On Some Aspects of the QCD Effective Locality

Thierry Grandou
Institut Non Linéaire de Nice
Université de Nice Sophia-Antipolis
06560 Valbonne, FRANCE

1 Introduction

'*The strong interactions comprise a richer field than the set of phenomena that we have learned to describe in terms of perturbative QCD or the (near-) static non-perturbative domain of lattice QCD (...) It may well be that interesting unusual occurrences happen outside the framework of perturbative QCD-happen in some collective, or intrinsically nonperturbative way.*'

It is in these terms that the *Resource Letter: Quantum Chromodynamics*, arXiv: 1002.5032v2 [hep-ph], of February 26, 2010, concludes its overall review of the most salient achievements realized in QCD.

In a recent article [1] a curious new property was observed concerning the QCD fermionic scattering amplitudes, which was dubbed 'effective locality'. That property can be phrased as follows: If one considers two scattering quarks, then the gauge invariant summation of all the non-abelian gluonic interactions between them, 'boils down' to a local interaction between the two fermionic currents, of the contact-type. This is of course a non-expected result because, ordinarily, integrations of elementary field degrees of freedom result in highly non-local and non-trivial structures, and the 'effective locality' denomination, which sounds like an oxymoron, accounts for this rather unusual circumstance.

Approximations were used in [1], though, such as the Eikonal approximation and the neglect of the fermionic determinant (*i.e.*, the approximation of 'quenching'). The question then naturally arises to know wether this 'effective locality' is or isn't an *artefact* of the approximations being used.

Remarkably enough, it turns out that all of the approximations that were used in [1] can be relaxed, each, and that the effective locality property of the QCD fermionic scattering amplitudes still holds [2]. The following statement can accordingly be proposed: In any Quark/Quark (or Anti-Quark) scattering amplitude, the full gauge-invariant sum of cubic and quartic vectorial gluonic interactions, including fermionic loops, results into a local contact-type interaction; this local interaction is mediated by a tensorial field structure which is antisymmetric both in Lorentz and color indices.

This non-perturbative property of QCD could therefore be worth exploring, in particular, in relation to the non-perturbative sector of QCD and its expected physical characteristics such as color confinement, chiral symmetry breaking, quark binding potentials etc..

In this presentation, however, we will rather focus on some of the technical aspects which come about in the derivation and exploitation of that QCD effective locality property, because they govern the consequences that can be drawn from them.

Some of the results to be dealt with in the sequel are sound and established, whereas other parts are still the matter of ongoing work, and should accordingly be taken as preliminary [4].

2 The QCD generating functional

Within standard functional notations, and standard functional manipulations [6], the QCD generating functional can be written as

$$\begin{aligned}\mathcal{Z}[j,\eta,\bar{\eta}] \;&=\; \mathcal{N}\, e^{\frac{i}{2}\int j\mathbf{D}_c(\zeta)j}\int \mathrm{d}[\chi]\; e^{\frac{i}{4}\int\chi^2}\; e^{\mathcal{D}_A}\; e^{\frac{i}{2}\int\chi\mathbf{F}+\frac{i}{2}\int A\mathbf{D}_c^{-1}(\zeta)A} \\ &\quad\times e^{i\int\bar{\eta}\mathbf{G}_c[A]\eta}\; e^{\mathbf{L}[A]}\Big|_{A=\int\mathbf{D}_c(\zeta)j}\end{aligned} \tag{1}$$

where $\mathcal{N}$ is a normalization constant, and the j^μ and $\eta_\alpha, \bar{\eta}_\delta$ stand for bosonic and fermionic sources respectively (the subscript c means 'causal', and doesn't differ from the most customary Feynman prescription). The 'Linkage Operator',

$$\mathcal{D}_A = -\frac{i}{2}\int \mathrm{d}^4x\; \mathrm{d}^4y\; \frac{\delta}{\delta A^a_\mu(x)}\; \mathbf{D}_c^{(\zeta)}\Big|^{ab}_{\mu\nu}(x-y)\,\frac{\delta}{\delta A^b_\nu(y)}$$

involves the covariant gluonic propagator, with gauge parameter ζ,

$$\mathbf{D}_c^{(\zeta)}\Big|^{ab}_{\mu\nu} = \delta^{ab}(-\partial^2)^{-1}\left[g_{\mu\nu}-(1-\zeta)\frac{\partial_\mu\partial_\nu}{\partial^2}\right] \tag{2}$$

The χ-field appearing in $\mathcal{Z}[j,\eta,\bar{\eta}]$ is here to *linearize* the original F^2- dependence of the original QCD Lagrangian density:

$$e^{-\frac{i}{4}\int F^2} = \mathcal{N}_\chi \int \mathrm{d}[\chi^a_{\mu\,\nu}]\, e^{\frac{i}{4}\int\chi^2+\frac{i}{2}\int\chi^{\mu\,\nu}_a F^a_{\mu\,\nu}} \tag{3}$$

Proceeding in this way, the A_μ-gauge field dependences are made gaussian instead of cubic and quartic, and the interest is that the linkage operations can be carried out exactly.

In (1), the original fermionic fields have been integrated out: This has given rise to the term involving the fermionic propagator $G_c[A]$ in the gauge field background configuration A, as well as to the logarithm of the fermionic determinant, $L[A]$, that is,

$$G_c(x, y\,|A) = \langle x|[m + \gamma^\mu\,(\partial_\mu - i\,g\,A^a_\mu\,\lambda_a)]^{-1}|y\rangle$$

and,

$$L[A] = \ln[1 - i\,g\,(\gamma\,A\,\lambda)\,S_c]\ ,\ S_c = G_c[0]$$

3 On a puzzling constraint of $\delta^{(4)}(\bar{u}(s_2) - u(s_1))$

The property of effective locality is not readable on the generating functional itself, but on the 'fermionic momenta' of the $\mathcal{Z}[j, \eta, \bar{\eta}]$-distribution, that is on the full set of $2n$-point fermionic Green's functions (though equivalent, and thus not a crucial difference at this stage, a form of the effective action is under investigation, on which effective locality can be read off directly: This will matter instead, when this formulation of the QCD amplitudes will possibly be promoted to the status of the most dual QCD formulation).

A point is that, in order to be able to display the property of effective locality, one must devise convenient enough representations for the functionals $G_c[A]$ and $L[A]$. Schwinger-Fradkin's representations have been used so far, with, for example, for a ' mixed' (configuration and momentum space) expression of $G_c[A]$,

$$\begin{aligned}
\langle p|\mathbf{G}_c[A]|y\rangle &= e^{-ip\cdot y}\ i\int_0^\infty ds\ e^{-ism^2}\ e^{-\frac{1}{2}\ Tr\ln(2h)} \\
&\times \int d[u]\{m - i\gamma\cdot[p - gA(y - u(s))]\}\ e^{\frac{i}{4}\int_0^s ds'\,[u'(s')]^2}\ e^{ip\cdot u(s)} \\
&\times \boxed{\left(e^{g\int_0^s ds'\sigma\cdot F(y-u(s'))}\ e^{-ig\int_0^s ds'\,u'(s')\cdot A(y-u(s'))}\right)_+}
\end{aligned}$$

and likewise for $L[A]$. In the representation above, one has

$$h(s_1, s_2) = s_1\Theta(s_2 - s_1) + s_2\Theta(s_1 - s_2)\ ,\quad h^{-1}(s_1, s_2) = \frac{\partial}{\partial s_1}\frac{\partial}{\partial s_2}\delta(s_1 - s_2)$$

and in the framed expression one has $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]$. Note that this expression ends up with a subscript '$+'$ meaning that time ordering with respect to s' is in order, because of the Lie-algebra valuation of the gauge fields A_μ. Eventually, $u(s')$ is the Fradkin's field variable,

$$u_\mu(s) = \int_0^s \mathrm{d}s'\,v_\mu(s')\ ,\qquad u_\mu(0) = 0 \tag{4}$$

On mass shell, $v_\mu(s)$ can be interpreted as the particle 4-velocity $p_\mu(s)/m$. Even in the simplified case of a quenched and eikonal approximation [1], the full derivation of the effective locality property cannot be repeated here: Its complete non-approximate derivation can be found in [2]. We will here focus on some important technical details whose control matters a lot if one wishes to start thinking of non-perturbative QCD out of reliable results [4].

As a typical and important part of a 4-point fermionic function, one gets in an exponential the argument

$$
\begin{aligned}
&+\frac{i}{2}g\int \mathrm{d}^4w \int_0^s \mathrm{d}s_1 \int_0^{\bar{s}} \mathrm{d}s_2\; u'_\mu(s_1)\,\bar{u}'_\nu(s_2) \\
&\times \Omega^a(s_1)\,\bar{\Omega}^b(s_2)\;\left(f\cdot\chi(w)\right)^{-1}\Big|^{\mu\nu}_{ab} \\
&\times \delta^{(4)}(w-y_1+u(s_1))\,\delta^{(4)}(w-y_2+\bar{u}(s_2))
\end{aligned}
\tag{5}
$$

This expression is obtained at the approximation of quenching and by neglecting quark's spins: The full non-approximate expression though, would manifest exactly the same technical intricacy as the one under consideration, and this is why the point can be dealt with on this simplified example.

Now, how should we think of

$$
\delta^{(4)}(w_1-y_1+u(s_1))\,\delta^{(4)}(y_1-y_2+\bar{u}(s_2)-u(s_1))
\tag{6}
$$

That is, basically, how should one interpret such a factor as

$$
\delta^{(4)}(\bar{u}(s_2)-u(s_1)) \qquad\qquad ?
\tag{7}
$$

At face value, at any given couple of values $(s_1,s_2)\in\,]0,s]\times]0,\bar{s}]$, and any couple of arbitrary functions $(u,\bar{u})$, each belonging to some infinite dimensional functional space, the probability of coincidence of $u(s_1)$ with $\bar{u}(s_2)$ is likely to be infinitesimally small if not zero.

On the other hand, a somewhat heuristic manipulation of (7) would suggest to write

$$
\delta(\bar{u}_0(s_2)-u_0(s_1))\delta(\bar{u}_{\mathrm{L}}(s_2)-u_{\mathrm{L}}(s_1)) = \frac{\delta(s_1)\delta(s_2)}{|u'_{\mathrm{L}}(0)||\bar{u}'_0(0)|}
\tag{8}
$$

where it is assumed that the $u,\bar{u}$ Fradkin's fields are $\mathcal{C}^1\left(]0,s],]0,\bar{s}]\to R^4\right)$. Out of $\delta^{(4)}(\bar{u}(s_2)-u(s_1))$, and in view of (4), this leads to a remaining constraint of

$$
\delta^{(2)}(\vec{y}_{1\perp}-\vec{y}_{2\perp}) := \delta^{(2)}(\vec{b})
$$

where $\vec{b}$ is the impact parameter, or transverse distance between the two scattering quarks. This $\delta^{(2)}(\vec{b})$ is of course an intriguing factor. For example, it has been suggested [2] that this strange factor has been generated through the calculation because of some lurking surreptitious element, inherent to an implicit perturbative hypothesis: That of the existence of asymptotic quark states. This point of view leads to a change of $\delta^{(2)}(\vec{b})$ into a modified impact parameter distribution of form,

$$\delta^{(2)}(\vec{b}) \to \varphi(b) = \frac{\mu^2}{\pi} \frac{1+\xi/2}{\Gamma(\frac{1}{1+\xi/2})} e^{-(\mu b)^{2+\xi}}, \quad \mathbf{R} \ni \xi \ll 1$$

where μ is a typical mass term used to calibrate the transverse momenta distributions of quarks inside a given bound state: In the transverse plane, bound quarks are endowed with transverse momenta taken as independent random variables, exponentially suppressed above a given mass parameter μ. First fits, with respect to the model pion $Q-\bar{Q}$ and the model nucleon QQQ, indicate a value of μ pretty close to the pion mass, and a value of ξ on the order of 0.1 [7].

That is, starting from quark propagation as ordinarily conceived, *à la* $G_c(x, y|A)$, in the non-perturbative bound context, one could be lead to think of quark propagation in terms of *Levy-flights*, a picture which in view of color confinement, may look rather appealing indeed (Levy flights distributions are generalizations of gaussian distributions for independent random variables, complying with the 'central limit theorem').

Whatever its interpretation, though, the latter will be ruined if (8) does not provide a reliable enough evaluation of (7).

In order to deal with that issue, one may skip to the most achieved realization of a functional space, that is to the Wiener functional space [5]. Then, the following theorem can be proven:

Theorem: For all couple $(s_1, s_2) \in\,]0, s]\times]0, \bar{s}]$,
$\mathrm{m}\otimes m\left(\{(u,\bar{u}) \in \mathcal{C}_0^{0,s} \times \mathcal{C}_0^{0,\bar{s}} \mid u(s_1) = \bar{u}(s_2)\}\right) = 0$
$\mathrm{m}\otimes m\left(\{(u,\bar{u}) \in \mathcal{C}_0^{0,s} \times \mathcal{C}_0^{0,\bar{s}} \mid u(0) = \bar{u}(0) = 0\}\right) = 1$
with m, the Wiener measure on $\mathcal{C}_0^{0,s}$ whereas $m \otimes m$ is taken as the Wiener measure on the product of spaces $\mathcal{C}_0^{0,s} \times \mathcal{C}_0^{0,\bar{s}}$, endowed with the topology product. This is made possible thanks to the independence of the random variables u and $\bar{u}$.

This theorem proves that the left hand side of (8) is indeed proportional to $\delta(s_1)\delta(s_1)$. Then dimensional and symmetry arguments complete the right hand side of (8) . The proof is as follows.

Let A be the set $\{(u,\bar{u}) \in \mathcal{C}_0^{0,s} \times \mathcal{C}_0^{0,\bar{s}} \mid u(s_1) = \bar{u}(s_2)\}$. One has $A = \bigcap_{n=1}^{\infty} A_n$, where

$$A_n = \{(u,\bar{u}) \in \mathcal{C}_0^{0,s} \times \mathcal{C}_0^{0,\bar{s}} \mid -\frac{1}{n} \leq\ u(s_1) - \bar{u}(s_2) \leq +\frac{1}{n}\}$$

Because of the obvious inclusion, $\forall n, A_{n+1} \subset A_n$, one can write,

$$m \otimes m(A) = \lim_{n\to\infty} m \otimes m(A_n)$$

Now, $X_n \equiv m\otimes m(A_n)$ is given by

$$\begin{aligned} X_n = & m\otimes m\{(u,\bar u)\in \mathcal{C}_0^{0,s}\times \mathcal{C}_0^{0,\bar s} \mid u(s_1)-\frac{1}{n}\leq \ \bar u(s_2)\leq u(s_1)+\frac{1}{n}\} \\ = & \int_{-\infty}^{+\infty}\frac{\mathrm{d}x}{\sqrt{2\pi s_1}}\ e^{-\frac{x^2}{2s_1}}\int_{x-\frac{1}{n}}^{x+\frac{1}{n}}\frac{\mathrm{d}y}{\sqrt{2\pi s_2}}\ e^{-\frac{y^2}{2s_2}} \\ \equiv & \int_{-\infty}^{+\infty}\frac{\mathrm{d}x}{\sqrt{2\pi s_1}}\ e^{-\frac{x^2}{2s_1}}\ f_n(x) \end{aligned}$$

Since

$$|\frac{1}{\sqrt{2\pi s_1}}\ e^{-\frac{x^2}{2s_1}}\ f_n(x)|\leq \frac{1}{\sqrt{2\pi s_1}}\ e^{-\frac{x^2}{2s_1}}$$

one gets $X_\infty = 0$, as a mere consequence of the *Dominated convergence theorem.*

The presence of this surprising factor of $\delta^{(2)}(\vec{b})$ is thus unavoidable, and that issue bounces back and forth with the related emergence of the ξ parameter: Again, if this parameter is really an unavoidable output of this approach, more work is in order so as to make explicit the relations of ξ appearing in the confining potential between a pair of quarks separated by a distance r, to wit,

$$V(r)\simeq \xi\mu(\mu r)^{1+\xi} \tag{9}$$

to the levy-flight propagation modes of confined quarks, as well as to an unexpected non-commutative geometry aspect of the transverse plane.

4 A fruitful equivalence

Let $\mathcal{F}(A)$ stand for some functional of the A_μ-gauge-field. In QED, one has the long known (abelian) equivalence of

$$e^{-\frac{i}{2}\int\frac{\delta}{\delta A}D_c\frac{\delta}{\delta A}}\ \mathcal{F}(A)_{|A=0} = N\int \mathrm{d[A]}\ \mathrm{e}^{\frac{i}{2}\int \mathrm{AD_c^{-1}A}}\,\mathcal{F}(\mathrm{A}) \tag{10}$$

which is nothing else, indeed, that an expression of a Wick's theorem underlying structure. For the very same reason (Wick's Theorem), the same equivalence extends to the non-abelian case of QCD where the abelian A_μ becomes A_μ^a, and where D_c is given by the gluonic propagator (2), in a covariant gauge.

• A consequence of some concern of (10) is that the QCD property of effective locality can be displayed by using the left hand side of (10), whereas the equivalent right hand side does not allow one to ever detect it [2].

• Itself, the QCD effective locality property, could allow one to recognize somewhat the intrinsic perturbative character of the QCD BRST generating functional, as recognized recently by C. Becchi himself [3]. In here, if one introduces Fadeev-Popov ghost fields so as to quantize the theory while preserving its full gauge-invariance, unitarity, and factoring out the infinite volume gauge group orbit, then, the effective locality property seems to become out of reach. And not very surprisingly then, a reminiscence of the original covariant gauge dependence shows up in the final expressions of the QCD amplitudes. This is most suggestive of the non-perturbative character of that effective locality property, as well as it could inspire some new approach to the Gribov-Singer copies long standing issue [4]. Note that the unfathomed problem of non-perturbative quantization is also lurking here ..

5 A non-abelian simplification

This brings about a further remarkable (non-abelian) simplification. Effective locality in effect, allows one to define the infinite dimensional functional integrations over Halpern's field $\chi^a_{\mu\nu}$ configuration space: That is, functional integrals with measure written symbolically as,

$$\int \mathrm{d}[\chi] = \prod_{i\in\mathcal{M}} \prod_{a=1}^{N^2-1} \prod_{\mu<\nu,0}^{3} \int \mathrm{d}[\chi^a_{\mu\nu}](x_i) \tag{11}$$

eventually reduce to ordinary Lebesgue integrations over finite-dimensional R^n-spaces. As well known, such a reduction is not possible in general, and is here made possible thanks to the effective locality property of QCD.

In order to catch some intuition of this interesting non-abelian simplification, one has to resort to the standard trick of the generating functional construction, splitting spacetime in an infinite series of spacetime cells centered at given points [8]. Effective locality determines a unique point, say w_0, where the interaction takes place, whereas all of the other spacetime cells will accordingly yield contributions that cancel out with their proper normalization. As a result, in the concrete realization of Wiener functional space, it is easy to see that the $6\times(N^2-1)$ infinite-dimensional functional integrations get reduced to a R^n integration because a functional like

$$\{\chi\} \mapsto F(\ldots \chi^a_{\mu\nu}(w_0)\ldots)$$

depend solely on the values n-functions take at a given point w_0.

Rather clear intuitively [1], a rigorous proof can be given in Wiener functional space by making use of an adapted form of the Wiener's integration formula [5],

$$\int_{\mathcal{W}} f\left(x(s_1),\ ..\ , x(s_n)\right) dm(x) = \int_{R^n} \frac{f\left(u_1,\ .., u_n\right) e^{-\sum_1^n \frac{(u_j-u_{j-1})^2}{2(s_j-s_{j-1})}}}{\sqrt{s_1..(s_n-s_{n-1})}} d\vec{U}_n \tag{12}$$

where $\vec{U_n}$ is a shorthand for a R^n vector which would be made out of the n u_j-values. In (12), it is made manifest that one has an infinite dimensional functional space on the left hand side, whereas integration is performed on a finite dimensional vectorial space on the right hand side (the key result making that interesting equality possible is the '*measure image theorem*' [5]).

Further on, since the $\chi^a_{\mu\nu}$ fields can be $SU(3)$- Lie-algebra valuated in the adjoint representation:

$$\chi^a_{\mu\nu} \rightarrow \sum_{a=1}^{N^2-1} \chi^a_{\mu\nu} T^a$$

it is the full calculational power of '*Random Matrix*' that can be used.

6 A short sequence of fecond equivalences

Let H be an element of $\mathcal{M}_n(C)$, the algebra of hermitian $n' \times n'$ traceless random matrices. Such a matrix can be parametrized with the help of its n eigenvalues, and l other parameters p_i, $1 \leq i \leq n^2 - n - 1$. Over the algebra $\mathcal{M}_n(C)$, we have the measure,

$$d[H] = \prod_{1 \leq i < j \leq n} (\Theta_i - \Theta_j)^2 \mathrm{d}\Theta_1 \,..\, \mathrm{d}\Theta_{n'} \\ \times \quad f(p)\ \mathrm{d}p_1 \,..\, \mathrm{d}p_l$$

where $f(p) = f(p_1,\ ..,p_l)$ is an unspecified probability distribution for the set of parameters $(p_1,\ ..,p_l)$. However, a most interesting feature of this translation in terms of random matrix, is that in the course of Green's functions actual calculations, the dependences over the $p_i's$- extra parameters with unknown $f(p)$ probability distribution, factor out and cancel in the normalization, leaving integrations over the eigenvalue's spectra only.

To summarize, we have the following equivalence

$$\int \mathrm{d[A]}\, \mathrm{e}^{\frac{\mathrm{i}}{2}\int \mathrm{AD_c^{-1}A}}\, \mathcal{F}(\mathrm{A}) \stackrel{A \rightarrow NA}{=} e^{-\frac{i}{2}\int \frac{\delta}{\delta A} D_c \frac{\delta}{\delta A}}\ \mathcal{F}(A)_{|A=0}$$

where the superscript $A \rightarrow NA$ means that the long known Abelian equivalence extends to the Non Abelian case of QCD. Then, in a second step, relying on this equivalence one can write

$$\int \mathrm{d[A]}\, \mathrm{e}^{\frac{\mathrm{i}}{2}\int \mathrm{AD_c^{-1}A}}\, \mathcal{F}(\mathrm{A}) \stackrel{EL}{=} N' \int_{\mathcal{W}^n} \prod \mathrm{d}\left[\chi^a_{\mu\nu}(w_0)\right] e^{\frac{i}{4}\sum(\chi^a_{\mu\nu}(w_0))^2}\ \ (\ldots)\ \ \mathcal{G}(\chi(w_0))$$

where the dots stand for subsidiary integrations on Fradkin fields and other Lagrange multipliers, and where the specific form of the right hand side expression is due to the effective locality property (meant by the superscript EL). At 4 spacetime dimensions and with $SU(3)$, the functional integration is to be performed on the direct product of $6 \times (N^2-1)$ 'standard Wiener functional spaces' [5].

Eventually, in the case of every $2n$-point fermionic correlators, by Lie algebra valuating the χ-fields, and with the help of some convenient changes of variables, one can re-express the QCD amplitudes under a form that takes advantage of '*Random Matrix*', and lends itself to concrete calculations

$$\int \mathrm{d[A]}\, \mathrm{e}^{\frac{\mathrm{i}}{2}\int \mathrm{A D_c^{-1} A}}\, \mathcal{F}(\mathrm{A}) \stackrel{F \to L}{=} N'' \int_{\mathcal{M}_n(C)} \mathrm{d}[H]\, \delta(Tr\, H)\; e^{\frac{i}{4C_A} Tr\,(H)^2} \;\; (\ldots)\, \mathcal{G}(H)$$

where $C_A = N$, and where the superscript of $F \to L$ means that we have passed from an infinite dimensional functional space (F) to a finite product of ordinary Lebesgue (L) integrals, in agreement with the non-abelian simplification of Section 5, itself a consequence of effective locality. It turns out that in the right hand side, only the spectrum of the random matrix H enters into play.

7 Conclusion

Effective locality seems to be an exact property of the full non-approximate QCD theory, which has been overlooked for decades because functional differentiation, instead of functional integration, is a necessary procedure to be followed in order to display it.

However remarkable in itself, what effective locality means is perhaps something wider: Indeed, a most dual expression of ordinary QCD amplitudes may have been achieved in the spirit of what had been proposed in [9], in the pure Yang Mills case. If one looks at things in this way, then, effective locality is but an inherent feature of a proper dual formalism, and nothing else.

Of course, the strict dual relations, like $g \leftrightarrow 1/g$, which are satisfied in the pure Yang Mills case of [9], have here to be enlarged so as to account for the fact that the full QCD amplitudes display a much richer structure in terms of scaling laws in g than in the pure Yang Mills case. To wit, we have

$$\frac{i}{2}\int d^4x\; \mathcal{Q}(x)\cdot\left(\mathcal{K}_{\mathcal{S}} + g(f\cdot\chi)\right)^{-1}(x)\cdot \mathcal{Q}(x)$$

with

$$\mathcal{O}(\mathcal{Q}) = \mathcal{O}(1) + \mathcal{O}(g)$$

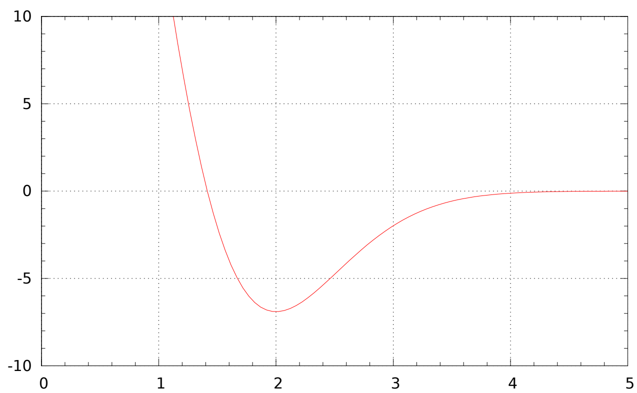
and where the spin related contributions scale as follows

$$\mathcal{O}(\mathcal{K}_{\mathcal{S}}) = \mathcal{O}(g^2)$$

where we have, in the pure Yang Mills case,

$$\frac{i}{2g}\int d^4x\ \partial^\lambda \chi^a_{\lambda\mu}(x)\cdot\Big((f\cdot\chi)^{-1}\Big)^{\mu\nu}_{ab}(x)\cdot\partial^\sigma\chi^b_{\sigma\nu}(x)$$

Clearly much work is needed in order to explore the consequences of that unexpected property of the QCD amplitudes, at both qualitative and quantitative levels. In the former case, for example, it is interesting to see the light that this non-perturbative approach to QCD sheds on the difference between pure Yang Mills and full QCD theories. To our knowledge at least, we do not know of any other approach able to discriminate between them in this way. Likewise, it is encouraging to observe that, aside from some structural properties that go in the sense of known results [10], first simple calculations come out in compliance with other known results too: Such is the case of the already mentionned confining potential of (9). But one may also propose an explicit construction of a model neutron-proton binding to form a deuteron, and the result can be displayed as the following picture:



$V(x) = V_0(2-x^2)\ \exp[-(1/2)\,x^2]$

The potential must go negative for large r, with parameters appropriate to form a deuteron of Binding Energy = 2.2 MeV. This result closely resembles the average of Jastrow's 1951 singlet and triplet potentials. To our knowledge, this is the first example of Nuclear Physics from basic QCD.

References

[1] H.M. Fried, Y. Gabellini, T. Grandou, Y.-M. Sheu, Eur. Phys. J. C.**65**, 395, 411 (2010).

[2] H. M. Fried, M. Gattobigio, T. Grandou, Y.-M. Sheu, arXiv:1003.2936 [hep-th].

[3] http://www.scholarpedia.org/article/Becchi-Rouet- Stora-Tyutin-symmetry.

[4] B. Candelpergher, T. Grandou, work in completion.

[5] G.W. Johnson and M.L. Lapidus, *The Feynman Integral and Feynman's Operational Calculus* (Oxford University Press, Oxford 2000).

[6] H.M. Fried, *Functional Methods and Eikonal Models* (Editions Frontières, 1990).

[7] H.M. Fried, Y. Gabellini, T. Grandou, Y.-M. Sheu, arXiv:hep-th/1104.4663.

[8] A. Zee, *Quantum Field Theory in a Nutshell*, 2nd Ed. (Princeton University Press, Princeton 2010).

[9] H. Reinhardt, K. Langfeld, L. v. Smekal, Phys. Lett. B **300**, 111, (1993).

[10] K. Huang, D.R. Stump, Phys. Rev. Lett. **37**, 545 (1976).

Discussion

E. Iancu (CEA Saclay): : The field $\chi^a_{\mu\nu}$ is anti-symmetric, isn't it?

Grandou: : Yes it is ... However the effective resulting structure melts together internal and external degrees into an overall symmetric structure (see E. Iancu's talk)

A. Jevicky (Brown University): : I think that V.P. Nair, in $2+1$ spacetime dimensions, obtained results that look similar to yours.

B. Müller (Duke University): : Do you have anything like a 'string tension' in term of which to calculate a meson mass, for example?

Grandou: : In the plane transverse to the collision axis, a mass parameter must be introduced so as to describe and limit the transverse momentum distribution of the scattering quarks, and masses can be obtained from this parameter. There are reasons to think that this mass term should be on the order of the π-meson's mass, still, this is just a conjecture. Calculations are certainly in order so as to check its reliability, but it is true that first approximate calculations fit it.

B. Müller (Duke University): : Can you calculate analytically?

Grandou: : At least as long as fermionic loops are neglected (if not quark's spins altogether, but this has to be checked), it is quite remarkable that calculations can be carried through analytically, by means of ‘ Random Matrices’. Beyond these two approximations, I do not know yet.