Nonradial Oscillations of Solid Quark Stars

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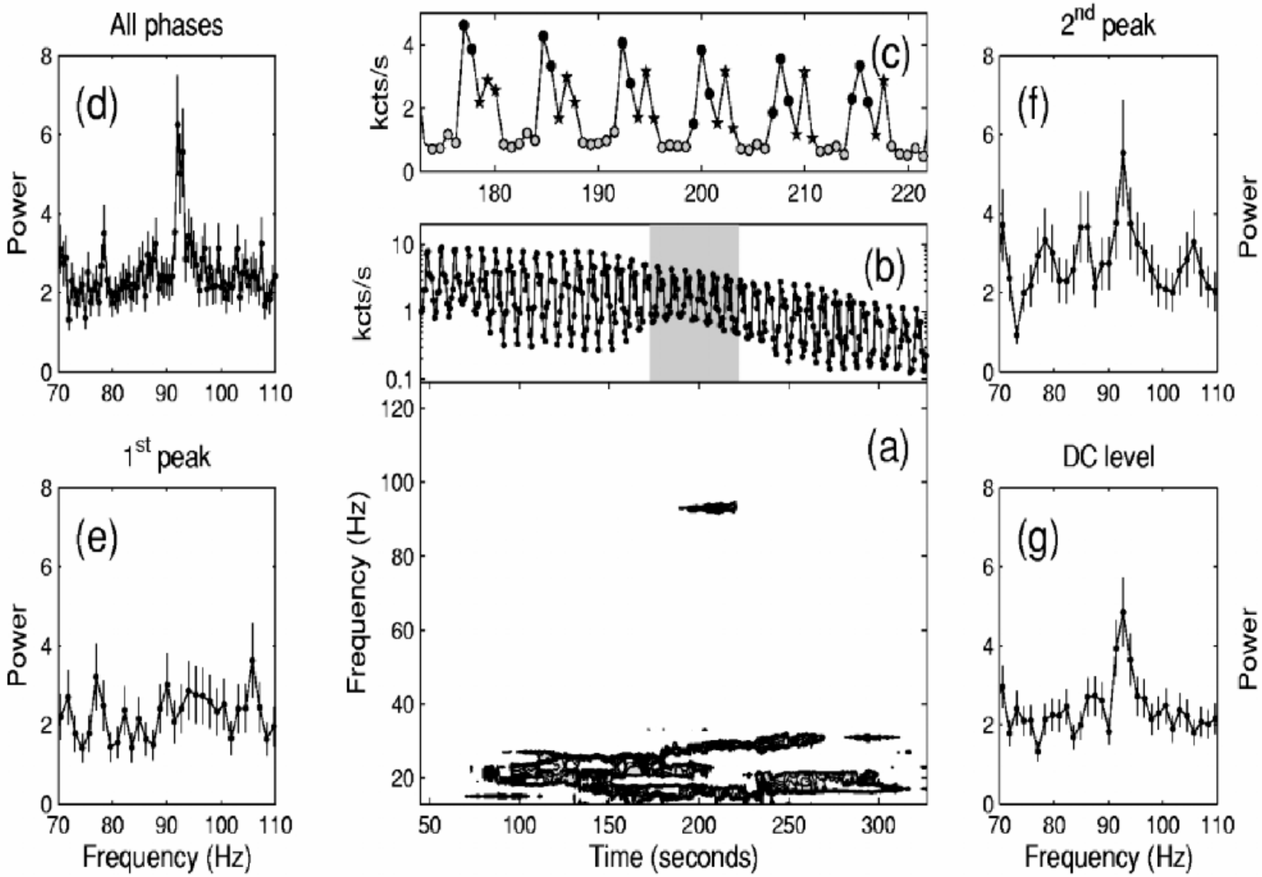
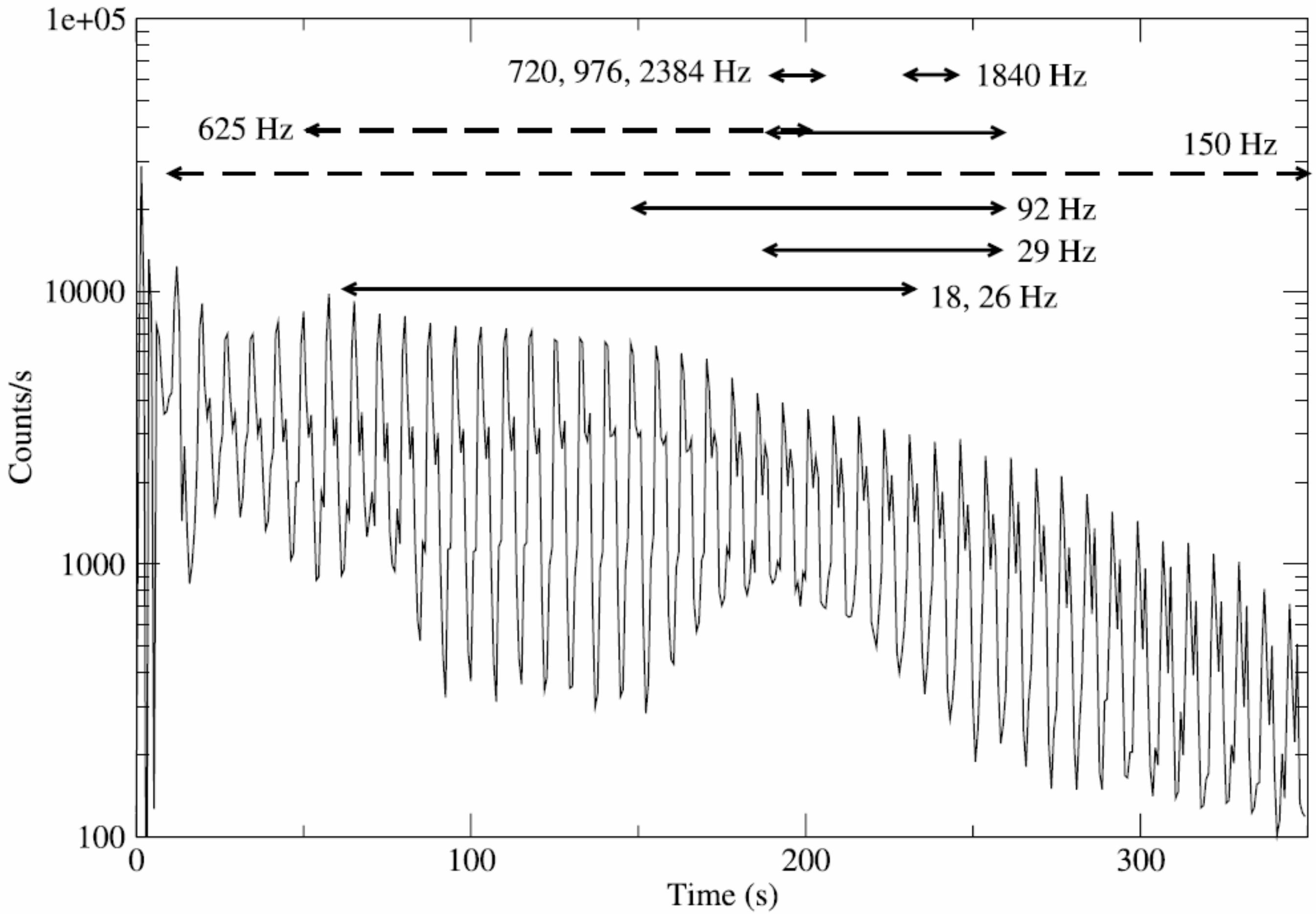
Introduction



Israel et al. (2005)

- Israel et al. (2005) found QPOs of 18 Hz, 30 Hz and 92 Hz in the decaying tail of the giant flare recorded on December 27th 2004 from SGR 1806-20.



Strohmayer & Watts (2006)

- Watts & Strohmayer (2006) and Strohmayer & Watts (2006) confirmed this discovery and they also found other QPOs: 26 Hz, 150 Hz, 625 Hz and 1840 Hz from the same SGR giant flare.

- Prompted by this discovery from SGR 1806-20, Strohmayer & Watts (2005) reanalyzed the data from the August 27th 1998 giant flare of SGR 1900+14 and also found QPOs at 28 Hz, 53 Hz, 84 Hz and 155 Hz.
- In the magnetar model the QPOs are interpreted as the torsional shear oscillations of the neutron star crust (Piro 2005; Glampedakis et al. 2006; Lee 2007; Samuelsson & Andersson2007; Vavoulidis et al. 2007; Sotani et al. 2007; Bastrukov et al. 2007; Timokhin et al. 2008).

SGR 1806-20	SGR 1900+14	Torsional shear mode identification
18*		
26*		
30*	28	$n = 0,\ l = 2$
	53	$n = 0,\ l = 4$
92*	84	$n = 0,\ l = 6$
150		$n = 0,\ l = 10$
	155	$n = 0,\ l = 11$
625*		$n = 1$
1840		$n = 3$

Watts & Strohmayer (2007)

- However, the torsional shear vibration model of the neutron star crust can not produce the two lowest frequency modes in the SGR 1806-20, i.e. the modes of 18 Hz and 26 Hz.

- In addition, the real state of the matter at supra-nuclear densities is still a matter of debate because of non-perturbative effect of QCD.
- It is worth noting that nuclear and quark matter states are only two speculations for the inner structure of pulsar-like stars.
- An interesting suggestion is that cold quark matter could be in a solid state, and the astrophysical necessities and consequences of the existence of the solid quark stars have been discussed by Xu (2003).

- Furthermore, from the theoretical side the solid quark matter being in crystalline color superconducting state has been proposed by Rajagopal & Sharma (2006) and the corresponding shear modulus of the crystalline color superconducting quark matter has been calculated theoretically by Mannarelli et al. (2007).
- Therefore, it is necessary to further study the consequences of the solid quark star in SGR QPOs.

Equations of Nonradial Pulsations

- The equations of nonradial oscillations include the continuity equation, the momentum equation and the Poisson's equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0,$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{1}{\rho}\nabla \cdot \vec{\sigma} - \nabla \Phi,$$

$$\nabla^2 \Phi = 4\pi G \rho,$$

- The perturbation of the shear stress under the adiabatic condition is

$$\Delta\sigma_{ik} = (\Gamma_1 p\, u_{ll})\, \delta_{ik} + 2\mu \left(u_{ik} - \frac{1}{3} u_{ll} \delta_{ik} \right)$$

- u_{ik} is the strain tensor and is related with the displacement vector u_i through

$$u_{ik} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right)$$

- There are two independent families of modes governed by the pulsation equations, i.e. the spheroidal and the toroidal. For the pheroidal mode, the space component of the displacement can be parameterized as

$$\xi_r = U(r) Y_{lm}, \qquad \xi_\theta = V(r) \frac{\partial Y_{lm}}{\partial \theta}, \qquad \xi_\phi = V(r) \frac{1}{\sin\theta} \frac{\partial Y_{lm}}{\partial \phi},$$

- Introducing new variables

$$y_1 = U, \qquad y_2 = \lambda\alpha + 2\mu\frac{dU}{dr},$$

$$y_3 = V, \qquad y_4 = \mu\left(\frac{dV}{dr} - \frac{V}{r} + \frac{U}{r}\right),$$

We can obtain

$$\frac{dy_1}{dr} = -\frac{2\lambda}{(\lambda+2\mu)\,r}y_1 + \frac{1}{(\lambda+2\mu)}y_2 + \frac{l(l+1)\lambda}{(\lambda+2\mu)\,r}y_3,$$

$$\frac{dy_2}{dr} = \left[-\rho\sigma^2 + \frac{4\mu(3\lambda+2\mu)}{(\lambda+2\mu)r^2} - \frac{4\rho g}{r} + 4\pi G\rho^2\right]y_1 - \frac{4\mu}{(\lambda+2\mu)r}y_2$$
$$+l(l+1)\left[\frac{\rho g}{r} - \frac{2\mu(3\lambda+2\mu)}{(\lambda+2\mu)r^2}\right]y_3 + \frac{l(l+1)}{r}y_4,$$

$$\frac{dy_3}{dr} = -\frac{1}{r}y_1 + \frac{1}{r}y_3 + \frac{1}{\mu}y_4,$$

$$\frac{dy_4}{dr} = \left[\frac{\rho g}{r} - \frac{2\mu(3\lambda+2\mu)}{(\lambda+2\mu)r^2}\right]y_1 - \frac{\lambda}{(\lambda+2\mu)\,r}y_2$$
$$+\left[-\rho\sigma^2 + \frac{4l(l+1)\mu(\lambda+\mu)}{(\lambda+2\mu)\,r^2} - \frac{2\mu}{r^2}\right]y_3 - \frac{3}{r}y_4,$$

- The toroidal modes are characterized by the vanishing of the divergence of the displacement as well as the radial component of the displacement, i.e. $\nabla \cdot \mathbf{u} = 0$ and $u_r = 0$. And the displacement of the toroidal modes reads

$$\xi_r = 0, \qquad \xi_\theta = W(r)\frac{1}{\sin\theta}\frac{\partial Y_{lm}}{\partial \phi}, \qquad \xi_\phi = -W(r)\frac{\partial Y_{lm}}{\partial \theta}$$

- We introduce two variables as

$$z_1 = \frac{W}{r}, \qquad z_2 = \mu\left(\frac{dW}{dr} - \frac{W}{r}\right)$$

The oscillation equations can be written as

$$\begin{aligned}
\frac{dz_1}{dr} &= \frac{1}{\mu r} z_2, \\
\frac{dz_2}{dr} &= \mu\left[\frac{l(l+1)-2}{r} - \frac{\rho\sigma^2 r}{\mu}\right] z_1 - \frac{3}{r} z_2,
\end{aligned}$$

Numerical Results

- The eigenfrequencies of the shear oscillations is close related with the speed of the shear waves, defined as

$$v_s = \left(\frac{\mu}{\rho}\right)^{1/2} = 10^8\,\mathrm{cm\,s^{-1}}\left(\frac{\mu}{10^{30}\,\mathrm{erg\,cm^{-3}}}\right)^{1/2}\left(\frac{\rho}{10^{14}\,\mathrm{g\,cm^{-3}}}\right)^{-1/2}$$

- For the toroidal modes, in a simple solid stellar model with homogeneous density and shear modulus, the eigenfrequencies for the fundamental toroidal modes can be obtained as (Bastrukov et al. 2007)

$$\frac{\sigma}{2\pi} = \frac{v_s}{2\pi R}\left[(2l+3)(l-1)\right]^{1/2},$$

$$= 42.1\mathrm{Hz}\left[\frac{(2l+3)(l-1)}{7}\right]^{1/2}\left(\frac{\mu}{10^{30}\,\mathrm{erg\,cm^{-3}}}\right)^{1/2}$$

$$\times\left(\frac{\rho}{10^{14}\,\mathrm{g\,cm^{-3}}}\right)^{-1/2}\left(\frac{R}{10^{6}\,\mathrm{cm}}\right)^{-1}.$$

- We construct a solid strange quark star with mass $M = 1.4M_{\odot}$ and radius $R = 10\ \mathrm{km}$, and adopt a homogeneous shear modulus with $\mu = 1.2\times10^{30}\ \mathrm{erg\,cm^{-3}}$

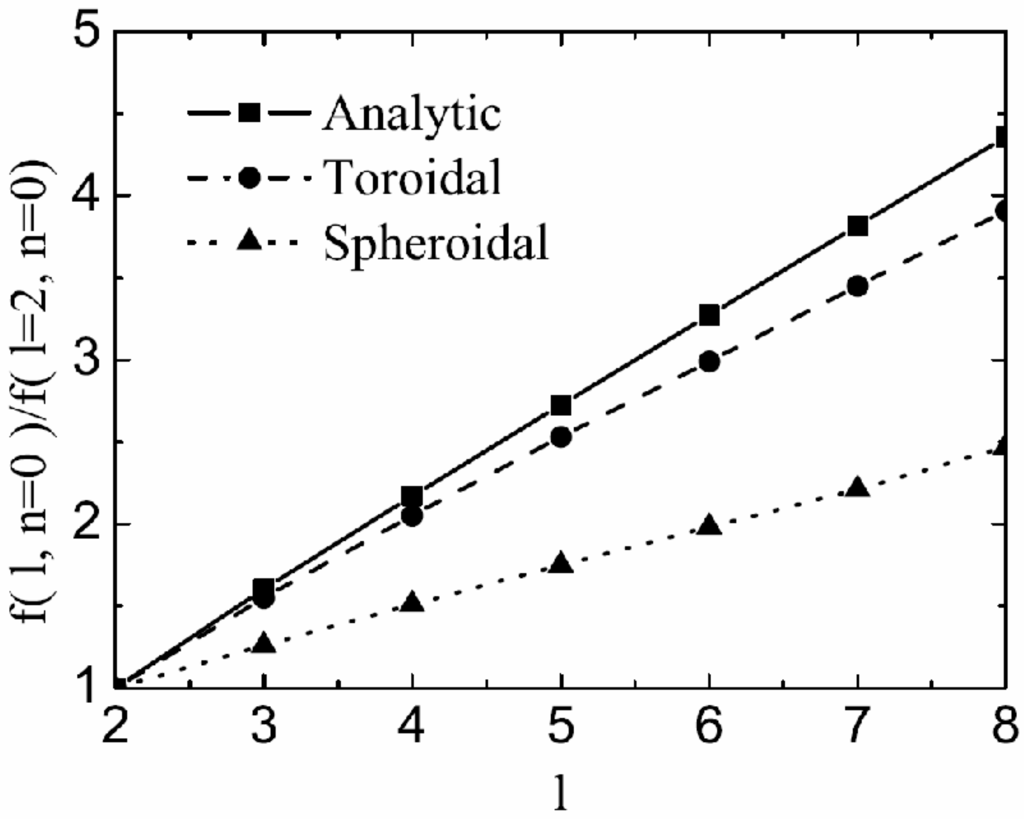


Fig. 1. Calculated eigenfrequencies $f = \sigma/2\pi$ of the fundamental toroidal (circular dots) and spheroidal (triangular dots) modes in our solid quark star model with $M = 1.4M_{\odot}$, $R = 10\,\mathrm{km}$ and $\mu = 1.2 \times 10^{30}\,\mathrm{erg\,cm^{-3}}$. For comparison we also depict the analytic results (square dots) for the toroidal modes in a solid stellar model with uniform density (Bastrukov et al. 2007), i.e. Eq. (28).

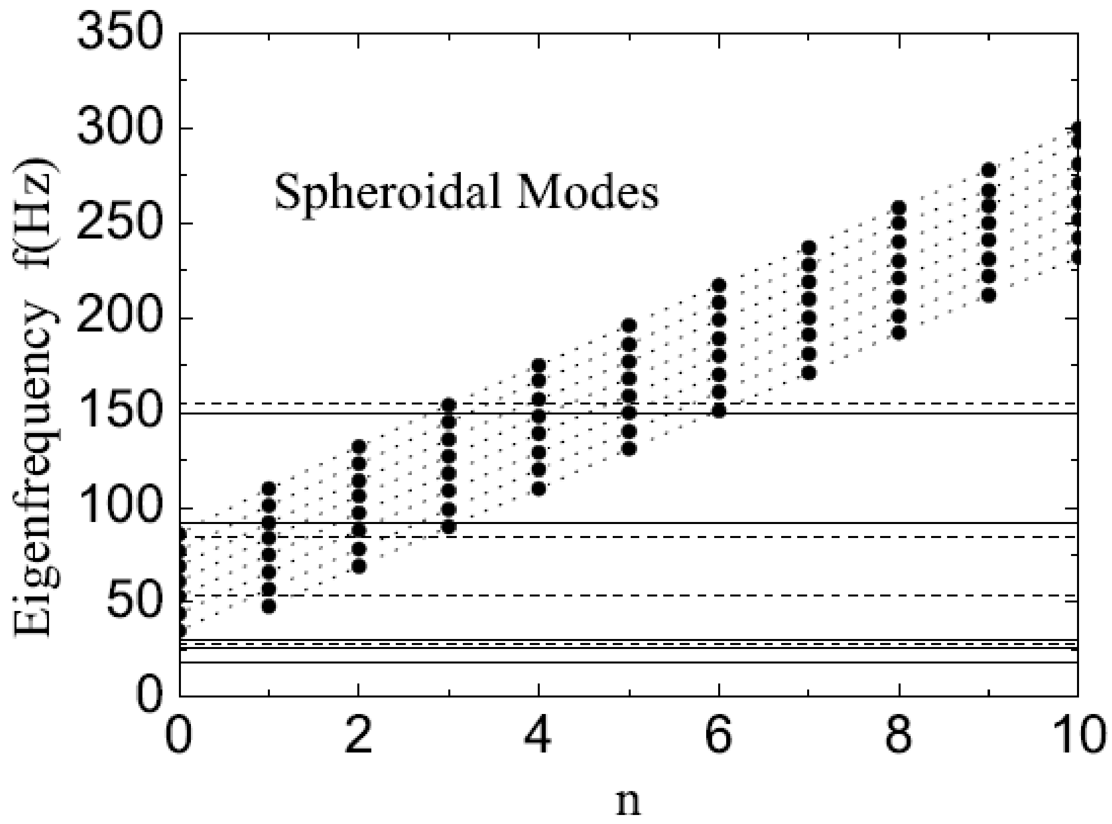
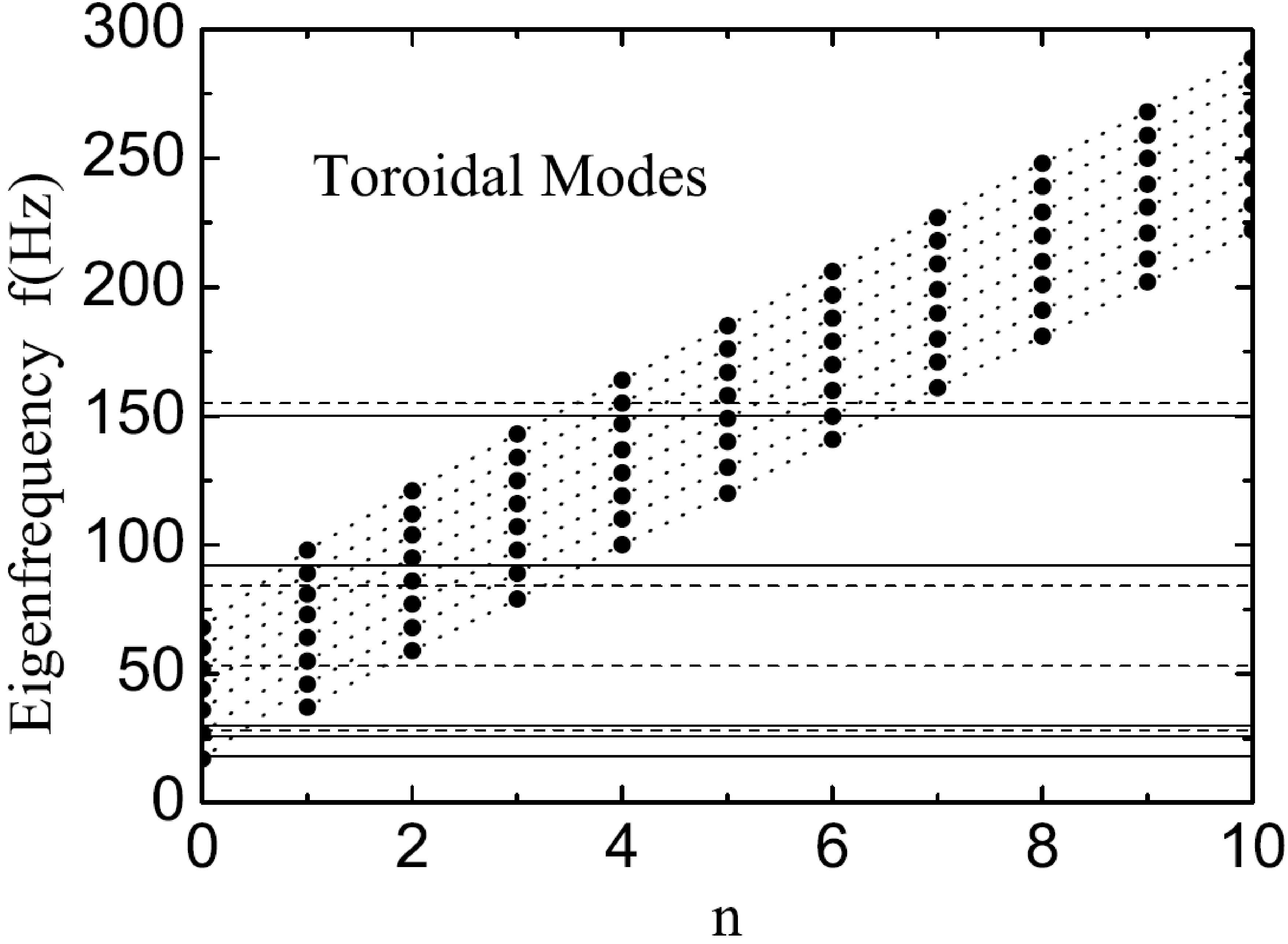


Fig. 2.— Calculated eigenfrequencies of spheroidal modes as functions of the radial node number n with angular harmonic number l ranging from 1 to 8, corresponding to the dotted lines from bottom to top. Here the solid quark star is modeled with a mass of $1.4M_{\odot}$, radius of 10 km and a homogeneous shear modulus of $1.2 \times 10^{30}\,\mathrm{erg\,cm^{-3}}$. QPO frequencies of 18, 26, 30, 92, and 150 Hz from SGR 1806-20 (Israel et al. 2005; Watts & Strohmayer 2006; Strohmayer & Watts 2006) and those of 28, 53, 84, and 155 Hz from SGR 1900+14 (Strohmayer & Watts 2005) are indicated with horizontal solid lines and dashed lines, respectively. More details are presented in the text.



Toroidal Modes
Eigenfrequency f(Hz)
n
300
250
200
150
100
50
0
0
2
4
6
8
10

Table 1: Calculated eigenfrequencies (in unit of Hz) of the spheroidal modes in a solid quark star model with $M = 1.4M_\odot$, $R = 10\,\mathrm{km}$ and a homogeneous shear modulus $\mu = 1.2 \times 10^{30}\,\mathrm{erg\,cm^{-3}}$, corresponding to the results in Fig. 2.

l	$_lS_0$	$_lS_1$	$_lS_2$	$_lS_3$	$_lS_4$	$_lS_5$	$_lS_6$	$_lS_7$	$_lS_8$	$_lS_9$	$_lS_{10}$
1	–	48	69	90	110	131	151	171	192	212	232
2	35	57	78	99	120	140	161	181	201	222	242
3	44	66	88	109	129	150	170	191	211	231	252
4	53	75	97	118	139	159	180	200	221	241	261
5	61	84	106	127	148	168	189	210	230	250	271
6	69	92	114	136	157	177	199	219	240	259	281
7	77	101	123	145	167	186	208	228	250	267	293
8	86	110	132	154	175	196	217	237	258	278	300

Table 2: Calculated eigenfrequencies (in unit of Hz) of the toroidal modes in a solid quark star model with $M = 1.4 M_{\odot}$, $R = 10\,\mathrm{km}$ and a homogeneous shear modulus $\mu = 1.2 \times 10^{30}\,\mathrm{erg\,cm^{-3}}$, corresponding to the results in Fig. 3.

l	${}_lt_0$	${}_lt_1$	${}_lt_2$	${}_lt_3$	${}_lt_4$	${}_lt_5$	${}_lt_6$	${}_lt_7$	${}_lt_8$	${}_lt_9$	${}_lt_{10}$
1		37	59	79	100	120	141	161	181	202	222
2	17	46	68	89	110	130	150	171	191	211	232
3	27	55	77	98	119	140	160	180	201	221	242
4	36	64	86	107	128	149	170	190	210	231	251
5	44	73	95	116	137	158	179	199	220	240	261
6	52	81	104	125	147	167	188	209	229	250	270
7	60	89	112	134	155	176	197	218	239	259	280
8	68	98	121	143	164	185	206	227	248	268	289

Table 3: Calculated eigenfrequencies (in unit of Hz) of the spheroidal and the toroidal modes in a solid quark star model with $M = 1.4 M_{\odot}$, $R = 10\,\mathrm{km}$ and a homogeneous shear modulus $\mu = 3.96 \times 10^{33}\,\mathrm{erg\,cm^{-3}}$.

l	${}_ls_0$	${}_ls_1$	${}_ls_2$	${}_ls_3$	${}_ls_4$	${}_ls_5$	${}_lt_0$	${}_lt_1$	${}_lt_2$	${}_lt_3$	${}_lt_4$	${}_lt_5$
1	–	2755	3963	5149	6326	7497	–	2137	3367	4560	5739	6913
2	1970	3184	4517	5703	6882	8056	999	2670	3909	5109	6294	7471
3	2449	3718	5088	6253	7432	8608	1549	3182	4436	5645	6836	8018
4	2903	4204	5714	6807	7979	9155	2049	3682	4952	6171	7370	8558

Summary and Discussion

- In this work, we have studied the possibility whether the global spheroidal and toroidal vibrations of a solid quark star can produce the QPOs in the decaying X-ray light curves of giant flares from SGR 1806-20 and SGR 1900+14.
- We find that when the shear modulus of the solid quark matter is comparable with that of the conventional solid neutron star crust, the two lowest frequencies 18 and 26 Hz from SGR 1806-20 can be assigned as the nodeless modes of the toroidal oscillations with *l* being 2 and 3, respectively, i.e.

$${}_2t_0, \quad {}_3t_0 \quad .$$

- The QPO frequency of 30 Hz from SGR 1806-20 or 28 Hz from SGR 1900+14 can be assigned as the nodeless quadrupole spheroidal mode ${}_2s_0$.
- However, the much higher QPO frequencies, such as 625 and 1840 Hz from SGR 1806-20 can not be interpreted in our solid quark star model. The origin of the high frequency QPOs in the solid quark star model deserves further investigations.

Thank you!