QCD and String Theory[1]

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Abstract

We give a very brief summary of the basic idea and some of the interesting results in holographic QCD. Using the gauge/string duality, we obtain a low energy effective theory of hadrons based on string theory. Mesons and baryons are obtained as open strings and D-branes, respectively, and a lot of properties of hadrons can be extracted using this new description.

1 Claim

The main claim in this talk is that string theory can be a theory of hadrons. Here, we are not saying QCD is wrong. Our claim is that QCD can have a dual description based on string theory, in which the string scale is around 1 GeV. This duality is one of the examples of gauge/string duality, which has been discussed extensively since the discovery of the AdS/CFT correspondence.[2] The string theory description is defined in ten dimensional curved space-time. For this reason, it is often called a holographic description of QCD.

Remarkably, hadrons in string theory are described without using quarks and gluons. As explained below, mesons and baryons correspond to open strings and D-branes wrapped on a non-trivial cycle in the background, respectively. Note that strings are of course the fundamental degrees of freedom in string theory, while D-branes are kind of solitons in string theory. Therefore, the way baryons are described is exactly analogous to what Skyrme proposed in the early 60's [7], in which baryons are described as solitons (Skyrmions) in a theory of mesons.

Our model has two parameters denoted as $M_{\rm KK}$ and λ. $M_{\rm KK}$ corresponds to a cut-off scale and λ is the coupling constant ('t Hooft coupling) at the scale of $M_{\rm KK}$. We know that QCD is better described at high energy ($\lambda \ll 1$) because of

[1]Main part of this talk is based on the works done in collaboration with T. Sakai, H. Hata, S. Yamato, K. Hashimoto and T. Imoto. [1, 2, 3, 4, 5]

[2]See, e.g., [6] for a review.

the asymptotic freedom. Although it is possible to analyze the low energy properties of hadrons based on QCD by using, e.g., lattice QCD, our intuition of quarks and gluons as free particles does not work and the calculations are more complicated and challenging at low energy. On the other hand, the string theory description becomes a good description at low energy ($\lambda \gg 1$). The low energy phenomena such as the quark confinement and dynamical chiral symmetry breaking are almost trivial and the low energy properties of hadrons can be analyzed quite easily in the string theory description. The price we have to pay is that the background used in the string theory description will become highly curved when λ is small and it is very difficult to recover the results in perturbative QCD with our current technology in string theory. So, we will focus on the low energy properties of hadrons in this talk.

The plan of the talk is as follows. In section 2, we give a tiny review of D-branes and gauge/string duality. If you are familiar with these terminologies you can skip this section. Our model is explained in section 3. If you just want to learn what we can obtain, you can go directly to section 4, in which our main results are summarized.

2 Gauge/string duality

The gauge/string duality is a duality between gauge theory and string theory. Although this is still a conjecture, there are good reasons to believe that this kind of duality really exists.

To explain the basic idea, we introduce an object called D-brane in string theory. A Dp-brane is defined as a $(p+1)$ dimensional object on which end points of open strings can be attached. It can be shown that gauge particles living on the Dp-brane are obtained as the massless modes of the open strings attached on it. Therefore, if we consider a flat Dp-brane, a $(p+1)$ dimensional gauge theory is realized on it. For the cases with N_c Dp-branes located at the same place, the gauge group is enhanced to $U(N_c)$.

The basic idea of gauge/string duality is as follows. Consider a D-brane system. As we have just explained, the low energy effective theory of open strings attached on the D-branes is a gauge theory. On the other hand, D-branes couple with gravity and the space-time around them will be curved. In fact, in some parameter region (large N_c and large λ in our case), the D-branes are better described as the corresponding curved background, which is analogous to the fact that a heavy point-like object in general relativity is described by Schwarzschild background. Therefore, there are two descriptions of the D-branes. One of them is a gauge theory and the other one is string theory in a curved background. Since they are describing the same object, these two descriptions should be equivalent.

The typical example of the gauge/string duality is the duality between 4 dimensional $\mathcal{N} = 4$ super Yang-Mills theory and type IIB string theory in $\mathrm{AdS}_5 \times S^5$

background. Since $\mathcal{N} = 4$ super Yang-Mills theory is a conformal field theory (CFT), and the background in the holographic dual description contains AdS factor, this type of duality is often called "AdS/CFT correspondence". Though the direct proof of the equivalence between these two descriptions is still an open problem, there are a lot of evidence supporting this duality and it is widely believed that they are physically equivalent. Our aim is to apply this idea to more realistic gauge theory (QCD) and see what we can learn from string theory.

3 Holographic QCD

A holographic description of Yang-Mills theory (without quarks) was proposed by Witten in [8]. He proposed to consider N_c D4-branes extended along $x^{0\sim 4}$ directions, where the x^4 direction is compactified to S^1 with an anti-periodic boundary condition for all the fermions in this system. The supersymmetry is completely broken by this boundary condition and it is possible to show that the low energy effective theory of the open strings attached on the D4-branes is $U(N_c)$ Yang-Mills theory. To add quarks to this system, we add N_f D8-$\overline{\text{D8}}$ pairs extended along $x^{5\sim 9}$ directions.[1] Then, we can show that the massless degrees of freedom obtained from the open strings stretched between D4-branes and D8(or $\overline{\text{D8}}$)-branes are N_f massless fermions in fundamental representation of the $U(N_c)$ gauge group. As a result, the system flows to $U(N_c)$ QCD with N_f massless quarks at low energy.

There are two parameters in the system. One of them is the radius of the S^1, denoted as M_{KK}^{-1}, and the other parameter is the 't Hooft coupling $\lambda = g_{\text{YM}}^2 N_c$. Note that there are Kaluza-Klein (KK) modes with masses of order M_{KK}, which cannot be interpreted as particles in QCD. Therefore, M_{KK} can be thought of as a cut-off scale below which the theory becomes QCD. Note, however, that we expect that the effect of this "cut-off" is much milder than that of lattice gauge theory, because we are not really cutting off the energy higher than M_{KK} and the Lorentz symmetry is not broken.

To obtain a holographic description, we replace the D4-branes with the corresponding curved background found in [8]. Then, the radius of the S^1 becomes r dependent, where r is the radial coordinate of the $x^{5\sim 9}$ plane, and we obtain a cigar-like geometry as depicted in Figure 1. The topology of the space-time is $\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$,

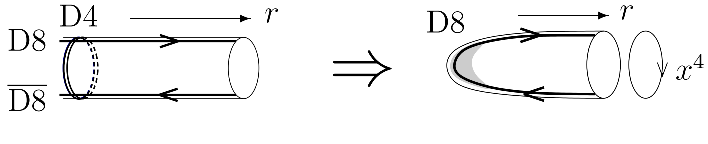

Figure 1: Replacing D4-brane with the corresponding curved background.

where $\mathbf{R}^{1,3}$ and $\mathbf{R}^2$ factors are parameterized by $x^{0\sim3}$ and (r, x^4), respectively, and S^4 corresponds to the angular directions of the $x^{5\sim9}$ plane. The D8-branes are treated as probe branes and we neglect their backreaction to the background. This approximation is analogous to the quench approximation that is widely used in lattice QCD, and it can be justified when $N_c \gg N_f$. Note that the D8-$\overline{\text{D8}}$ pairs are now smoothly connected as in the right side of Figure 1. This phenomenon corresponds to the chiral symmetry breaking in QCD. In this description, it can be shown that the string coupling is proportional to $1/N_c$ and the string length is proportional to $\lambda^{-1/2}$. Therefore, this description can be trusted when N_c and λ are large enough. Since QCD is an asymptotically free theory, the latter condition ($\lambda \gg 1$) implies that this description works better at low energy.

4 Surprise

Using the holographic description explained in the previous section, a lot of properties of hadrons can be extracted. Here, we just show some of the interesting results without trying to explain the details. The readers should keep in mind that the approximations we made may not be accurate enough. In most of our calculations, we have only estimated the leading terms in the $1/N_c$ and $1/\lambda$ expansions. Since we have not estimated the $1/N_c$ and $1/\lambda$, corrections, we cannot tell how much we can trust our results. In our quantitative results, we set M_{KK} to be around 1 GeV, which is not high enough to justify the decoupling of unwanted KK modes. In addition, we have neglected the effect of the quark masses. Therefore, we should not expect too much. If the order of magnitudes in our calculations turns out to be comparable to the experimental results, we should consider it as a success. But, please don't be too pessimistic. As we will show in the following, it turns out that the results are much better than what one would naively expect.

First of all, hadrons are nicely realized in our framework. As depicted in Figure 2, glueballs, mesons, and baryons in QCD are realized as closed strings, open strings attached on the D8-branes, and D4-branes wrapped on the S^4, respectively. Since the

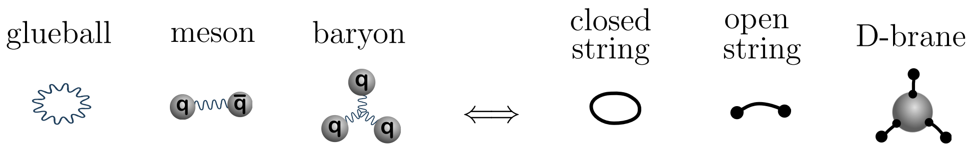

Figure 2: Hadrons in the model

end points of the open strings are attached on the D8-branes, each open string carries two flavor indices, which is consistent with our interpretation as a meson. Although we will not explain the details here, it is known that the D4-brane wrapped on the S^4

has to have N_c open strings attached on it because of the RR flux in the background, [9, 10] and hence it is natural to interpret it as a bound state of N_c quarks.

The effective theory of mesons can be obtained from the effective theory of open strings attached on the D8-branes. Using the standard D8-brane effective action, and integrating over the S^4 direction, we obtain the following 5 dimensional $U(N_f)$ Yang-Mills-Chern-Simons theory:

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$$S_{\text{YM}} = \kappa \int d^4x dz \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) , \quad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_{\text{5dim}} \omega_5(A) , \tag{1}$$

where $h(z) = (1+z^2)^{-1/3}$ and $k(z) = 1 + z^2$. Here $\mu, \nu = 0 \sim 3$ correspond to the coordinates of our 4 dimensional world, and $-\infty < z < \infty$ is the coordinate parameterizing the D8-brane world-volume in the right side of Figure 1. The coefficient κ in S_{YM} is a constant proportional to λN_c and $\omega_5(A)$ in S_{CS} is the Chern-Simons 5 form. The 5 dimensional gauge field can be decomposed as

$$A_\mu(x^\mu, z) = \sum_{n \geq 1} B_\mu^{(n)}(x^\mu) \psi_n(z) , \quad A_z(x^\mu, z) = \sum_{n \geq 0} \varphi^{(n)}(x^\mu) \phi_n(z) , \tag{2}$$

using complete sets $\{\psi_n\}_{n\geq 1}$ and $\{\phi_n\}_{n\geq 0}$ of functions of z. We can show that $B_\mu^{(n)}$ with odd (even) n correspond to vector (axial-vector) fields, and $\varphi^{(0)}$ is a massless pseudo scalar field. $\varphi^{(n)}$ with $n \geq 1$ are absorbed by $B_\mu^{(n)}$ to make them massive. Now, we interpret $B_\mu^{(1)}$, $B_\mu^{(2)}$, $B_\mu^{(3)}$, etc., as the ρ-meson, a_1-meson, ρ'-meson, etc., respectively, and $\varphi^{(0)}$ as the pion. It can be easily checked that the spin, parity and charge conjugation parity are consistent in this interpretation.

Inserting these expansions into the 5 dimensional action (1) and integrating over z, we obtain a traditional 4 dimensional effective action of the mesons π, ρ, a_1, etc.:

$$S_{\text{5dim}}(A) = S_{\text{4dim}}(\pi, \rho, a_1, \rho', a_1', \cdots) . \tag{3}$$

Remarkably, the meson effective theory obtained in this way reproduces a lot of old phenomenological models of hadrons, such as Skyrme model, vector meson dominance model, Gell-Mann - Sharp - Wagner model, hidden local symmetry model, a bottom up holographic model proposed in [11] etc., without making any phenomenological assumptions. The CS-term S_{CS} correctly reproduces the chiral anomaly in QCD and yields the WZW term. Furthermore, the masses and couplings calculated in (3) roughly agree with the experimental data. (See Table 1.) Other mesons, including higher spin mesons, are obtained as excited string states. For example, $a_2(1320)$, $b_1(1235)$, $\pi(1300)$, $a_0(1450)$, etc., are interpreted as the first excited open string states, and $\rho_3(1690)$, $\pi_2(1670)$, etc., are interpreted as the second excited states. The lightest

mass	our model	experiment
ρ	[776 MeV]	776 MeV
a_1	1189 MeV	1230 MeV
ρ'	1607 MeV	1465 MeV

coupling	our model	experiment
f_π	[92.4 MeV]	92.4 MeV
L_1	0.58×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	1.2×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	-3.5×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	8.7×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	-8.7×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	4.8	6.0
g_ρ	0.16 GeV^2	0.12 GeV^2
$g_{a_1\rho\pi}$	4.6 GeV	$2.8 \sim 4.2$ GeV

Table 1: Masses and couplings calculated in our model.[1, 2] Here, $M_{\rm KK}$ and λ are fixed by fitting the ρ meson mass and the pion decay constant f_π.

spin J mesons with $J \geq 1$ are $(J-1)$-th excited open string states. See [5] for more details.

As a typical example, let us briefly explain what we can learn for the ω-meson decay.[2] Here we consider the cases with $N_f = 2$. Then, the vector meson field $B_\mu^{(1)}$ obtained above is a 2×2 hermitian matrix. The trace part (isoscalar) corresponds to the ω-meson, while the other three components (isovector) correspond to the ρ-meson (ρ^0 and $\rho^\pm$). We are interested in the decay channels $\omega \to \pi\gamma$ and $\omega \to \pi\pi\pi$. The relevant diagrams for these decay channels obtained from the effective action (3) are depicted in Figure 3. Note that there is no direct ω-π-γ and ω-π-π-π couplings, and

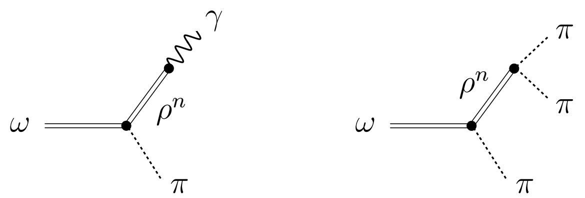

Figure 3: The relevant diagrams for $\omega \to \pi\gamma$ and $\omega \to \pi\pi\pi$. ρ^n is the isovector part of the vector meson $B_\mu^{(2n-1)}$.

these decay modes are always mediated by the vector mesons ρ^n. This structure is exactly what Gell-Mann - Sharp - Wagner proposed in [12]. Furthermore, using the completeness condition for $\{\psi_n\}_{n\geq 1}$, we can show a sum rule:

$$\sum_{n=1}^{\infty} \frac{g_{\omega\rho^n\pi} g_{\rho^n\gamma}}{m_{\rho^n}^2} = g_{\rho\pi\pi} \ , \tag{4}$$

where $g_{\omega\rho^n\pi}$, $g_{\rho^n\gamma}$ and $g_{\rho\pi\pi}$ are the coupling constants for ω-ρ^n-π, ρ^n-γ and ρ-π-π interections, respectively, and m_{ρ^n} is the mass of the vector meson ρ^n. Using this sum rule, the decay width of the $\omega \to \pi\gamma$ can be calculated as

$$\Gamma(\omega \to \pi\gamma) = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2}\left(\sum_{n=1}^{\infty}\frac{g_{\omega\rho^n\pi}g_{\rho^n\gamma}}{m_{\rho^n}^2}\right)^2 |\mathbf{p}_\pi|^3 = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2}\, g_{\rho\pi\pi}^2 |\mathbf{p}_\pi|^3 \ , \tag{5}$$

which reproduces the expression given in [13].

As mentioned above, baryons are obtained as D4-branes wrapped on S^4. It is known that a D4-brane embedded in D8-brane world-volume is equivalent to a soliton in the gauge theory realized on the D8-brane. In our 5 dimensional gauge theory (1), baryons are described as a soliton carrying non-trivial instanton number

$$\frac{1}{8\pi^2}\int_{\mathbf{R}^4} \mathrm{Tr}(F \wedge F) = N_B \ , \tag{6}$$

where $\mathbf{R}^4$ is a four dimensional space $\{(x^{1\sim 3}, z)\}$ and N_B is an integer interpreted as the baryon number. This is analogous to the description of baryons as solitons (Skyrmions) in Skyrme model [7], and more directly related to the proposal in [14], in which the Skyrmions are obtained from the instanton configurations in a five dimensional Yang-Mills theory. Applying the techniques developed in the Skyrme model [15] to our system, we can analyze various properties of baryons. The spectrum of the baryon is obtained as in Figure 4. Interestingly our result suggests that second

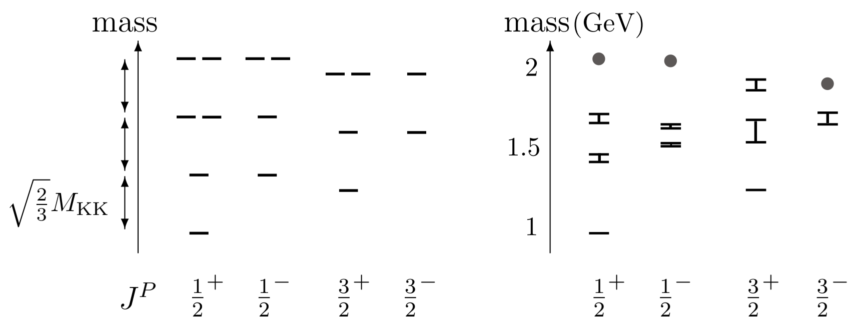

Figure 4: Left side is the baryon spectrum obtained in our model [3], while the right side is the spectrum of the baryons with $I = J$ taken from particle listings by Particle Data Group. Here I and J are the isospin and spin, respectively, and and P is the parity.

lightest baryon with $J^P = \frac{1}{2}^+$ (Roper) and the lightest one with $J^P = \frac{1}{2}^-$ are degenerate. This feature cannot be reproduced from the naive quark model and has been a mystery in hadron physics for a long time.[3] The results for the charge radii and magnetic moments for nucleons obtained in [4] are listed in Table 2.

	our model	experiment
$\langle r^2\rangle_{I=0}$	$(0.74\ \mathrm{fm})^2$	$(0.81\ \mathrm{fm})^2$
$\langle r^2\rangle_{I=1}$	$(0.74\ \mathrm{fm})^2$	$(0.94\ \mathrm{fm})^2$
$\langle r^2\rangle_{A}$	$(0.54\ \mathrm{fm})^2$	$(0.67\ \mathrm{fm})^2$
$g_{I=0}$	1.7	1.8
$g_{I=1}$	7.0	9.4
g_A	0.73	1.3

Table 2: $\langle r^2\rangle_{I=0}$, $\langle r^2\rangle_{I=1}$ and $\langle r^2\rangle_{A}$ are isoscalar, isovector and axial mean square radii, respectively. $g_{I=0}$ and $g_{I=1}$ are isoscalar and isovector g-factors. g_A is the axial coupling.

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