1 Introduction

It was realized a long time ago, that by observing cosmic ray particles it is possible to learn about hadronic cross sections at the highest accessible energies [1]. In the beginning the energy reach of these analyses was around $10^{12}$ eV but rapidly went up to $10^{17}$ eV and nowadays reaching beyond $10^{19}$ eV in total cosmic ray primary energy. A compilation of proton-air cross section measurements from cosmic ray experiments is given in Figure 1.

At any given time cosmic ray analyses were far beyond the reach of any conventional Earth-based particle accelerator experiments. For example, characteristics of the energy dependence of the hadronic cross section have been deduced with the help of cosmic ray data [2, 3]. Today, the LHC is expected to explore hadronic interactions around $\sqrt{s} \sim 14$ TeV, while cosmic ray analyses can reach up to $\sqrt{s} > 100$ TeV.

However, it always was the question which part of the total hadronic cross section can really be accessed by cosmic ray experiments [9, 21]. Clearly cosmic ray analyses are only sensitive to the fraction of interactions that have a significant impact on the formation of extensive air showers. These are interactions with a minimum inelasticity $k_{\text{inel}} = 1 - E_{\text{max}}/E_{\text{tot}}$, excluding elastic, diffractive and quasielastic interactions. The conversion of the inelastic part of the cross section to the total hadronic cross section is highly non trivial and theoretical uncertainties might absolutely dominate these calculations [21, 22, 23]. Many cosmic ray cross section analyses were not careful in specifying the definition of their cross section and also about the eventual conversion to total cross sections.

2 Total Cross Section and Glauber Theory

The relation between inelastic and total cross section is typically evaluated within the Glauber theory [24, 25]. The multiple scattering approximation by Glauber combined with the optical theorem provides a framework that connects the nucleon-nucleon scattering properties $\sigma_{\text{tot}}$, $\rho$ and $B_{\text{ela}}$ to inelastic, total, elastic and quasi-
elastic cross sections of nucleus-nucleus scattering. In this framework model predictions for hadronic cross sections at cosmic ray energies can be deduced from the extrapolation of well measured nucleon-nucleon scattering parameters at much lower energies. However, these extrapolations are themselves subject to large systematic uncertainties, since the parametric shape of the extrapolation has to be taken from phenomenology and is not well constraint. Several important characteristics of these extrapolation are unknown and are discussed in the literature (mostly saturation effects, see e.g. [26, 27]). Currently this results in relatively large uncertainties for example when interpreting ultra-high energy cosmic ray data in terms of their primary mass composition. When the uncertainties of the extrapolation of the nucleon-nucleon data is propagated into the proton-air cross section using the Glauber approach, the uncertainty of the predicted cross section at $10^{19}$ eV might get as large as $\sim 50\%$ (c.f. Fig. 2). This is much larger than suggested by the spread of the predictions of the hadronic interaction models.

The impact of a faster rise of the cross section on the predicted location of the depth of the shower maxima $X_{\text{max}}$ is demonstrated in Fig. 3 and clearly can have a major impact on the interpretation of the air shower data in terms of the primary mass composition.
Figure 2: Resulting uncertainty from extreme assumptions on the extrapolation of nucleon-nucleon scattering parameters on the proton-air production cross section (from [28]).

In order to use the composition information of cosmic ray data to constrain astrophysical source and propagation scenarios this uncertainty must be reduced drastically. The LHC will certainly help a lot by providing another high precision data point of nucleon-nucleon scattering parameters at energies that are already significant in terms of cosmic ray physics. For the still needed extrapolation to energies above $E_{\text{lab}} \sim 10^{17}$ eV it is required to rely on cosmic ray data itself to constrain the proton-air cross section.

3 Comparison of Techniques to Derive the Proton-Air Cross Section from Cosmic Ray Data

All analysis techniques to derive the proton-air cross section from cosmic ray data at the relevant energies above $E_{\text{lab}} \sim 10^{17}$ eV are based on the interpretation of air shower data. This makes any attempt to learn about the cross section of the primary cosmic ray particle a highly indirect procedure, since all the air shower development with itself relatively large uncertainties is in between the process under study and the measurement apparatus. It is thus very important to careful evaluate and incorporate
Figure 3: Measurements of the mean depth of the shower maximum [29, 30, 31, 32] compared to simulations with a steeper rise of the proton-air cross section extrapolation beyond $E_{\text{lab}} \sim 10^{15}$ eV (20%, 40% and 60% larger cross section than the original SIBYLL model) (from [28]).

For the scope of this work, we focus now on four measurement techniques:

**Constant $N_e - N_\mu$** Is based on the independent measurement of the electron and muon number sizes of air showers at observation level $X_{\text{obs}}$. The observation of the attenuation of the shower frequency for constant $N_\mu$ (⇒ constant energy) and constant $N_e$ (⇒ constant stage of shower development, $X_{\text{obs}}/\cos \theta - X_1$) with zenith angle can be related to the proton-air cross section.

**$X_{\text{max}}$-tail** The slope of the tail at large depths is related to the proton-air cross section.

**$X_{\text{max}}$-unfolding** The HiRes Collaboration proposed to unfold the $X_{\text{max}}$-distribution with a kernel distribution of $\Delta X = X_{\text{max}} - X_1$ derived from simulations.

**$X_{\text{max}}$-model** This approach parameterizes the impact of a modified extrapolation of the proton-air cross section on the resulting air shower development, and thus provides a model for $X_{\text{max}}$-distribution with the proton-air cross section as a free parameter.
Figure 4: Comparison of the performance of four techniques to derive the proton-air cross section form air shower data at ultra-high energies. Shown is the ability to derive a deviating cross section extrapolation from the original model prediction at $10^{19}$ eV, where $\Delta \sigma_{\text{rec}} = \sigma_{\text{rec}} - \sigma_{\text{modified}}$ (from [33]).
In Figure 3 the performance and model-dependence of these techniques is tested and compared to each other. Considering the impact of the measured proton-air cross section already on the level of the air shower simulations needed for the interpretation of the data, as done by the $X_{\text{max}}$-model, leads to a strong increase of the performance of the analysis and a significant reduction of the model dependence of the results, see Fig. 3 (bottom).

4 Summary

The uncertainty of the extrapolation of the hadronic production cross section to ultra-high energies is a major problem for the interpretation of existing cosmic ray data for example in terms of the primary mass composition.

The real uncertainty of these extrapolations is unknown and the spread of the predictions by existing hadronic interaction models might underestimate it.

A steeper rise of the cross section after $\sim 10^{15}$ eV would result into the prediction of a lighter primary cosmic ray composition.

The upcoming data from the LHC experiments will reduce the existing uncertainties significantly, but not solve the extrapolation to even higher energies and phase space regions not explored at the LHC. Only cosmic ray data itself can be used to learn about the relevant features of hadronic interactions.

With improved analysis techniques and high quality air shower data it will be possible to evaluate the proton-air cross section around $10^{19}$ eV with systematic uncertainties of the order of 10%. This is a dramatic improvement compared to existing results at these energies and will help to constrain the energy dependence of the proton-air cross section up to the highest cosmic ray energies.

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References


