Anomalies and Transport Coefficients: The Chiral Gravito-Magnetic Effect

Karl Landsteiner[a], **Eugenio Megías**[a,b] **and Francisco Pena-Benitez**[a,c]
[a]*Instituto de Física Teórica UAM-CSIC, Universidad Autónoma de Madrid, Cantoblanco E-28049 Madrid, Spain*
[b]*Grup de Física Teòrica and IFAE, Departament de Física, Universitat Autònoma de Barcelona, Bellaterra E-08193 Barcelona, Spain*
[c]*Departamento de Física Teórica, Universidad Autónoma de Madrid, Cantoblanco E-28049 Madrid, Spain*

1 Introduction

At finite temperature and density anomalies give rise to new non-dissipative transport phenomena in the hydrodynamics of charged relativistic fluids [1, 2, 3, 4]. As is beautifully explained in the RHIC-made video [5] magnetic fields and vortices in the fluid induce currents via the so-called chiral magnetic and chiral vortical conductivities. Although there have been many early precursors that found manifestations of this phenomena in the physics of neutrinos [6, 7, 8, 9], the early universe [10] and condensed matter systems [11], the recent surge of interest is clearly related to the physics of the quark gluon plasma. It has been suggested that the observed charge separation in heavy ion collisions is related to a particular manifestation of these anomalous transport phenomena: the chiral magnetic effect [1, 12]. The latter describes how a (electro-magnetic) B-field induces via the axial anomaly an electric current parallel to the magnetic field. The first application of holography to the anomalous hydrodynamics is [13] where the anomalous transport effects due to R-charge magnetic fields have been examined. Later studies showed that there is also a related vortical effect [2, 3], i.e. a vortex in the fluid induces a current parallel to the axial vorticity vector $\Omega^\mu = \epsilon^{\mu\nu\rho\lambda} u_\nu \partial_\rho u_\lambda$, and related effects of the presence of angular momentum had been discussed before in a purely field theoretical setup in [8] and [14]. Studies of the chiral magnetic effect using holography have appeared in [15, 16, 17, 18, 19, 20, 21, 22, 23, 24] and using lattice field theory in [25, 26, 27]. The experimental status of the observed charge separation in heavy ion collision is discussed in [28, 29]. A related effect is the so called chiral separation effect that induces an axial current in a magnetic field [30]. It has been argued to lead to enhanced production of high spin hadrons in [31]. In [4] the authors showed that purely hydrodynamic considerations based on demanding a positive definite divergence of the entropy current fix the chiral magnetic and chiral vortical conductivities almost uniquely. In [32] it was however pointed out that there are ambiguities stemming from integration constants that allow additional temperature dependence. It is precisely this temperature dependence that we can fix by calculating the transport coefficients via Green-Kubo formulas and discover a somewhat surprising relation to the gravitational anomaly.

Anomalies belong to the most interesting and most subtle properties of relativistic quantum field theories. They are responsible for the breakdown of a classical symmetry due to quantum effects. The Adler-Bardeen non-renormalization theorem guarantees that this breakdown is saturated at the one-loop level. Therefore the presence of anomalies can be determined through simple algebraic criteria on the representations under which the chiral fermions of a particular theory transform. In vacuum the anomaly appears as the non-conservation of a classically conserved current in a triangle diagram with two additional currents. In four dimension two types of anomalies can be distinguished according to whether only spin one currents appear in the triangle [33, 34] or if also the energy-momentum tensor participates [35, 36]. We will call the first type of anomalies simply chiral anomalies and the second type gravitational anomalies. To be precise, in four dimension we should actually talk of mixed gauge-gravitational anomalies since triangle diagrams with only energy-momentum insertions are perfectly conserved (see e.g. [37]). In four dimensional Minkowski space massless fermions can always be written in a basis of only left-handed fermions. If they transform in a representation T_A of a symmetry the presence of chiral anomalies is detected by the non-vanishing of $d_{ABC} = \frac{1}{2}\mathrm{Tr}(T_A\{T_B, T_C\})$ whereas the presence of a gravitational anomaly is detected by the non-vanishing of $b_A = \mathrm{Tr}(T_A)$.

2 Kubo Formulas

Transport coefficients can be computed in linear response theory via so-called Green-Kubo formulas. The response of a system in equilibrium to an external perturbation is encoded in the retarded Green's functions. If we apply an electric field to our system we will generate an electric current. If the electric field is (sufficiently) weak the magnitude of the current will be proportional to the applied electric field and the constant of proportionality is simply the electric conductivity $\sigma^{\mathcal{E}}$.

$$\vec{J} = \sigma^{\mathcal{E}} \vec{\mathcal{E}} \,. \tag{1}$$

We can derive a Green-Kubo formula by noting that the electric field is $\vec{\mathcal{E}} = -i\omega\vec{\mathcal{A}}$ in terms of a vector potential $\vec{A}$. We now interpret above equation as an expression for the vacuum expectation value of the current in the background of an external gauge field $\vec{\mathcal{A}}$. Since the vector potential acts as a source for the current we can functionally differentiate with respect to it and get an expression for the (retarded) two-point function of currents. In particular we can obtain in this way the Green-Kubo formula for the DC-conductivity

$$\sigma^{\mathcal{E}} = \lim_{\omega\to 0} \frac{i}{\omega} \langle J_i J_i \rangle|_{\vec{k}=0}\,, \tag{2}$$

(no summation over the spatial index i here).

Let us now jump to the chiral magnetic effect. It describes the generation of a current due to a magnetic field, so

$$\vec{J} = \sigma^{\mathcal{B}} \vec{\mathcal{B}}\,. \tag{3}$$

Proceeding as before it is straightforward to see that the Green-Kubo formula for the chiral magnetic conductivity $\sigma^{\mathcal{B}}$ is

$$\sigma^{\mathcal{B}} = \lim_{k_n\to 0} \sum_{i,j} \epsilon_{ijn} \frac{i}{2k_n} \langle J_i J_j \rangle|_{\omega=0}\,. \tag{4}$$

So far we have only discussed the response to electro-magnetic $\mathcal{E}$- and $\mathcal{B}$-fields. From now on we want to be a bit more general and imagine to have an arbitrary symmetry group with generators T_A and couple them to non-dynamical, i.e. external gauge fields $\mathcal{A}_A$ such that their variation inserts the currents J_A into correlation functions. The obvious generalization of the chiral magnetic conductivity is then

$$\sigma^{\mathcal{B}}_{AB} = \lim_{k_n\to 0} \sum_{i,j} \epsilon_{ijn} \frac{i}{2k_n} \langle J^i_A J^j_B \rangle|_{\omega=0}\,. \tag{5}$$

That is $\sigma^{\mathcal{B}}_{AB}$ describes the generation of the current $\vec{J}_A$ if an external field $\vec{\mathcal{B}}_B = \vec{\nabla}\times\vec{\mathcal{A}}_B$ is switched on[1].

If the fluid under consideration is charged with respect to some of the (classically) conserved charges there will necessarily be an energy transport related to the charge transport induced by the current $\vec{J}_A$. We know that a variation in the charge distribution will cost us an energy of the form $\delta\epsilon = \mu_A \delta Q_A$. If we imagine a test charge δQ_A moving through the charged plasma it will therefore generate a current $\delta\vec{J}_A$ and induce also an energy current of the form $\delta T^{0i} = \mu_A \delta J^i_A$. For a finite current we should integrate this over μ_A and obtain $T^{0i} = \int \mu'_A \frac{dJ^i}{d\mu'_A} d\mu'_A$. It follows therefore that the energy flux due to an external $\mathcal{B}_A$ field is measured by the transport coefficient

$$\sigma^{\mathcal{B}}_{B} = \lim_{k_n\to 0} \sum_{i,j} \epsilon_{ijn} \frac{i}{2k_n} \langle T^{0i} J^j_B \rangle|_{\omega=0} = \int \mu_A d\sigma^{\mathcal{B}}_{AB} + \text{const.}\,. \tag{6}$$

We explicitly introduced a undetermined integration constant here. Evaluating the Kubo formula we will see that the integrations constant is $\propto T^2$ and non-zero only if gravitational anomalies are present.

[1]In principle we should include also commutator terms in $\mathcal{B}_B$. They are however not important if we want to study only two point functions of currents with vanishing external fields.

The correlators are retarded ones but evaluated at zero frequency. For this reason the order of the operators can be reversed and we we also can define

$$\sigma^{\mathcal{V}}_A = \lim_{k_n\to 0} \sum_{i,j} \epsilon_{ijn} \frac{i}{2k_n} \langle J^i_A T^{0j} \rangle|_{\omega=0} \tag{7}$$

Although it is clear that $\sigma^{\mathcal{B}}_A = \sigma^{\mathcal{V}}_A$ they describe different transport phenomena. Whereas $\sigma^{\mathcal{B}}_A$ describes the generation of an energy flux due to an external $\mathcal{B}_A$ field, $\sigma^{\mathcal{V}}_A$ describes the generation of the current $\vec{J}_A$ due to an external field that sources T^{0i}. It is not difficult to convinces oneself that the field in question is the so-called gravito-magnetic field defined by a variation of the (flat) metric of the form

$$ds^2 = -dt^2 + 2\vec{\mathcal{A}}_g dt d\vec{x} + d\vec{x}^2 \,. \tag{8}$$

If we linearize gravity with this metric $\vec{\mathcal{A}}_g$ acts indeed like a normal abelian vector potential. We might therefore call the new transport coefficient $\sigma^{\mathcal{V}}_A$ a "chiral gravito-magnetic" conductivity giving rise to a "chiral gravito-magnetic effect". In a fluid a second, and somewhat more down-to earth interpretation is available. The flow of a relativistic fluid is characterized by the fluid four-velocity u^μ. In the restframe of the fluid but in the background of a gravito-magnetic potential we have $u_\mu = (-1, \vec{v}) = (-1, \vec{\mathcal{A}})$. Therefore the gravito-magnetic field can also be calculated as the curl of the velocity field $\vec{\mathcal{B}}_g = \vec{\nabla} \times \vec{v}$. It is therefore natural to interpret $\sigma^{\mathcal{V}}_A$ as the response in the current due to a vortex in the fluid, i.e. as a "chiral vortical conductivity".

Since now we have convinced ourselves that vortices (or gravito-magnetic fields) in the fluid might generate currents it comes as no surprise that they also will generate an energy flux and that the corresponding conductivity will be given by

$$\sigma^V = \lim_{k_n\to 0} \sum_{i,j} \epsilon_{ijn} \frac{i}{2k_n} \langle T^{0i} T^{0j} \rangle|_{\omega=0} \tag{9}$$

In a hydrodynamic framework we can summarize our findings in the constitutive relations

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P\eta^{\mu\nu} + Q^\mu u^\nu + Q^\nu u^\mu \,, \tag{10}$$
$$J^\mu_A = n_A u^\mu + N^\mu_A \,. \tag{11}$$

with the first order in derivatives terms

$$N^\mu_A = \sigma^{\mathcal{B}}_{AB} \mathcal{B}^\mu_B + \sigma^{\mathcal{V}}_A \Omega^\mu \,, \tag{12}$$
$$Q^\mu = \sigma^{\mathcal{B}}_A \mathcal{B}^\mu_A + \sigma^{\mathcal{V}}_A \Omega^\mu \,. \tag{13}$$

For simplicity of the expressions we have dropped here the usual dissipative terms related to shear and bulk viscosities or electric conductivity. The equilibrium quantities

ϵ, P, n_A are energy density, pressure and charge densities and we defined the (covariant) magnetic fields $\mathcal{B}^\mu_A = \epsilon^{\mu\nu\rho\lambda} u_\nu \partial_\rho A_{\lambda,A}$ and vorticity vector $\Omega^\mu = \epsilon^{\mu\nu\rho\lambda} u_\nu \partial_\rho u_\lambda$. The gravito-magnetic Lorentz force on a test particle of mass m is [2]

$$\vec{F} = m.\vec{v} \times \vec{\mathcal{B}}_g \,. \tag{14}$$

It follows that the work done by the gravito-magnetic field on a test particle is $W = \int \vec{F}.\vec{v}dt = 0$ just as in the case of a usual magnetic field. Since neither a magnetic nor a gravito-magnetic field (or a vortex) do work on the system these constitutive relations describe dissipationless transport. This property and the related T-invariance of the transport have recently been emphasized in [39, 40] where these constitutive relations have been generalized to higher dimensions as well.

We also note that the fluid velocity in the hydrodynamic derivative expansion suffers from ambiguities. Indeed we can always redefine $u^\mu \to u^\mu + \delta u^\mu$ and declare δu^μ to be of the same order as Q^μ and N^μ. Choosing $\delta u^\mu = -Q^\mu/(\epsilon + P)$ effectively removes Q^μ from the constitutive relation of the energy-momentum tensor and defines the so called Landau frame. The current is then given by

$$N^\mu = \xi^{\mathcal{B}}_{AB} \mathcal{B}_B + \xi^{\mathcal{V}}_A \Omega^\mu \tag{15}$$

with the Landau frame transport coefficients

$$\xi^{\mathcal{B}}_{AB} = \sigma^{\mathcal{B}}_{AB} - \frac{n_A}{\epsilon + P} \sigma^{\mathcal{B}}_B \,, \tag{16}$$

$$\xi^{\mathcal{V}}_A = \sigma^{\mathcal{V}}_A - \frac{n_A}{\epsilon + P} \sigma^{\mathcal{V}} \,, \tag{17}$$

2.1 Weak coupling

Our aim is now to evaluate these Green-Kubo's formulas in a theory of free right-handed fermions Ψ^f transforming under a global symmetry group G generated by matrices $(T_A)^f{}_g$. We denote the generators in the Cartan subalgebra by H_A. Chemical potentials μ_A can be switched on only in the Cartan subalgebra. Furthermore the presence of the chemical potentials breaks the group G to a subgroup $\hat{G}$. Only the currents that lie in the unbroken subgroup are conserved (up to anomalies) and participate in the hydrodynamics. The chemical potential for the fermion Ψ^f is given by $\mu^f = \sum_A q^f_A \mu_A$, where we write the Cartan generator $H_A = q^f_A \delta^f{}_g$ in terms of its eigenvalues, the charges q^f_A. The unbroken symmetry group $\hat{G}$ is generated by those matrices $T^f_{A\ g}$ fulfilling

$$T^f_{A\ g} \mu^g = \mu^f T^f_{A\ g} \,. \tag{18}$$

There is no summation over indices in the last expression. From now on we will assume that all currents $\vec{J}_A$ lie in directions indicated in (18). We define the chemical

[2] See [38] but note that our definition of the gravito-magnetic potential differs by a factor of 2.

potential through boundary conditions on the fermion fields around the thermal circle [41], $\Psi^f(\tau) = -e^{\beta\mu^f}\Psi^f(\tau-\beta)$ with $\beta = 1/T$. Therefore the eigenvalues of ∂_τ are $i\tilde{\omega}_n + \mu^f$ for the fermion species f with $\tilde{\omega}_n = \pi T(2n+1)$ the fermionic Matsubara frequencies. A convenient way of expressing the currents is in terms of Dirac fermions and writing

$$J^i_A = \sum_{f,g=1}^{N} T_{A\ f}^{g}\overline{\Psi}_g\gamma^i\mathcal{P}_+\Psi^f \,, \tag{19}$$

$$T^{0i} = \frac{i}{2}\sum_{f=1}^{N}\overline{\Psi}_f(\gamma^0\partial^i + \gamma^i\partial^0)\mathcal{P}_+\Psi^f \,, \tag{20}$$

where we used the chiral projector $\mathcal{P}_\pm = \frac{1}{2}(1\pm\gamma_5)$. The fermion propagator is

$$S(q)^f{}_g = \frac{\delta^f{}_g}{2}\sum_{t=\pm}\Delta_t(i\tilde{\omega}^f,\vec{q})\mathcal{P}_+\gamma_\mu\hat{q}^\mu_t \,, \tag{21}$$

$$\Delta_t(i\tilde{\omega}^f,q) = \frac{1}{i\tilde{\omega}^f - tE_q} \,, \tag{22}$$

with $i\tilde{\omega}^f = i\tilde{\omega}_n + \mu^f$, $\hat{q}^\mu_t = (1, t\hat{q})$, $\hat{q} = \frac{\vec{q}}{E_q}$ and $E_q = |\vec{q}|$. We can easily include left-handed fermions as well.

The relevant Green's function for the chiral magnetic can be evaluated with standard finite temperature techniques (see [42] for details). The results for the different conductivities are neatly summarized as

$$\sigma^{\mathcal{B}}_{AB} = \frac{1}{4\pi^2}d_{ABC}\mu^C \,, \tag{23}$$

$$\sigma^{\mathcal{V}}_{A} = \frac{1}{8\pi^2}d_{ABC}\mu^B\mu^C + \frac{T^2}{24}b_A \,, \tag{24}$$

$$\sigma^{\mathcal{V}} = \frac{1}{12\pi^2}d_{ABC}\mu^A\mu^B\mu^C + \frac{T^2}{12}b_A\mu^A \,. \tag{25}$$

The result shows that these conductivities are non-zero if and only if the theory features anomalies. Let us come back now to the above mentioned integration constants. We see that in they are fixed to a particular form proportional to T^2 with a coefficient that coincides with the gravitational anomaly coefficient. Additional terms of the form T^3 are allowed on dimensional grounds in $\sigma^{\mathcal{V}}$ [32], CPT invariance forbids these terms however and indeed they don't show up in the Green-Kubo formulas.

In vacuum the anomaly appears on the level of three point functions. In the presence of external sources for the energy momentum tensor and the currents this is conveniently expressed through [35, 36, 37]

$$\nabla_\mu J^\mu_A = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{ABC}}{32\pi^2}F^B_{\mu\nu}F^C_{\rho\lambda} + \frac{b_A}{768\pi^2}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right) \,. \tag{26}$$

Note that the gravitational anomaly is actually fourth order in derivatives and therefore a naive counting argument would suggest that it can not contribute to first order hydrodynamics. It is however more useful to count derivatives acting on the connections $\mathcal{A}_a^\mu$ and $\Gamma^\rho_{\mu\nu}$. In this way the Riemann tensor is first order in derivatives just like the gauge field strength. Since this transport coefficients seem to be intimately related to anomalies it is tempting to speculate that they are not renormalized as we switch on interactions. We will investigate this question in the next section.

2.2 Strong coupling

As is well known gauge theories at infinitely strong coupling can be investigated through the gauge-gravity correspondence. It is therefore interesting to see if the anomalous conductivities can be obtained also from a strong coupling calculations based on the action

$$\begin{aligned}
S &= \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN}\right. \\
&\quad \left. +\epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)\right] + S_{GH} + S_{CSK}\,, && (27)\\
S_{GH} &= \frac{1}{8\pi G}\int_\partial d^4x\sqrt{-h}\,K\,, && (28)\\
S_{CSK} &= -\frac{1}{2\pi G}\int_\partial d^4x\sqrt{-h}\,\lambda n_M\epsilon^{MNPQR}A_N K_{PL}D_Q K^L_R\,, && (29)
\end{aligned}$$

where S_{GH} is the usual Gibbons-Hawking boundary term and $D_A = h_A^B\nabla_B$ is the covariant derivative on the four dimensional boundary. The most important fact about this action is the presence of two Chern-Simons terms, one a pure gauge field CS term and the second one a mixed gauge-gravitational CS-term. The action depends therefore explicitly on the gauge field A_M and is gauge invariant only up to a boundary term of the form

$$\delta S = \frac{1}{16\pi G}\int_\partial d^4x\sqrt{-h}\epsilon^{mnkl}\left(\frac{\kappa}{3}\hat{F}_{mn}\hat{F}_{kl} + \lambda\hat{R}^i{}_{jmn}\hat{R}^j{}_{ikl}\right)\,. \qquad (30)$$

where $\hat{R}$ is the induced curvature on the boundary. This is indeed of the form of the anomaly and we can match the CS couplings to the anomaly coefficients

$$\begin{aligned}
-\frac{\kappa}{48\pi G} &= \frac{b}{96\pi^2}\,, && (31)\\
-\frac{\lambda}{16\pi G} &= \frac{d}{768\pi^2}\,. && (32)
\end{aligned}$$

In this simple model there is only one $U(1)$ symmetry and therefore the anomaly coefficients are simply $b = \sum q_i$ and $d = \sum q_I^3$. where q_i are the charges of the chiral fermions of the dual field theory.

As background we chose the charged AdS black brane solution with metric and gauge field

$$ds^2 = r^2\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{r^2}{f(r)r^2} \quad , \quad f(r) = \left(1 - \frac{M}{r^4} + \frac{Q^2}{r^6}\right) , \tag{33}$$

$$A = \left(\beta - \frac{\mu r_H^2}{r^2}\right) dt\,. \tag{34}$$

The chemical potential is related to the charge by $\mu = \sqrt{3}Q/r_H^2$ and is defined as the energy needed to bring a unit charge from the boundary behind the horizon. The background value of the temporal component of the boundary gauge field β has to be distinguished from it along the lines explained in [19].

Standard gauge-gravity techniques [43, 44, 45] can now be employed in order to calculate the relevant Green's functions. It is straightforward although slightly tedious due to the complexities of the gauge-gravitational CS-term (see [46]). In any case it is reassuring to find precisely the form of (24)-(25) specialized to the case of a single $U(1)$ theory. This can be taken as a strong hint pointing towards a non-renormalization of the anomalous conductivities.

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