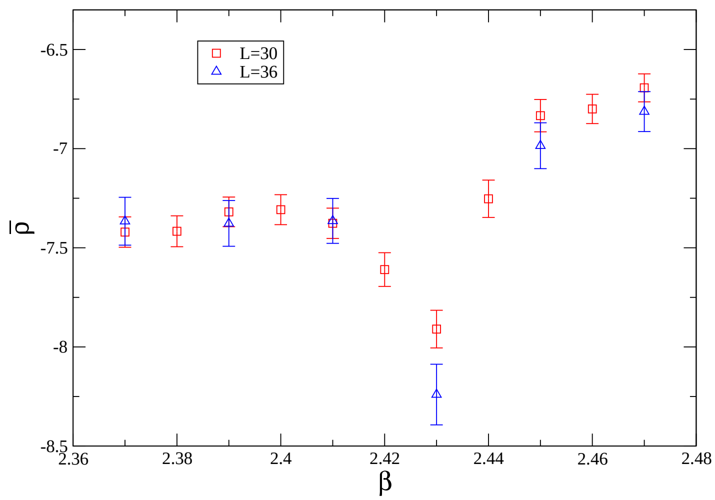

Figure 7: $\overline{\rho}$ for the $SU(2)$ lattice gauge theory near the deconfinement transition ($N_t = 6$) calculated for the Wu-Yang monopole of charge 4.

to condensation of vortices, as defined in analogy with μ[26] can suffer of the same disease. We will use them as a test for the new order parameter.

The numerical simulations have been performed on GRID resources provided by INFN and in particular on the CSNA Cluster located in Pisa.

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