We extract directly (for the first time) the heavy-baryons (spin 1/2 and 3/2) mass-splittings due to SU(3) breaking using double ratios of QCD spectral sum rules (QSSR) in full QCD, which are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios commonly used for determining the heavy baryon masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. We extract directly (for the first time) the heavy-baryons (spin 1/2 and 3/2) mass-splittings due to SU(3) breaking using double ratios of QCD spectral sum rules (QSSR) in full QCD, which are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios commonly used for determining the heavy baryon masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. We extract directly (for the first time) the heavy-baryons (spin 1/2 and 3/2) mass-splittings due to SU(3) breaking using double ratios of QCD spectral sum rules (QSSR) in full QCD, which are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios commonly used for determining the heavy baryon masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. We extract directly (for the first time) the heavy-baryons (spin 1/2 and 3/2) mass-splittings due to SU(3) breaking using double ratios of QCD spectral sum rules (QSSR) in full QCD, which are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios commonly used for determining the heavy baryon masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses. As a result, we provide (relatively) robust predictions of the Ω∗Q masses.

1. Introduction

QSSR [1,2] à la SVZ [3] has been used earlier in full QCD [4–6] and in HQET [7] for understanding charming and beautiful baryons masses. Recent observations at Tevatron of families of b-baryons [8,9] and of the Λ∗ baryon by Babar and Belle [10] have stimulated different recent theoretical activities for understanding their nature [11–16,18]. QSSR results are in quite good agreement with recent experimental findings but with relatively large uncertainties. The inaccuracy of these results is mainly due to the value of the heavy quark mass and of its ambiguous definition when working to LO in the radiative αs corrections in full QCD and HQET [1], where the heavy quark mass is the main driving term in the QCD expression of the baryon two-point correlator used in the QSSR analysis. Another source of uncertainty is the effect of the QCD continuum which parametrizes the higher baryon masses contributions to the spectral function and the ad hoc choices of interpolating baryon currents used in different literatures. In this paper, we shall concentrate on the analysis of the heavy baryons mass-splittings due to SU(3) breaking using double ratios (DR) of QCD spectral sum rules (QSSR), which are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios used in the literature to determine the absolute value of heavy baryon masses.

• For the spin 1/2 baryons, and following Ref. [4], we work with the lowest dimension currents:

\[
\begin{align*}
\eta_{Q} &= \epsilon_{abc} \left[ (q_a T C \gamma_5 s_b) + b(q_3 T C s_b) \gamma_5 \right] Q_c,
\eta_{\Lambda_Q} &= \eta_{Q} \quad (s \to q),
\eta_{B_Q} &= \epsilon_{abc} \left[ (s_a T C \gamma_5 Q_b) + b(s_3 T C Q_b) \gamma_5 \right] s_c,
\eta_{\Sigma_Q} &= \eta_{B_Q} \quad (s \to q),
\end{align*}
\]

where b is a priori an arbitrary mixing parameter. Its value has been found to be: \(b = -1/5\),

in the case of light baryons [29] and in the range [4–6]:

\[-0.5 \leq b \leq 0.5\],

(3)

for non-strange heavy baryons. The corresponding two-point correlator reads:

\[
S(q) = i \int d^4x \, e^{iqx} \langle 0 | \bar{Q} \eta_Q(x) \eta_Q(0) | 0 \rangle = \bar{q} F_1 + F_2,
\]

(4)

where \(F_1\) and \(F_2\) are two invariant functions.

• For the spin 3/2 baryons, we follow Ref. [5] and work with the interpolating currents:

\[
\begin{align*}
\eta_{\Lambda_Q}^{\mu} &= \frac{1}{\sqrt{2}} \left[ (q \gamma_5 \gamma_{\mu} Q) s + (s \gamma_5 C \gamma_{\mu} Q) q \right]
\equiv \eta_{\Lambda_Q}^{(0)},
\eta_{B_Q}^{\mu} &= \frac{1}{\sqrt{2}} \left[ (q_{3} \gamma_5 \gamma_{\mu} Q) s + (s_{3} \gamma_5 C \gamma_{\mu} Q) q \right]
\equiv \eta_{B_Q}^{(0)},
\eta_{\Sigma_Q}^{\mu} &= \frac{1}{\sqrt{2}} \left[ (q \gamma_{\mu} Q) s + (s \gamma_{\mu} C Q) q \right]
\equiv \eta_{\Sigma_Q}^{(0)},
\end{align*}
\]

where an anti-symmetrization over colour indices is understood. The normalization in Eq. (5) is chosen in such a way that in all cases one gets the same perturbative contribution. The corresponding two-point correlator reads:

\[
S^{\mu \nu}(q) = i \int d^4x \, e^{iqx} \langle 0 | \bar{Q} \eta_Q^{(0)}(x) \eta_Q^{(0)}(0) | 0 \rangle
\equiv g^{\mu \nu} (\bar{q} F_1 + F_2) + \ldots
\]

(6)

2. The spin 1/2 two-point correlator in QCD

In this letter, we extend the previous analysis in [4,5] by including the new SU(3) breaking \(m_s\) correction terms.

• The \(\Lambda_Q(Q qq)\) and \(\Xi_Q(Q sq)\) baryons

The expression for \(\Lambda_Q\) has been (first) obtained in the chiral limit \(m_q = 0\) in [5], and the one of \(\Xi_Q\) including \(SU(3)\) breaking in [14]. One can notice that due to the expression of the current the \(m_s\) corrections vanish to leading order in \(\alpha_s\) for the perturbative term, while the \(\Delta = 6\) condensates for the \(SU(2)\) case of [5] needs the following replacement in the \(SU(3)\) case:

\[
\rho(\bar{q} q)^2 \rightarrow \rho(\bar{q} q)(\bar{s} s),
\]

(7)
where \( \rho = 2 \sim 3 \) indicates the violation of the four-quark vacuum saturation \([31, 1, 27]\). The additional \( SU(3) \) breaking corrections for the \( \Xi_Q \) are \([14]\):
- \( F_1 \):
  \[
  \text{Im} F_{1s}^{m_{s}} |_{s_s} = -\frac{m_s}{2\pi} \left( 1 - x^2 \right) \left[ (1 - b^2) \langle \bar{q}q \rangle - \frac{(1 - b^2)}{2} \langle \bar{s}s \rangle \right].
  \]
  \[
  F_{1s}^{m_{s}} |_{m_{s}} = \frac{m_s}{2^{5/2} \pi m_Q^2 - q^2} \left( \langle \bar{s}G_s \rangle \left( 1 + b^2 \right) + \langle \bar{q}Gq \rangle (1 - b^2) \right),
  \]
  \[
  F_{1s}^{m_{s}} |_{m_{s}} = \frac{m_s m_Q}{2^{3/2} \pi \left( m_Q^2 - q^2 \right)} \left( \langle \bar{s}G_s \rangle \left( 1 + b^2 \right) + \langle \bar{q}Gq \rangle (1 - b^2) \right),
  \]

- \( F_2 \):
  \[
  \text{Im} F_{2s}^{m_{s}} |_{s_s} = -\frac{m_s m_Q}{2^{5/2} \pi} \left( 1 - x \right) \left[ (1 + b^2) \langle \bar{q}q \rangle - \frac{(1 - b^2)}{2} \langle \bar{s}s \rangle \right],
  \]
  \[
  F_{2s}^{m_{s}} |_{m_{s}} = \frac{m_s m_Q}{2^{5/2} \pi \left( m_Q^2 - q^2 \right)} \left( \langle \bar{s}G_s \rangle \left( 1 + b^2 \right) + \langle \bar{q}Gq \rangle (1 - b^2) \right),
  \]

where \( x \equiv m_Q^2 / s \) and \( \langle \bar{s}G_s \rangle \equiv g \langle \bar{s} \sigma_{\mu \nu} \lambda_a / 2 G_{\mu \nu} \rangle \).

\* The \( \Sigma_Q(Qq\bar{q}) \) and \( \Omega_Q(Qs\bar{s}) \) baryons

The expression for \( \Sigma_Q \) has been (first) obtained in \([4]\). The additional \( SU(3) \) breaking terms for the \( \Omega_Q \) are:
- \( F_1 \):
  \[
  \text{Im} F_{s}^{m_{s}} |_{pert} = \frac{3 m_s m_Q^3}{2^{8/3} \pi^3} \left( 1 - b^2 \right) \times \left[ \frac{2}{x} - 3 + 6(x + x^2 + 6 \ln x) \right],
  \]
  \[
  \text{Im} F_{s}^{m_{s}} |_{s_s} = -\frac{3 m_s \langle \bar{s}G_s \rangle}{2^{1/2} \pi} \left( 1 + b^2 \right) \left( 1 - x^2 \right),
  \]
  \[
  F_{s}^{m_{s}} |_{m_{s}} = -\frac{m_s \langle \bar{s}G_s \rangle}{2^{1/2} \pi \left( m_Q^2 - q^2 \right)} \left( \frac{1}{m_Q^2 - q^2} \right) \times \left[ 7 + 22b + 7b^2 \right] - 6(1 + b^2) \left( 1 - x^2 \right) \int_{0}^{1} \frac{da}{m_Q^2 - (1 - a)q^2} \left[ (1 - 3a) \langle \bar{s}s \rangle + \langle \bar{q}q \rangle \right],
  \]
  \[
  F_{s}^{m_{s}} |_{D=6} = -\frac{m_s m_Q \rho(\bar{s}s \langle \bar{q}q \rangle)}{8 \left( m_Q^2 - q^2 \right)^2}.
  \]
- \( F_2 \):
  \[
  \text{Im} F_{2s}^{m_{s}} |_{pert} = \frac{3 m_s m_Q^5}{2^{8/3} \pi^3} \left( 1 - b^2 \right) \times \left[ \frac{1}{x^2} - 6 \left( 6 + 3 + 6 \ln x \right) \right],
  \]
  \[
  \text{Im} F_{2s}^{m_{s}} |_{s_s} = -\frac{3 m_s m_Q \langle \bar{s}G_s \rangle}{2^{1/2} \pi} \left( 3 + 2b + 3b^2 \right) \left( 1 - x \right),
  \]
  \[
  F_{2s}^{m_{s}} |_{m_{s}} = \frac{m_s m_Q \langle \bar{s}G_s \rangle}{2^{1/2} \pi \left( m_Q^2 - q^2 \right)} \times \left[ \frac{1}{m_Q^2 - q^2} \right] \left( 25 + 22b + 25b^2 \right)
  \]

We have checked the existing results in \([4]\) obtained in the chiral limit and all our previous results agree with these ones.

\* The \( \Sigma_Q(Qq\bar{q}) \) and \( \Xi_Q(Qs\bar{s}) \) baryons

The additional terms and replacement due to \( SU(3) \) breaking for the \( \Xi_Q \) compared with the one of the \( \Sigma_Q(Qq\bar{q}) \) in \([5]\) are:
- \( F_1 \):
  \[
  \text{Im} F_{1s}^{m_{s}} |_{pert} = \frac{m_s m_Q^3}{4 \pi^2 \left( m_Q^2 - q^2 \right)^2} \left[ \frac{2}{x} + 3 - 6x + x^2 + 6 \ln x \right],
  \]
  \[
  \text{Im} F_{1s}^{m_{s}} |_{s_s} = \frac{m_s m_Q}{6 \pi} \left( \langle \bar{q}q \rangle + \langle \bar{s}s \rangle \right) \left( 1 - x \right)^2,
  \]
  \[
  \text{Im} F_{1s}^{m_{s}} |_{m_{s}} = \frac{m_s m_Q^5}{144 \pi^2 \left( m_Q^2 - q^2 \right)^2} \left( 12 \langle \bar{q}q \rangle - 9 \langle \bar{s}s \rangle \right) + \left( 1 - x \right)^3 \langle \bar{s}s \rangle + \langle \bar{q}q \rangle \right),
  \]
  \[
  \text{Im} F_{1s}^{m_{s}} |_{m_{s}} = \frac{7 m_Q^5}{32 \pi^2 \left( m_Q^2 - q^2 \right)^2} \left( \langle \bar{q}q \rangle + \langle \bar{s}s \rangle \right) \frac{x^2}{m_Q^2},
  \]
  \[
  F_{1s}^{m_{s}} |_{D=6} = \frac{m_s m_Q^5}{144 \pi^2 \left( m_Q^2 - q^2 \right)^2} \left( \frac{12 \langle \bar{q}q \rangle - 9 \langle \bar{s}s \rangle}{m_Q^2 - q^2} \right) + \left( 1 - x \right)^3 \langle \bar{s}s \rangle + \langle \bar{q}q \rangle \right),
  \]

- \( F_2 \):
  \[
  \text{Im} F_{2s}^{m_{s}} |_{pert} = \frac{m_s m_Q^5}{192 \pi^3 \left( m_Q^2 - q^2 \right)^2} \left[ \frac{3}{x^2} - \frac{16}{x} + 12x - 12 \ln x \right],
  \]
  \[
  \text{Im} F_{2s}^{m_{s}} |_{s_s} = \frac{m_Q^5}{18 \pi} \left( \langle \bar{q}q \rangle + \langle \bar{s}s \rangle \right) \frac{2 - x^2}{x^2 - 3 + x^2},
  \]
  \[
  \text{Im} F_{2s}^{m_{s}} |_{m_{s}} = \frac{m_s m_Q^5}{12 \pi} \left( \langle \bar{q}q \rangle + \langle \bar{s}s \rangle \right) \left( 1 + 3 \langle \bar{s}s \rangle \right) \left( 1 - x \right),
  \]
  \[
  F_{2s}^{m_{s}} |_{D=6} = \frac{m_s m_Q M_0^2}{72 \pi} \left( \frac{3 \langle \bar{q}q \rangle - \langle \bar{s}s \rangle}{m_Q^2 - q^2} \right) \left( 1 + 3 \langle \bar{s}s \rangle \right).
  \]
\[ \langle \bar{q} q \rangle = \int_0^1 \frac{d\alpha}{m_Q^2 - (1 - \alpha)q^2} , \]

\[ F_2|_{D=6} = \frac{2 m_Q \rho(\bar{s}s) \langle \bar{q} q \rangle}{3 (m_Q^2 - q^2)} , \]

\[ F_{m^*}|_{D=6} = \frac{2 m_s m_Q^2 \rho(\bar{s}s) \langle \bar{q} q \rangle}{9 (m_Q^2 - q^2)^2} , \]

where \( x = m_Q^2/s \) and \( \langle \bar{s} G s \rangle = g \langle \bar{s} \sigma_{\mu\nu} \lambda_\alpha/2G^\mu\nu s \rangle = M_Q^2(\bar{s}s) \).

- The \( \Omega^*_Q(\bar{Q}s) \) baryons

Compared with the expression of the \( \Sigma_Q^*(\bar{Q}q) \) in [5], the additional \( SU(3) \) breaking terms for the \( \Omega^*_Q \) are:

\[ F_{1|_{\text{mix}}} = \frac{m_s m_Q^2}{24\pi^2} \left[ \frac{2}{x^2} + 6 \ln x \right] , \]

\[ F_{m^*|_{\text{mix}}} = -\frac{m_s M_Q^2(\bar{s}s)}{2\pi^2} \left[ \frac{3}{m_Q^2 - q^2} + \int_0^1 \frac{d\alpha}{m_Q^2 - (1 - \alpha)q^2} \right] , \]

\[ F_{m^*|_{D=6}} = -\frac{4 m_s m_Q \rho(\bar{s}s)^2}{9 (m_Q^2 - q^2)^2} . \]

- \( F_2 \):

\[ \text{Im} F_{2|_{\text{pert}}} = \frac{m_s m_Q^2}{96\pi^3} \left[ \frac{3}{x^2} - \frac{16}{x} + 12 + x^2 - 12 \ln x \right] , \]

\[ \text{Im} F_{2|_{\text{mix}}} = -\frac{m_s m_Q (\bar{s}s)}{6\pi} \left[ 5 - 6x + x^2 \right] , \]

\[ F_{2|_{\text{mix}}} = \frac{m_s m_Q M_Q^2(\bar{s}s)}{36\pi^2} \left[ \frac{6}{m_Q^2 - q^2} + \int_0^1 \frac{d\alpha}{m_Q^2 - (1 - \alpha)q^2} \right] , \]

\[ F_{2|_{D=6}} = -\frac{4 m_s m_Q^2 \rho(\bar{s}s)^2}{9 (m_Q^2 - q^2)^2} . \]

We have checked the existing results in [5] obtained in the chiral and \( SU(2) \) limits and agree with these ones.

4. Form of the sum rules and QCD inputs

We parametrize the spectral function using the standard duality ansatz: “one resonance” plus “QCD continuum”. The QCD continuum starts from a threshold \( t_c \) and comes from the discontinuity of the QCD diagrams. Transferring its contribution to the QCD side of the sum rule, one obtains the finite energy Laplace/Borel sum rules:

\[ |\lambda_{B_Q^1(\bar{Q})}|^2 M_{B_Q^1(\bar{Q})} e^{-M_{B_Q^1(\bar{Q})^2 \tau}} = \int_{t_c}^{t_q} ds e^{-s\tau} \frac{1}{\pi} \text{Im} F_2(s) , \]

\[ |\lambda_{B_Q^2(\bar{Q})}|^2 M_{B_Q^2(\bar{Q})} e^{-M_{B_Q^2(\bar{Q})^2 \tau}} = \int_{t_c}^{t_q} ds e^{-s\tau} \frac{1}{\pi} \text{Im} F_1(s) , \]

where \( \lambda_{B_Q^1(\bar{Q})} \) and \( M_{B_Q^1(\bar{Q})} \) are the heavy baryon residue and mass; \( \tau \equiv 1/M^2 \) is the sum rule variable. Consistently, we also take into account the \( SU(3) \) breaking at the quark and continuum threshold:

\[ \sqrt{t_c}|SU(3) \simeq (\sqrt{t_c}|SU(2) \equiv m_Q) + \bar{m}_{q_1} + \bar{m}_{q_2} , \]

\[ \sqrt{t_c}|SU(3) \simeq (\sqrt{t_c}|SU(2) \equiv \sqrt{t_c} + \bar{m}_{q_1} + \bar{m}_{q_2} , \]

where \( q_{1,2} \equiv q \) or \( s \) depending on the channel. \( \bar{m}_{q_1} \) are the running light quark masses. \( m_Q \) is the heavy quark mass, which we shall take in the range covered by the running and on-shell mass (see Table 1) because of its ambiguous definition when working to LO. One can estimate the baryon masses from the following ratios:

\[ R_i^q = \frac{\int_{t_c}^{t_q} ds e^{-s\tau} \text{Im} F_i(s)}{\int_{t_c}^{t_q} ds e^{-s\tau} \text{Im} F_i(s)} , \]

\[ R_{21}^q = \frac{\int_{t_c}^{t_q} ds e^{-s\tau} \text{Im} F_2(s)}{\int_{t_c}^{t_q} ds e^{-s\tau} \text{Im} F_1(s)} , \]

where at the \( \tau \)-stability point:

\[ M_{B_Q^1(\bar{Q})} \simeq \sqrt{R_2^q} \simeq R_{21}^q \]

These quantities have been used in the literature for getting the baryon masses and lead to a typical uncertainty of 15-20\% [4-6]. In order to circumvent these problems, we work with the double ratio of sum rules (DR)[20]:

\[ r_i^{sd} = \sqrt{R_i^q} , \quad r_{21}^{sd} = \frac{R_{21}^q}{R_{21}^q} , \]

which take directly into account the \( SU(3) \) breaking effects. These quantities are obviously less sensitive to the choice of the heavy quark masses and to the value of the continuum threshold than the simple ratios \( R_i \) and \( R_{21} \). Analogous DR quantities have been used successfully (for the first time) in [20] for studying the mass ratio of the \( 0^{++}/0^{-} \) and \( 1^{++}/1^{-} \) B-mesons, in [21] for extracting \( f_{B_s}/f_B \), in [22] for estimating the \( D \to K/D \to \pi \) semi-leptonic form factors and in [23] for extracting the strange quark mass from the \( e^+e^- \to \pi = 1.0 \) data. For the numerical analysis we shall introduce the RGI quantities \( \tilde{m} \) and \( m \) [24]:

\[ \tilde{m}_q(\tau) = \frac{\bar{m}_q}{(-\log \sqrt{t_A})^{2/\beta_1}} , \]

\[ \langle \bar{q} q \rangle(\tau) = \frac{\bar{\mu}_3}{(-\log \sqrt{t_A})^{2/\beta_1}} , \]

\[ \langle \bar{q} G q \rangle(\tau) = \frac{\bar{\mu}_3}{(-\log \sqrt{t_A})^{1 - 3\beta_1} M_Q^2} \]

where \( \beta_1 \) is the first coefficient of the \( \beta \) function for \( n \) flavours. We have used the quark masses.

\[ \tilde{m}_q(\tau) = \frac{\bar{m}_q}{(-\log \sqrt{t_A})^{2/\beta_1}} , \]

\[ \langle \bar{q} G q \rangle(\tau) = \frac{\bar{\mu}_3}{(-\log \sqrt{t_A})^{1 - 3\beta_1} M_Q^2} \]

where \( \beta_1 \) is the first coefficient of the \( \beta \) function for \( n \) flavours. We have used the quark mass.
mass and condensate anomalous dimensions reviewed in [1]. We shall use the QCD parameters in Table 1. At the scale where we shall work, and using the parameters in the table, we deduce:

$$\rho = 2.1 \pm 0.2 , \quad (22)$$

which controls the deviation from the factorization of the four-quark condensates. We shall not include the $1/g^2$ term discussed in [25,26] which is consistent with the LO approximation used here as the latter has been motivated for a phenomenological parametrization of the larger order terms of the QCD series.

Table 1
QCD input parameters. For the heavy quark masses, we use the range spanned by the running $\overline{\text{MS}}$ mass $\overline{m}_0(M_2)$ and the on-shell mass from QSSH compiled in page 602,603 of the book [1].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda$</td>
<td>$(353 \pm 15)$ MeV</td>
<td>[27,9]</td>
</tr>
<tr>
<td>$\overline{m}_d$</td>
<td>$(6.1 \pm 0.5)$ MeV</td>
<td>[28,1,9]</td>
</tr>
<tr>
<td>$\overline{m}_s$</td>
<td>$(114.5 \pm 20.8)$ MeV</td>
<td>[28,1,9]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$(263 \pm 7)$ MeV</td>
<td>[28,1]</td>
</tr>
<tr>
<td>$\kappa \equiv \langle \overline{s}s \rangle / (\overline{d}d)$</td>
<td>$(0.7 \pm 0.1)$ GeV$^2$</td>
<td>[28,1]</td>
</tr>
<tr>
<td>$M_0^2$</td>
<td>$(6.8 \pm 0.1)$ GeV$^2$</td>
<td>[29,30,20]</td>
</tr>
<tr>
<td>$\langle \alpha_s G^2 \rangle$</td>
<td>$(4.5 \pm 0.3) \times 10^{-4} \text{ GeV}^6$</td>
<td>[27,31]</td>
</tr>
<tr>
<td>$\rho_{c,s} (\overline{d}d)^2$</td>
<td>$(1.18 \sim 1.47)$ GeV</td>
<td>[1,28,36,9]</td>
</tr>
<tr>
<td>$m_c$</td>
<td>$(4.18 \sim 4.72)$ GeV</td>
<td>[1,28,36,9]</td>
</tr>
</tbody>
</table>

5. The masses of the spin 1/2 octet baryons
As a preliminary step of the analysis, we check the different results obtained in full QCD and in the chiral limit [4,5]:

$$M_{\Sigma^+} = (2.45 \sim 2.94) \text{ GeV} ,$$
$$M_{\Sigma^0} = (5.70 \sim 6.62) \text{ GeV} ,$$
$$M_{\Sigma^+} - M_{\Lambda^0} \leq 207 \text{ MeV} ,$$
$$M_{\Sigma^0} - M_{\Lambda^0} \leq 163 \text{ MeV} , \quad (23)$$

which we confirm. However, we have not tried to improve these results due to the ambiguity in the definition of the heavy quark mass input mentioned earlier at LO.

$\Xi_c (csq)/\Lambda_c (cqq)$

- Choice of the sum rules: in so doing we choose (after iterations) $b = 0$ and study in Fig. 1a) and Fig. 1b), the $\tau$-behaviour of the double ratio of sum rules (DR) for two extremal values of $t_c$ ($M_{\Sigma^+}^2$: beginning of $\tau$-stability and 12 GeV$^2$: beginning of $t_c$-stability). The appearance of the extremas for $\tau \sim 0.3 \text{ GeV}^{-2}$ depends strongly on the value of $t_c$ at which one cannot extract an optimal result. Therefore, among the three DR, we retain $r_{2d}^{sd}$ continuous (red) which is the most stable in $\tau$ and $t_c$.

- Choice of the currents: we show in Fig. 1c), by fixing $\tau = 0.9 \text{ GeV}^{-2}$ and $t_c = 12 \text{ GeV}^2$ the $b$-behaviour of the result which is very stable like in the range given in Eq. (3). However, this generous range does not favour the ad hoc choice around 1 used in the existing literature [14,15]. The absolute extremum happens at $b = 0$, which is compatible with the one $b = -1/5$ in Eq. (3).

Figure 1. $\Xi_c / \Lambda_c$: a) $\tau$-behaviour of the double ratio of sum rules (DR) given $b = 0$ and $t_c = M_{\Sigma^+}^2 = 6 \text{ GeV}^2$: $r_{2d}^{sd}$ dashed-dotted (blue), $r_{2d}^{dd}$ dotted (green), $r_{2d}^{1d}$ continuous (red); b) the same as a) but for $t_c = 12 \text{ GeV}^2$; c) $b$-behaviour of $r_{2d}^{sd}$ for given values of $\tau = 0.9 \text{ GeV}^{-2}$ and $t_c = 12 \text{ GeV}^2$; d) $t_c$-behaviour of the DR given $b = 0$ and $\tau = 0.9 \text{ GeV}^{-2}$.

for the light baryon systems [29]. For definiteness, we shall work with:

$$b = - (1/5 \sim 0) . \quad (24)$$

- $t_c$ stabilities: we show in Fig. 1d) the $t_c$ behaviours of $r_{2d}^{sd}$ at fixed $\tau = 0.9 \text{ GeV}^{-2}$ and $b = 0$.

- Results: we can deduce the DR:

$$r_{2d}^{sd} = 1.075(0.1)(0.6)(0.4)(3.4)(11)(17)(6) , \quad (25)$$

where we have considered the mean value from $t_c = 6 \text{ GeV}^2$ (beginning of $\tau$-stability) to the beginning of $t_c$-stability (12 GeV$^2$). The errors are due respectively to the values of $\tau = (0.9 \pm 0.2) \text{ GeV}^{-2}$, $b$, $t_c$, $m_c$, $m_s$, the ratio $\kappa = \langle \overline{s}s \rangle / (\overline{d}d)$ and the factorization of the four-quark condensate $\rho$. $\kappa$ gives the most important error while the ratio of masses increases when the one of the quark condensate decreases. The other QCD parameters give negligible errors. Using as input the data [9]:

$$M_{\Lambda_c}^{exp} = (2286.46 \pm 0.14) \text{ MeV} , \quad (26)$$
and adding the different errors quadratically, one can deduce:

\[ M_{\Xi_c} = (2449 \pm 49) \text{ MeV} \],

which agrees with the data [9]:

\[ M_{\Xi_c}^{\exp} = (2467.9 \pm 0.4) \text{ MeV} \].

(28)

This result is also given in Table 2.

• \( \Xi_b(bqq)/\Lambda_b(bqq) \)

We repeat the previous analysis in the case of the \( b \) quark. The analysis of the ratio of sum rules shows similar curves than for the charm case except the obvious change of scale. It also indicates that \( r_{21}^{sd} \) has the best \( \tau \)- and \( t_c \)-stabilities, from which we deduce:

\[ r_{21}^{sd} = 1.048(4)(2)(3)(1)(7)(9)(7), \]

(29)

where we have considered the mean value from \( t_c = 34 \text{ GeV}^2 \) (beginning of \( \tau \)-stability) to the beginning of \( t_c \)-stability (60 GeV\(^2\)). We have used the optimal value \( \tau = 0.35 \pm 0.05 \text{ GeV}^{-2} \). The sources of the errors are the same as in the charm quark case. Using the data [9]:

\[ M_{\Lambda_b}^{\exp} = (5620.2 \pm 1.6) \text{ MeV} \],

(30)

and adding the different errors quadratically, one get:

\[ M_{\Xi_b} = (5888 \pm 81) \text{ MeV} \],

(31)

which agrees with the data [9]:

\[ M_{\Xi_b}^{\exp} = (5792.4 \pm 3.0) \text{ MeV} \].

(32)

Although the errors look quite large in the two cases of charm and bottom compared with the data, the predictions are more robust than the recent estimates in [14,15], where we expect that the errors have been underestimated.

• \( \Omega_c(cqq)/\Sigma_c(cqq) \)

We do an analysis similar to the one in the previous section. The result for the \( c \)-quark is shown in Fig 2. One can notice that the optimal choice of the current is the same as in Eq. (2) which we fix to the value \( b=0 \). One can notice from in Fig 2a) and Fig 2b), that \( r_{21}^{sd} \) does not give a result consistent with the one from \( r_{i}^{sd} \) \( (i = 1,2) \) and it is also less stable in \( \tau \) than the two others. The appearance of a (false) extremum at small \( \tau \)-values is also strongly affected by \( t_c \) at which one cannot extract any optimal result. We shall not also retain \( r_{21}^{sd} \) as it is not stable versus \( b \). The final result from \( r_{21}^{sd} \) is:

\[ r_{21}^{sd} = 1.141(12)(0.4)(17)(10)(13)(29)(1), \]

(33)

and come from the mean of \( t_c \) values from 6 (beginning of \( \tau \)-stability) to 12 GeV\(^2\) (beginning of \( t_c \)-stability). The sources of the errors are the same as before and come from \( \tau = (0.8 \pm 0.2) \text{ GeV}^{-2}, b, t_c, \ k, \rho \) (deviation from factorization of the four-quark condensate). The other QCD parameters gives negligible errors. Using this previous result together with the experimental averaged value [9]:

\[ M_{\Sigma_c}^{\exp} = 2453.6 \text{ MeV} \],

(34)

one can deduce the result in Table 2 in agreement with the data.

\[ \Omega_b(bq)\Sigma_b(bqq) \]

We repeat the previous analysis in the case of the \( b \)-quark. The curves present the same qualitative behaviour as in the case of the charm, where, only \( r_{21}^{ds} \) survives the different tests of stabilities. Here, the \( t_c \)-behaviour is almost flat from \( t_c = 34 \text{ GeV}^2 \) (beginning of \( \tau \)-stability). The optimal value is taken at the extremum \( \tau = (25 \pm 0.05) \text{ GeV}^{-2} \). Then, we obtain:

\[ r_{21}^{sd} = 1.051(2)(0.5)(1)(4)(3)(11)(1), \]

(35)

with the same sources of errors as before. Using this value together with the experimental averaged value [9]:

\[ M_{\Sigma_b}^{\exp} = 5811.2 \text{ MeV}, \]

(36)

one can deduce the result in Table 2 in agreement with the data [37], which, however, needs to be confirmed by some other experiments.
6. The masses of the spin 3/2 decuplet baryons

As a preliminary step of the analysis, we check the different results obtained in [5]:

\[
M_{\Omega^*_{c}} = (2.15 \sim 2.92) \text{ GeV}, \\
M_{\Omega^*_{c}} - M_{\Omega_{c}} = 3.3 \text{ GeV},
\]

(37)

and confirm them. However, like in the octet case, we have not tried to improve these (old) results.

- \(\Xi^*_c(csq)/\Sigma^*_c(cqq)\)

We repeat the previous DR analysis for the case of the \(\Xi^*_c\). We show in Fig. 3a) and Fig. 3b) the \(\tau\)-behaviour of the mass predictions for two extremal values of \(t_c\) between \(M^2_{\Xi^*_c}\) (beginning of the \(\tau\)-stability) and 12 GeV\(^2\) (beginning of \(t_c\)-stability). We do not retain \(r_{12}^{sd}\) which differs completely from \(r_{12}^{ds}\) and \(r_{12}^{ds}\), while we do not consider \(r_{12}^{ds}\) which becomes \(\tau\)-instable when \(t_c\) increases. We show in Fig. 1c) the \(t_c\)-behaviour of \(r_{21}^{ds}\) given \(\tau=0.7\) GeV\(^{-2}\).

\[
r_{21}^{sd} = 1.065(1)(10)(4)(4)(17)(5),
\]

(38)

The errors are due respectively to the values of \(\tau = (0.7 \pm 0.2) \text{ GeV}^{-2}\), \(t_c\), \(\kappa\), \(m_s\), \(\kappa \equiv \langle \bar{s}s \rangle/\langle \bar{d}d \rangle\) and \(\rho\) (factorization of the four-quark condensate). The ones due to some other parameters are negligible. Using the data [9]:

\[
M_{\Xi^*_c}^{exp} = (2517.97 \pm 1.17) \text{ MeV},
\]

(39)

and adding the different errors quadratically, we deduce the results in Table 2.

- \(\Xi^*_b(bsq)/\Sigma^*_b(bqq)\)

We extend the analysis to the case of the bottom quark. The curves are qualitatively similar to the charm case. We deduce:

\[
r_{21}^{sd} = 1.024(0.4)(2.5)(1)(1)(1)(0.6).
\]

(40)

The sources of the errors are the same as for the \(\Xi^*_c\), where here \(\tau = (0.22 \pm 0.04) \text{ GeV}^{-2}\) and \(t_c\) between \(M^2_{\Sigma^*_b}\) and 70 GeV\(^2\) (beginning of \(t_c\)-stability). The ones due to some other parameters are negligible. Using the averaged data:

\[
M_{\Sigma^*_b}^{exp} = (5832.7 \pm 6.5) \text{ MeV},
\]

(41)

and adding the different errors quadratically, we deduce the result in Table 2.

- \(\Omega^*_c(css)/\Sigma^*_c(cqq)\)

We pursue the analysis to the case of the \(\Omega^*_c(css)\). We show the \(\tau\)-behaviour of the different DR in Fig. 4a).

From this figure, we shall not retain \(r_{21}^{ds}\) which differs
and compatibility with the other ones. From the mean of associated non-strange heavy baryons.

Table 2

<table>
<thead>
<tr>
<th>Baryons</th>
<th>( r_{1d}^d )</th>
<th>Mass</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Octet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Xi_c )</td>
<td>1.075(21)</td>
<td>2458(50)</td>
<td>2467.9 \pm 0.4</td>
</tr>
<tr>
<td>( \Omega_c )</td>
<td>1.141(39)</td>
<td>2800(96)</td>
<td>2697.5 \pm 2.6</td>
</tr>
<tr>
<td>( \Xi_b )</td>
<td>1.048(15)</td>
<td>5888(81)</td>
<td>5792.4 \pm 3.0</td>
</tr>
<tr>
<td>( \Omega_b )</td>
<td>1.051(12)</td>
<td>6108(71)</td>
<td>6165.0 \pm 13</td>
</tr>
<tr>
<td>Decuplet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Xi_b^* )</td>
<td>1.065(21)</td>
<td>2682(53)</td>
<td>2646.1 \pm 1.3</td>
</tr>
<tr>
<td>( \Omega_b^* )</td>
<td>1.135(37)</td>
<td>2858(92)</td>
<td>2768.3 \pm 3.0</td>
</tr>
<tr>
<td>( \Xi_b^{*\prime} )</td>
<td>1.024(8)</td>
<td>5973(44)</td>
<td>–</td>
</tr>
<tr>
<td>( \Omega_b^{*\prime} )</td>
<td>1.051(17)</td>
<td>6130(99)</td>
<td>–</td>
</tr>
</tbody>
</table>

\( \Omega_b \) predictions observed in some quark models [11,7].

Acknowledgements

R.M.A and M.N. acknowledge the financial support from FAPESP and CNPq (Brazil). S.N. has been partially supported by the CNRS-IN2P3 within the project “Hadron Physics in QCD” and within the French-China Particle Physics Laboratory (FCPPL). We thank Jean-Marc Richard for some discussions.

- Combining the previous predictions for the decuplet with the ones for the octet, we give in Table 3 predictions of the hyperfine mass-splittings. These results agree quite well with the data and with some expectations from quark models.
- Like in the case of the light baryons [29], it is remarkable to notice that the leading term controlling the mass-splittings is the ratio \( \kappa \equiv \langle \bar{s}s \rangle / \langle \bar{d}d \rangle \) of the condensate rather than the running mass \( \bar{m}_s \). This ratio gives, after the choice of the continuum threshold \( t_c \), the largest errors in \( r_{1d}^d \).
- One can notice that for \( SU(3) \) symmetric quark condensates \( \langle \bar{s}s \rangle \simeq \langle \bar{d}d \rangle \), the predictions tend to be lower than the present predictions which deteriorate the agreement with the observed masses in different channels. This feature might explain the underestimate of the \( \Omega_b \) predictions observed in some quark models [11,7].
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