Neutrino Mass Models

- Why BSM?
- Neutrino mass models roadmap
- Survey of approaches
- TBM, A_4, CSD
- Family symmetry and GUTs
- Sum rules and predictions

Great interest in neutrino theory, e.g. Melbourne Participants:

Kev Abazajian (Maryland)
Carl Albright (Fermilab)
Evgeny Akhmedov (Max Planck, Heidelberg)
Matthew Baring (Rice)
Pasquale Di Bari (Padova)
Nicole Bell (Melbourne)
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Andreu Esteban-Pretel (Valencia)
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Gerard Stephenson (UNM)
Alexander Studenikin (Moscow State University)
Jayne Thompson (Melbourne)
Shoichi Uchinami (Tokyo Metropolitan U.)
Raoul Viollier (Cape Town)
Ray Volkas (Melbourne)
Renata Zukanovich-Funchal (São Paulo)

Why Beyond Standard Model?

1. There are no right-handed neutrinos ν_R
2. There are only Higgs doublets of $SU(2)_L$
3. There are only renormalizable terms

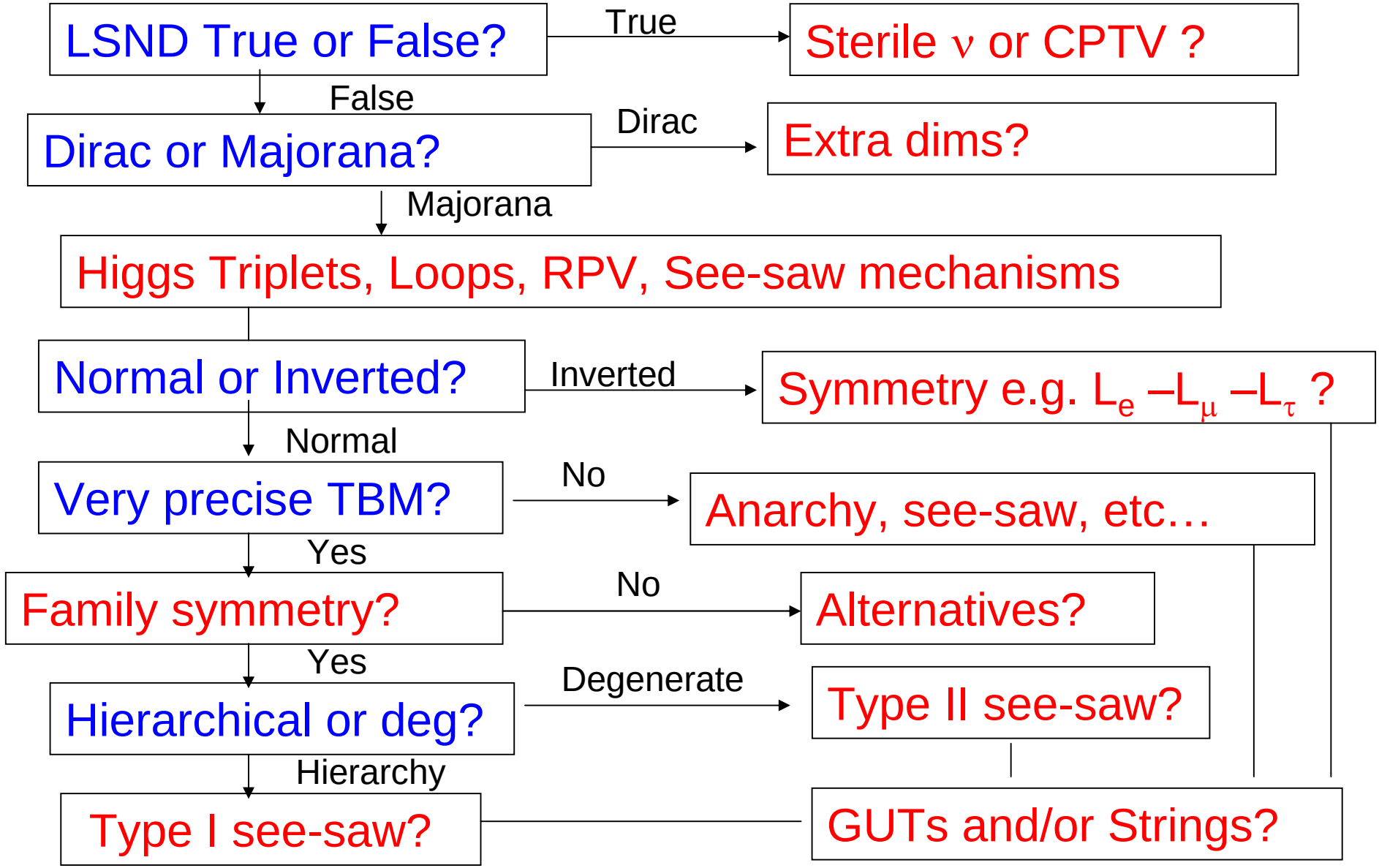
In the Standard Model these conditions all apply so neutrinos are massless, with ν_e , ν_μ , ν_τ distinguished by separate lepton numbers L_e, L_μ, L_τ

Neutrinos and anti-neutrinos are distinguished by the total conserved lepton number $L=L_e+L_\mu+L_\tau$

To generate neutrino mass we must relax 1 and/or 2 and/or 3

Staying within the SM is not an option – but what direction?

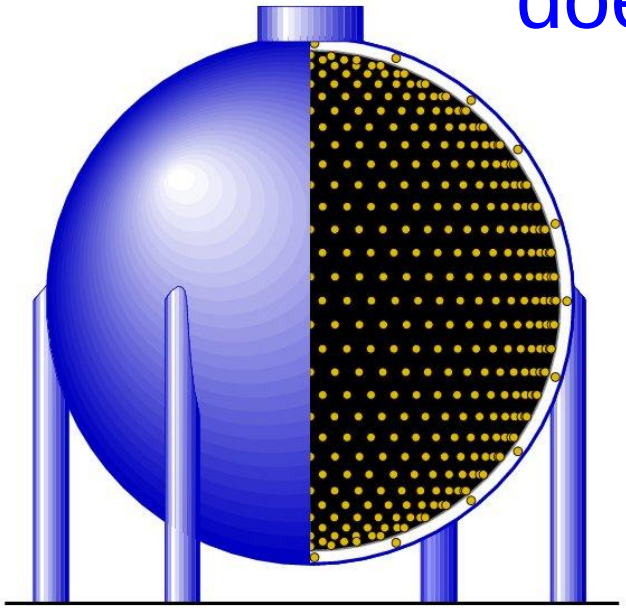
Neutrino mass models roadmap

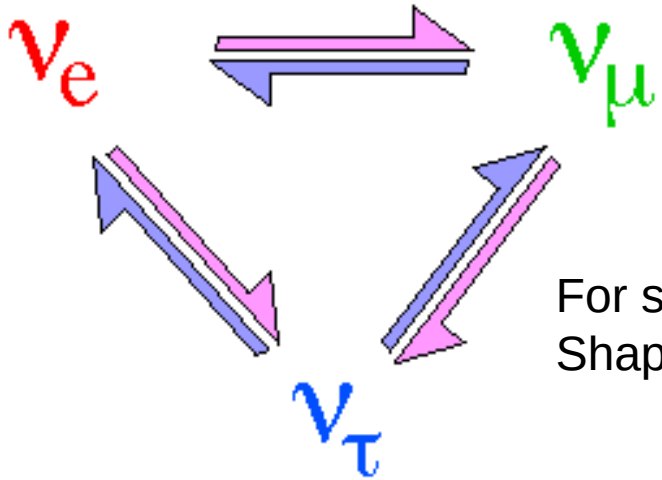


LSND True or False?

MiniBoone does not support LSND result

does support three neutrinos





For steriles see Shaposhnikov talk

In this talk we assume that LSND is false

Dirac or Majorana?

Majorana masses

CP conjugate

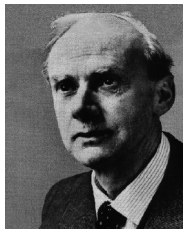
$$m_{LL}\bar{\nu}_L \nu_L^c$$

$$M_{RR}\bar{\nu}_R \nu_R^c$$

Violates L

Violates L_e, L_μ, L_τ

Neutrino=antineutrino



$$m_{LR}\bar{\nu}_L \nu_R$$

Conserves L

Violates L_e, L_μ, L_τ

Neutrino $\neq$ antineutrino

Dirac mass

1st Possibility: Dirac

Recall origin of electron mass in SM with $L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad e_R^-, \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$

$$\lambda_e \bar{L} H e_R^- = \lambda_e \left\langle H^0 \right\rangle \bar{e}_L^- e_R^-$$

Yukawa coupling λ_e must be small since $\langle H^0 \rangle = 175$ GeV

$$m_e = \lambda_e \left\langle H^0 \right\rangle \approx 0.5\, MeV \Leftrightarrow \lambda_e \approx 3.10^{-6}$$

Introduce right-handed neutrino ν_{eR} with zero Majorana mass

$$\lambda_\nu \bar{L} H^c \nu_{eR} = \lambda_\nu \left\langle H^0 \right\rangle \bar{\nu}_{eL} \nu_{eR}$$

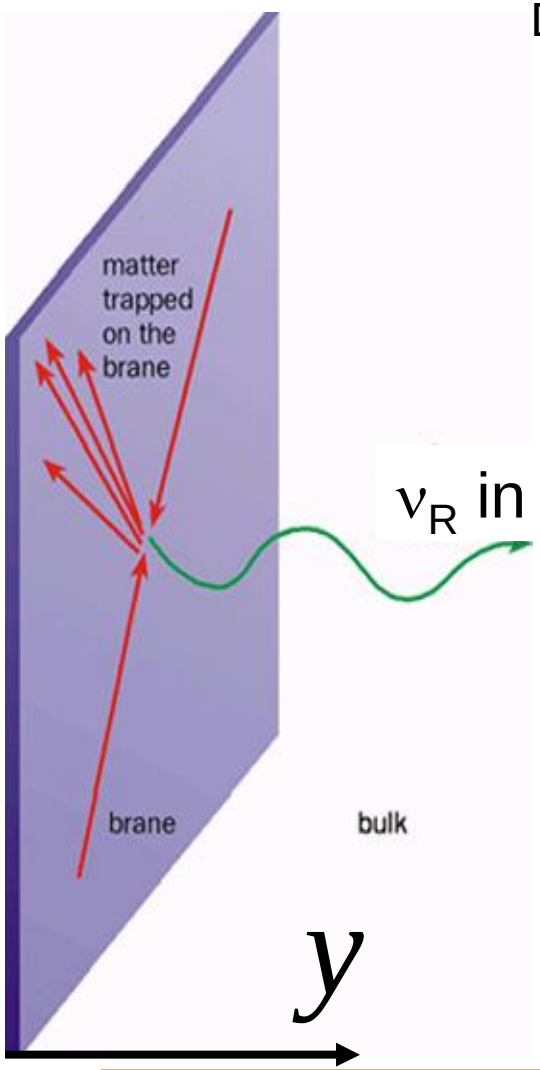
then Yukawa coupling generates a Dirac neutrino mass

$$m_{LR}^\nu = \lambda_\nu \left\langle H^0 \right\rangle \approx 0.2\, eV \Leftrightarrow \lambda_\nu \approx 10^{-12}$$

Why so small?
– extra dimensions

Flat extra dimensions with RH neutrinos in the bulk

Dienes, Dudas, Gherghetta; Arkhani-Hamed, Dimopoulos, Dvali, March-Russell



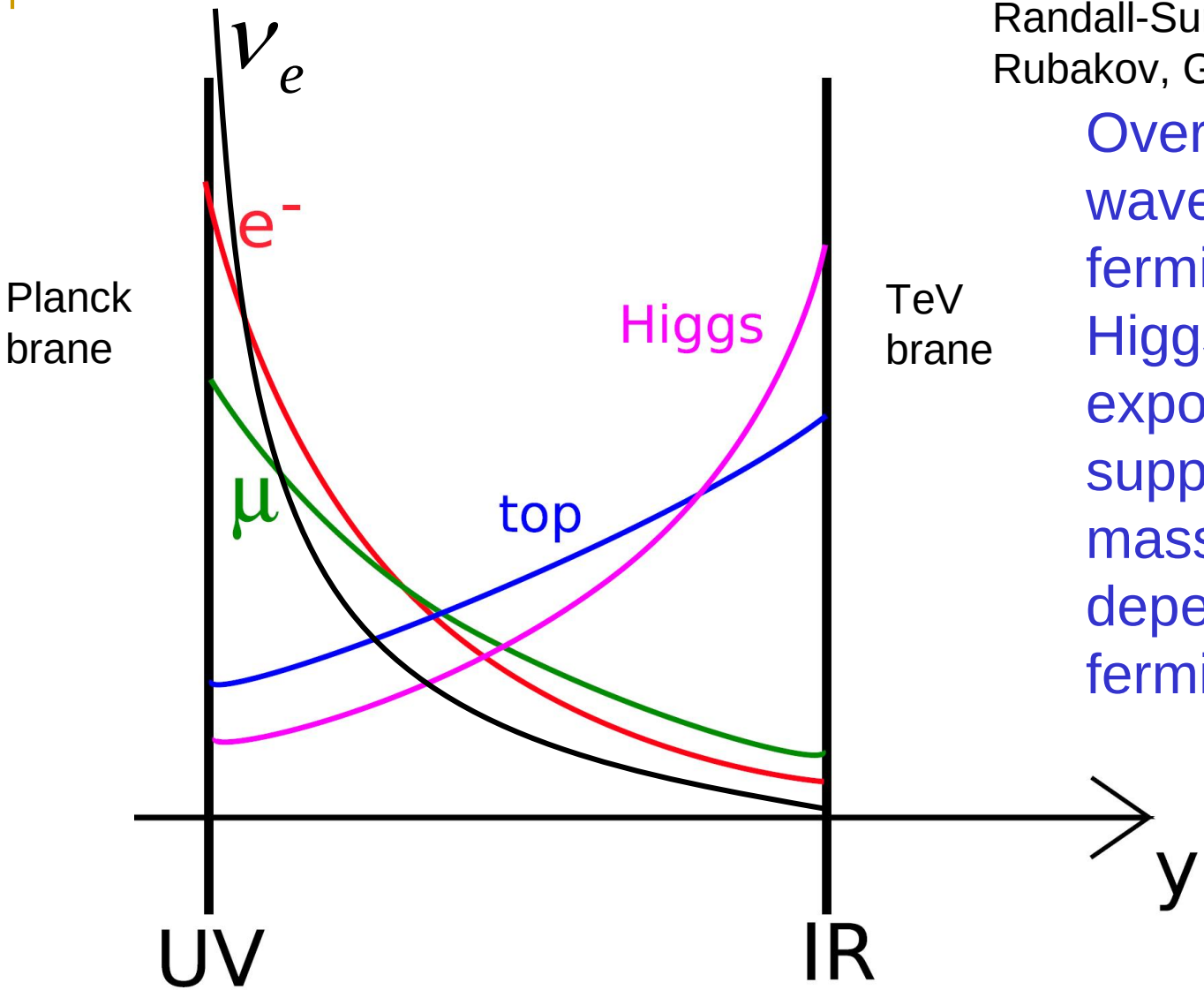
For one extra dimension y the ν_R wavefunction spreads out over the extra dimension, leading to a volume suppressed Yukawa coupling at y=0

$$\rightarrow m_{LR}^{\nu} = \frac{\lambda \langle H^0 \rangle}{\sqrt{V}} = \lambda \langle H^0 \rangle \frac{M_{string}}{M_{Planck}}$$

$$e.g. \quad \frac{M_{string}}{M_{Planck}} = \frac{10^7}{10^{19}} = 10^{-12}$$

Warped extra dimensions with SM in the bulk

Randall-Sundrum;
Rubakov, Gherghetta,…



Overlap wavefunction of fermions with Higgs gives exponentially suppressed Dirac masses, depending on the fermion profiles

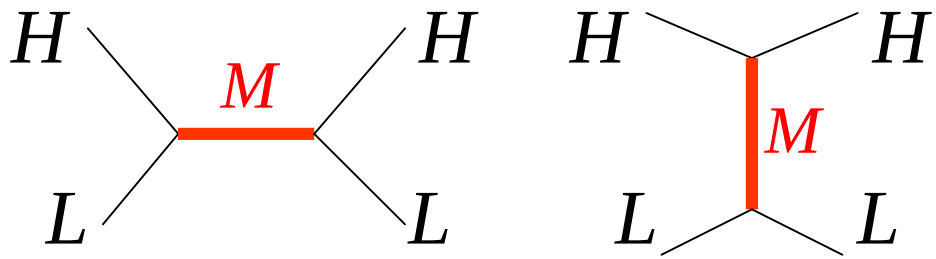
2nd Possibility: Majorana

Renormalisable
$\Delta L = 2$ operator $\lambda_\nu L L \Delta$ where Δ is light Higgs triplet with VEV < 8GeV from ρ parameter

Non-renormalisable
$\Delta L = 2$ operator $\frac{\lambda_\nu}{M} L L H H = \frac{\lambda_\nu}{M} \langle H^0 \rangle^2 \bar{\nu}_{eL} \nu^c_{eL}$ Weinberg

This is nice because it gives naturally small Majorana neutrino masses $m_{LL} \sim \langle H^0 \rangle^2 / M$ where M is some high energy scale

The high mass scale can be associated with some heavy particle of mass M being exchanged (can be singlet or triplet)

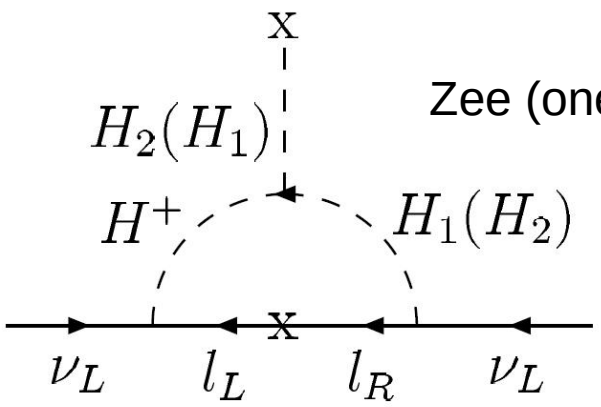


- Loop models
- RPV SUSY
- See-saw mechanisms

• Loop models

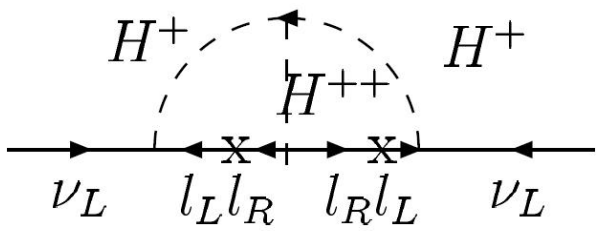
Introduce Higgs singlets and triplets with couplings to leptons

$$-\mathcal{L}^{yuk} = f_{ij} H^{++} l_i l_j + g_{ij} H^{+} l_i \nu_j + h_{ij} H^{0} \nu_i \nu_j$$



Zee (one loop)

Babu (two loop)

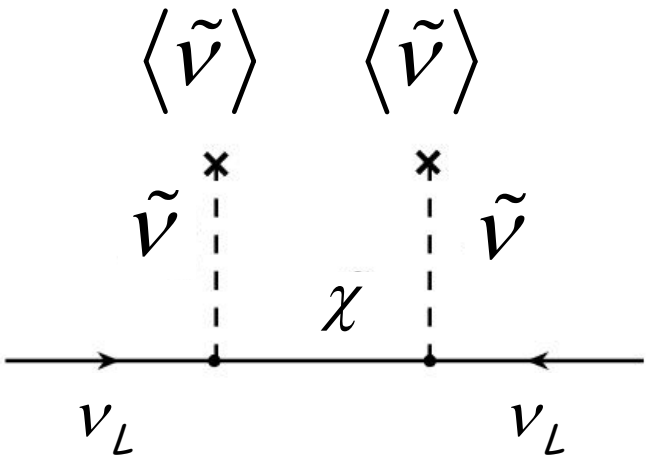


• RPV SUSY

Another way to generate Majorana masses is via SUSY

Scalar partners of lepton doublets (slepton doublets) have same quantum numbers as Higgs doublets

If R-parity is violated then sneutrinos may get (small) VEVs inducing a mixing between neutrinos and neutralinos χ



$$m_{LL}^{\nu} \approx \frac{\langle\tilde{\nu}\rangle^2}{M_\chi} \approx \frac{MeV^2}{TeV} \approx eV$$

Also need loops

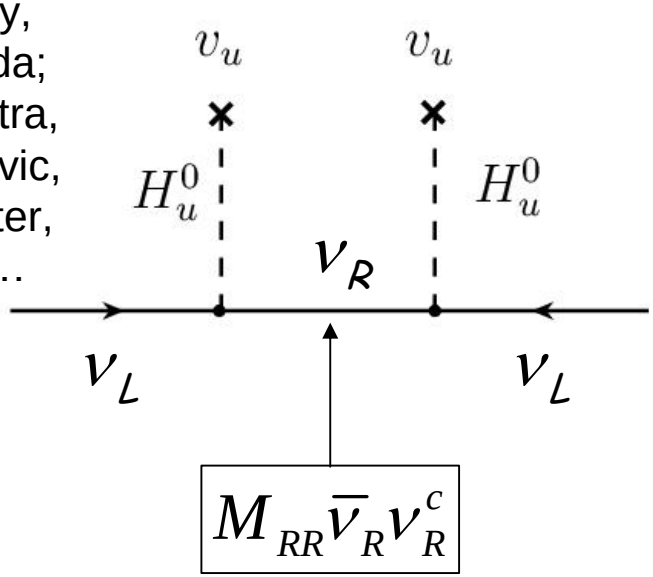
Drees,Dreiner, Diaz, Hirsch, Porod, Romao,Valle,…

See Senjanovic talk for type III

•Type I and II see-saw mechanism

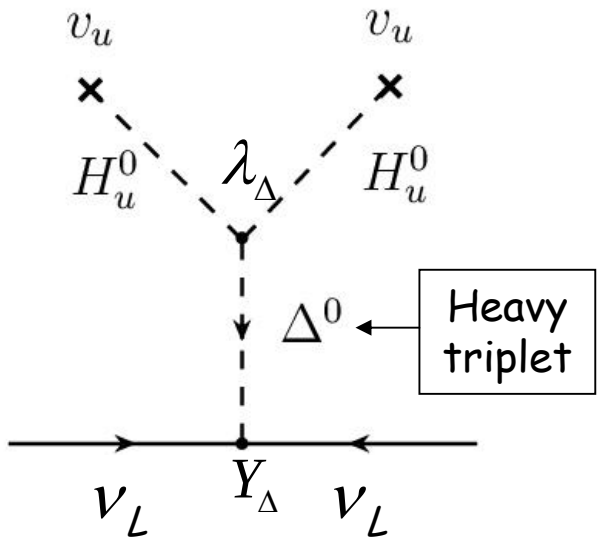
P.Minkowski, Gell-Mann, Ramond, Slansky, Yanagida; Mohapatra, Senjanovic, Schechter, Valle,…

Type I see-saw mechanism



$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type II see-saw mechanism

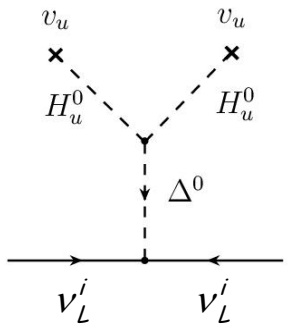


$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

Lazarides, Magg, Mohapatra, Senjanovic, Shafi, Wetterich (1981)

•Type II upgrade of type I models

Antusch, SFK

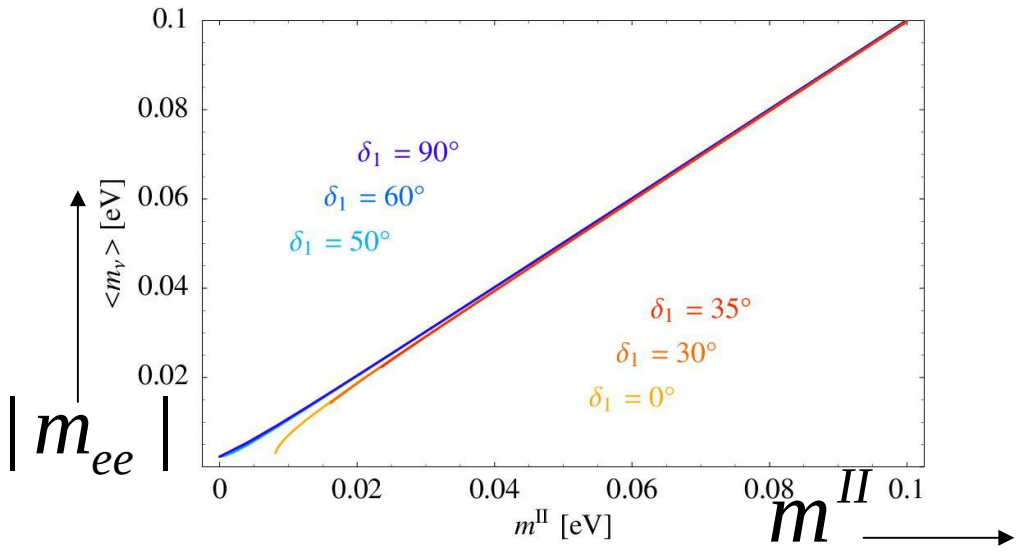


$$m_{LL}^{\nu} = m^{II}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} - m_{LR}M_{RR}^{-1}m_{LR}^{T}$$

Unit matrix type II contribution from an SO(3) family symmetry

Hierarchical type I contribution controls the neutrino mixings and mass splittings

Type II contribution governs the neutrino mass scale and renders neutrinoless double beta decay observable



Very precise Tri-bimaximal mixing (TBM) ?

Harrison, Perkins, Scott

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\theta_{12} = 35^\circ, \qquad \theta_{23} = 45^\circ, \qquad \theta_{13} = 0^\circ.$$

c.f. data

$$\theta_{12} = 33.8^\circ \pm 1.4^\circ, \theta_{23} = 45^\circ \pm 3^\circ, \ \theta_{13} < 12^\circ$$

- Current data is consistent with TBM
- But no convincing reason for exact TBM – expect deviations

It is useful to consider the following parametrization of the PMNS mixing matrix in terms of deviations from TBM

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a)$$

$$0 < r < 0.22, \; -0.11 < s < 0.04, \; -0.12 < a < 0.13.$$

r = reactor s = solar a = atmospheric

SFK; see also Pakvasa, Rodejohann, Wyler; Bjorken, Harrison, Scott, Parke,…

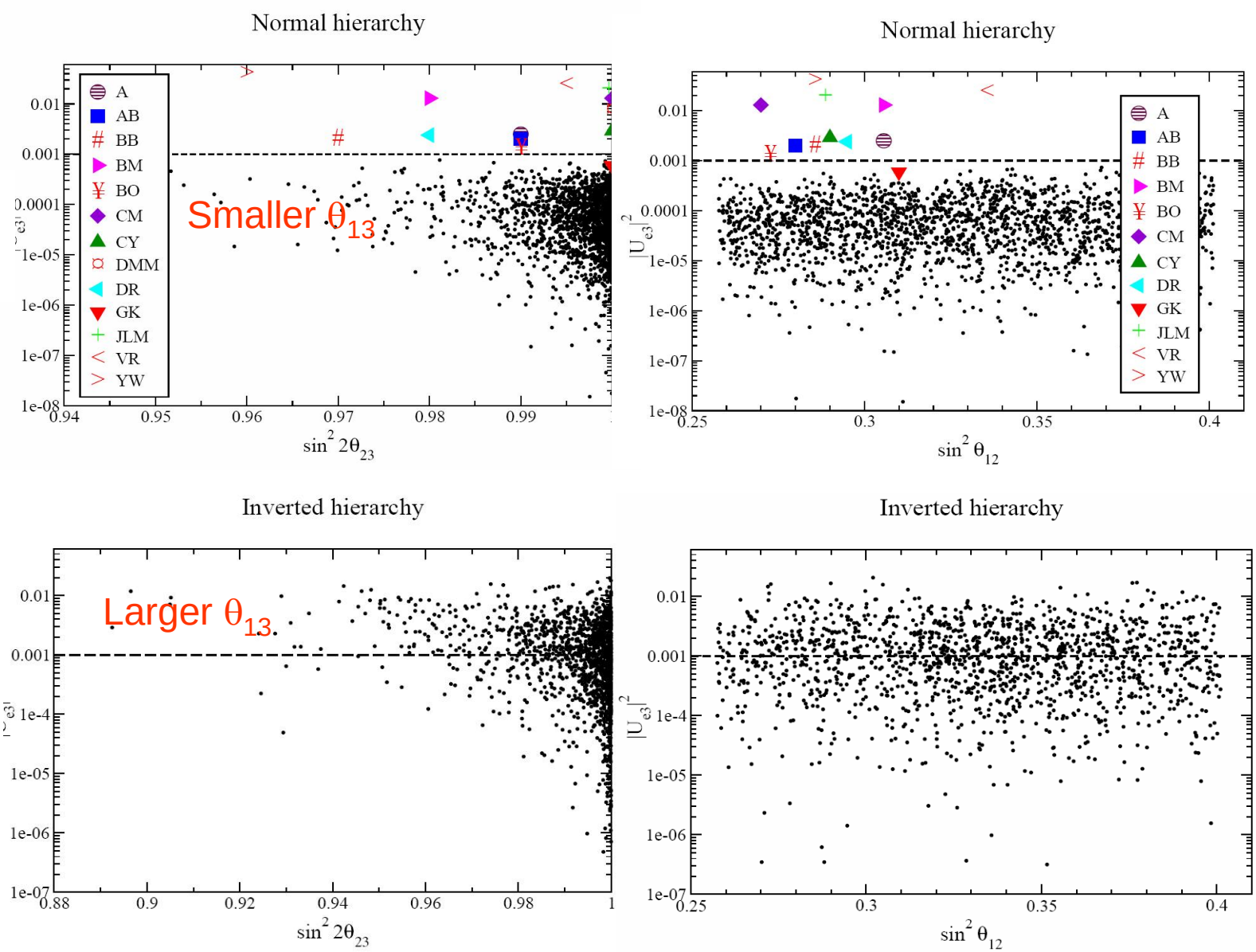
$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$

For a list of oscillation formulae in terms of r,s,a see SFK **arXiv:0710.0530**

Perturbing the TBM neutrino mass matrix

Albright, Rodejohann

$|U_{e3}|^2$



TBM mass matrices in three different bases

1. Diagonal charged lepton basis $U_{MNS} = V^{\nu_L \dagger}$

$$m_{LR}^{E} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \qquad m_{LL} = \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} + \frac{m_1}{6}\begin{pmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix}$$

2. Cabibbo-Wolfenstein basis $U_{MNS} = V^{E_L} V^{\nu_L \dagger}$ $\omega = e^{2\pi i/3}$

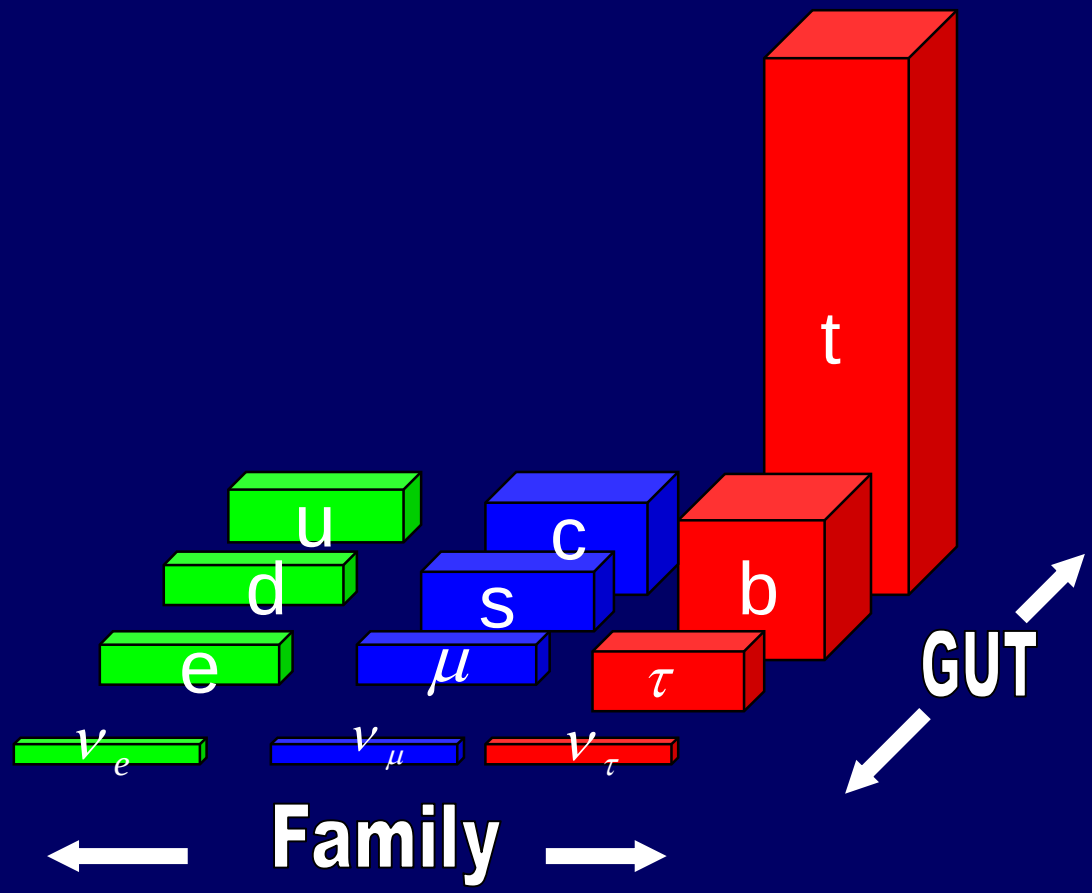
$$m_{LR}^{E} = \begin{pmatrix} m_e & m_\mu & m_\tau \\ m_e & \omega^2 m_\mu & \omega m_\tau \\ m_e & \omega m_\mu & \omega^2 m_\tau \end{pmatrix}, \; m_{LL} = \begin{pmatrix} m_2 & 0 & 0 \\ 0 & m_2 & \Delta \\ 0 & \Delta & m_2 \end{pmatrix} \rightarrow V^{E_L} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \; V^{\nu_L \dagger} = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{-i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \end{pmatrix}$$

3.Diagonal neutrino basis $U_{MNS} = V^{E_L}$

$$m_{LR}^{E} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} m_e & -\sqrt{\frac{1}{6}} m_\mu & \sqrt{\frac{1}{6}} m_\tau \\ \sqrt{\frac{1}{3}} m_e & \sqrt{\frac{1}{3}} m_\mu & -\sqrt{\frac{1}{3}} m_\tau \\ 0 & \sqrt{\frac{1}{2}} m_\mu & \sqrt{\frac{1}{2}} m_\tau \end{pmatrix} \qquad m_{LL} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Low energy physics doesn't care about the choice of basis, but the high scale theory does

Family symmetry



The basic idea of family symmetry is to assign each family a new type of colour charge

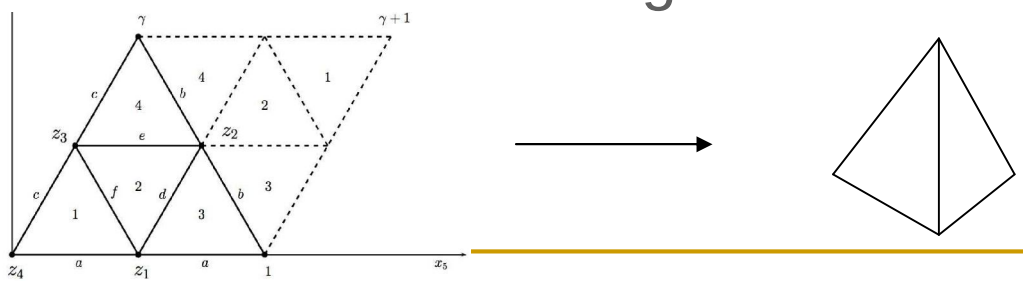
The magic symmetry A_4

- A_4 is symmetry group of the tetrahedron (Plato's ``fire'') reproduces the TBM form of the charged lepton mass matrix in the *Cabibbo-Wolfenstein* basis 2 Ma, Rajasakaran

- TBM form of the neutrino mass matrix then requires a delicate Higgs vacuum alignment Ma; Altarelli,Feruglio

- A_4 may also be used to give the TBM neutrino mass matrix in the *Flavour* basis 1 Altarelli,Feruglio

- A_4 may arise from 6D orbifolding Altarelli,Feruglio,Lin



Deriving TBM from see-saw mechanism

SFK

Diagonal RH nu basis

$$M_{\rm RR} = \begin{pmatrix} X & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & Z \end{pmatrix}$$

columns

$$Y^{\nu}_{\rm LR} = \begin{pmatrix} A & B & C \end{pmatrix}$$

See-saw I

$$m^{\nu}_{LL} = \frac{AA^T}{X} + \frac{BB^T}{Y} + \frac{CC^T}{Z}$$

Sequential dominance

Dominant m_3 | Subdominant m_2 | Decoupled m_1

$$\left.\begin{aligned} |A_1| &= 0, \\ |A_2| &= |A_3|, \\ |B_1| &= |B_2| = |B_3|, \\ A^\dagger B &= 0 \end{aligned}\right\}$$

Constrained SD

$$m^{\nu}_{LL} = \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

TBM mass matrix ($\sim$ 2RHN)

This requires a non-Abelian family symmetry

Need $$Y^{\nu}_{LR} = \begin{pmatrix} 0 & B_1 & - \\ A_2 & B_2 & - \\ A_3 & B_3 & - \end{pmatrix}$$ with $$\begin{aligned} |A_1| &= 0, \\ |A_2| &= |A_3|, \\ |B_1| &= |B_2| = |B_3|, \\ A^{\dagger} B &= 0 \end{aligned}$$

$2 \leftrightarrow 3$ symmetry (from maximal atmospheric mixing)

$1 \leftrightarrow 2 \leftrightarrow 3$ symmetry (from tri-maximal solar mixing)

Several examples of suitable non-Abelian Family Symmetries:

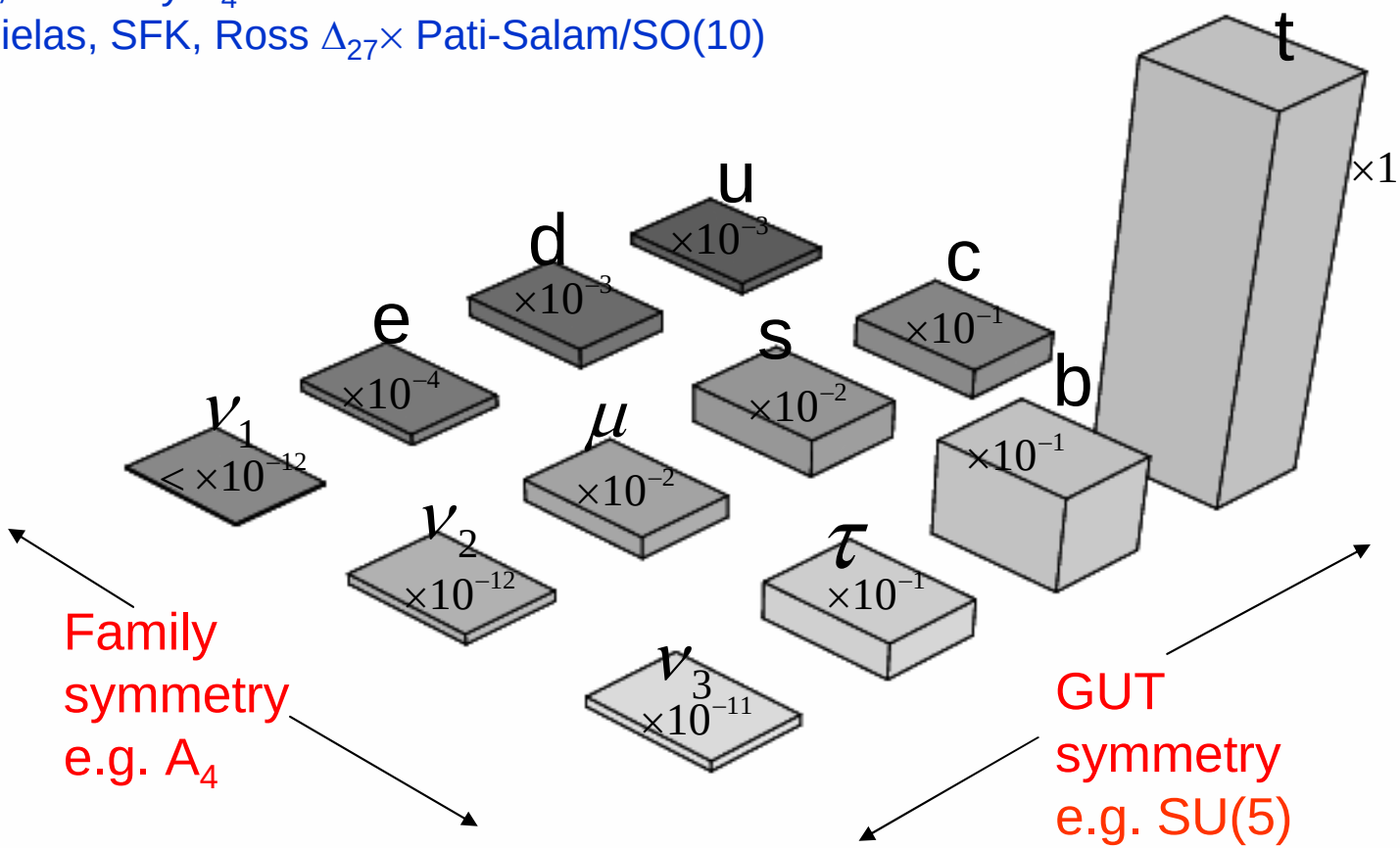
SFK, Ross; Velasco-Sevilla; Varzelias $SU(3)$ Δ_{27}

SFK, Malinsky $SO(3)$ A_4

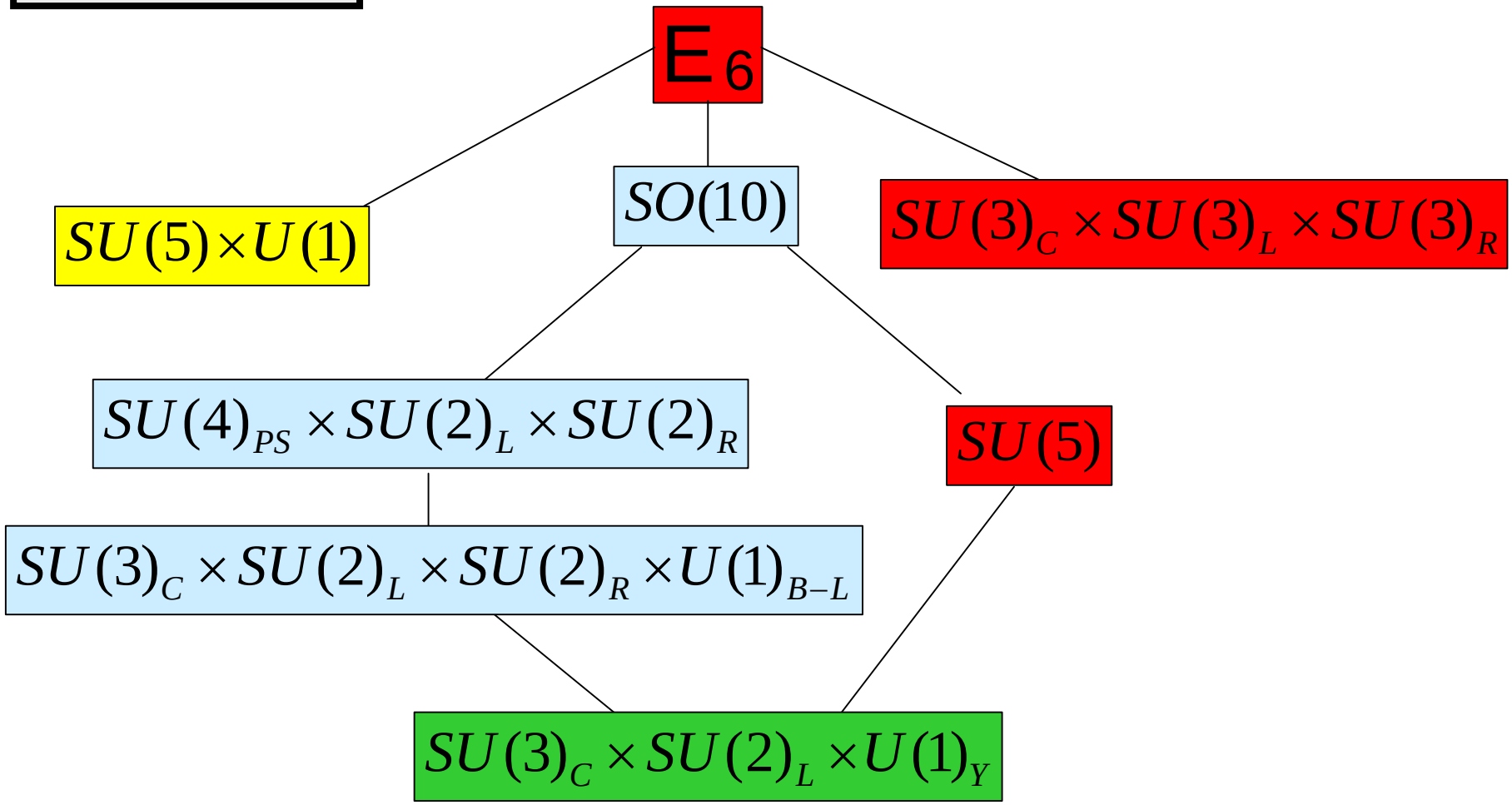
Discrete subgroups preferred by vacuum alignment

Family × GUT symmetry

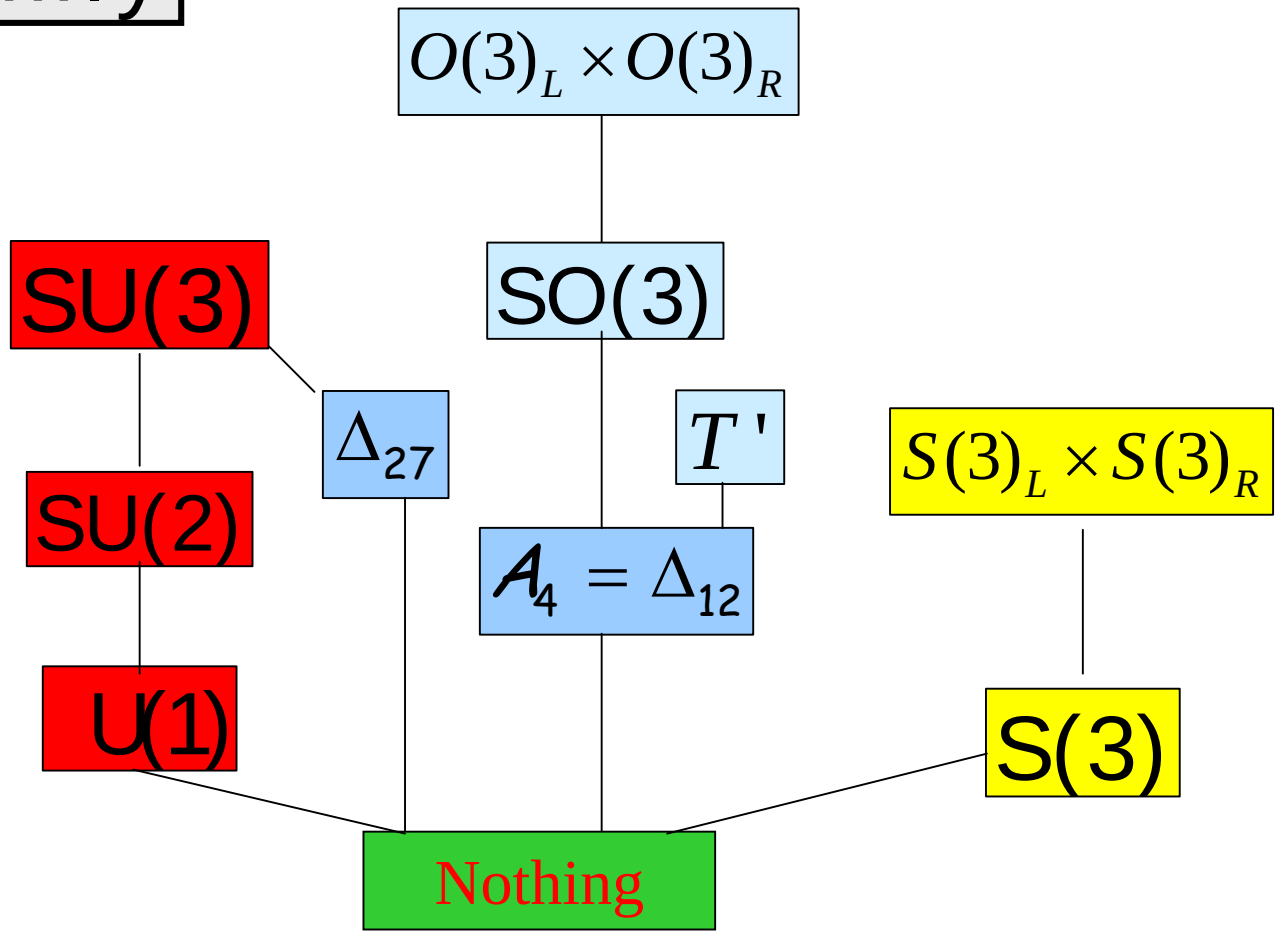
e.g. Chen and Mahanthappa T'× SU(5)
Altarelli, Feruglio, Hagedorn A_4 × SU(5) (in 5d)
SFK, Malinsky A_4 × Pati-Salam
Varzielas, SFK, Ross Δ_{27}× Pati-Salam/SO(10)



G_{GUT}


E_6
$SU(5)\times U(1)$
$SO(10)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(4)_{PS} \times SU(2)_L \times SU(2)_R$
$SU(5)$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$

G_{Family}


$O(3)_L \times O(3)_R$
SU(3)
SO(3)
Δ_{27}
T'
$S(3)_L \times S(3)_R$
SU(2)
$A_4 = \Delta_{12}$
U(1)
S(3)
Nothing

General Strategy

Choose a GUT and family symmetry and write down the reps

Asssign quarks, leptons, Higgs to reps $L_i = \begin{pmatrix} \nu_i \\ E_i^- \end{pmatrix}, \quad E_j^c, \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$

Renormalizable Yukawas requires extended Higgs H→ H^{ij}

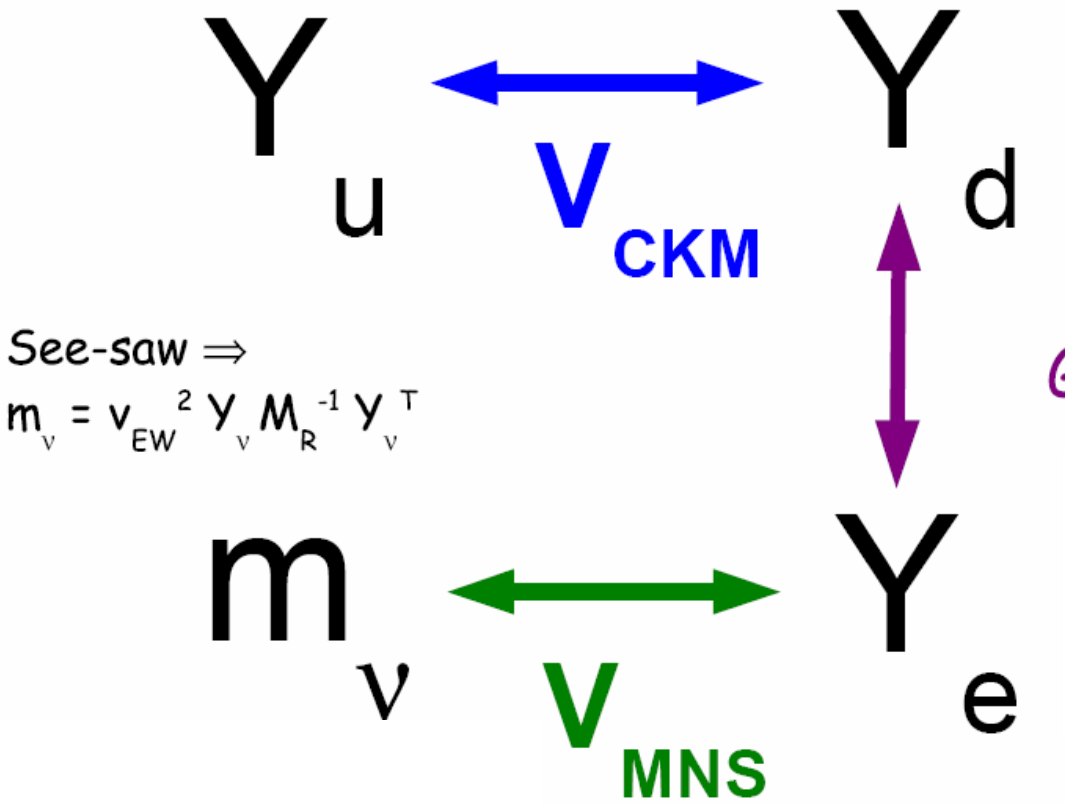
$$\lambda_{ij} H L_i E_j^c \rightarrow \lambda H^{ij} L_i E_j^c$$ Machado,Pleitez

Alternatively promote Yukawas to non-renormalizable terms involving the usual Higgs H plus SM singlet *flavon fields* ϕ

$$\lambda_{ij} H L_i E_j^c \rightarrow \lambda \frac{\phi^{ij}}{M} H L_i E_j^c \quad \text{or} \quad \lambda_{ij} H L_i E_j^c \rightarrow \lambda \frac{\phi^i \phi^j}{M^2} H L_i E_j^c$$

Koide; Stech SFK,Ross

GUT relations



Class of models: $\theta_{12}^{d} \gg \theta_{12}^{u}$

$\theta_{12}^{d} \approx \theta_c$

Georgi-Jarlskog

GUT relation

See-saw $\Rightarrow$

$m_\nu = v_{EW}^{2}\, Y_\nu M_R^{-1} Y_\nu^{T}$

$$\theta_{12}^{e} \approx \frac{\theta_{12}^{d}}{3} \approx \frac{\theta_c}{3}$$

Can this lead to Quark-Lepton Complementarity (QLC)?

$\theta_{12}+\theta_C=45^o$ Petcov,Smirnov; Raidal;Ohlsson,Seidl

Sum Rules

$$U_{MNS} = V^{E_L} V^{\nu_L \dagger}$$

Cabibbo-like

Bimaximal or
Tri-bimaximal

Bjorken; Ferrandis, Pakvasa; SFK

$$\theta_{13} \approx \frac{\theta_{12}^{e}}{\sqrt{2}} \approx \frac{\theta_C}{3\sqrt{2}} \approx 3^{\circ},$$

$$\theta_{12} = 45^{o}(35)^{o} + \frac{\theta_C}{3\sqrt{2}} \cos\delta$$

Antusch,SFK

Oscillation phase

$$\boxed{\theta_{12}^{o} = 45^{o}(35)^{o} + \theta_{13}^{o} \cos\delta}$$

Sum Rule

SFK; Antusch,SFK; Masina

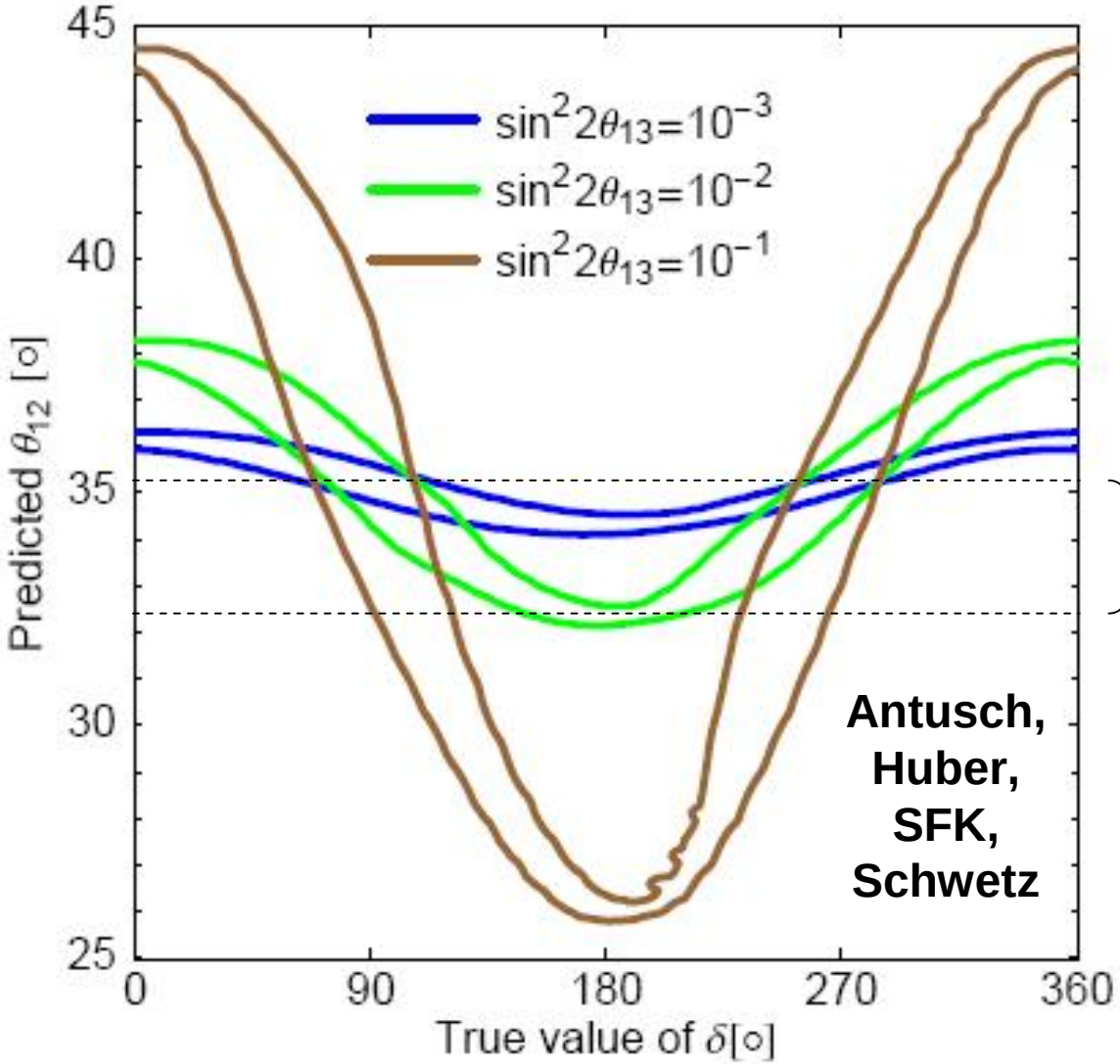
Bimaximal sum rule with 45° requires $\theta_{13} \approx \theta_C$ and $\delta \approx \pi$

→QLC is only achieved for a special phase and large θ_{13}

Antusch,SFK,Mohapatra

What about tri-bimaximal sum rule with 35° ?

Tri-bimaximal sum rule $\theta_{12} \approx 35.3^o + \theta_{13}\cos\delta$



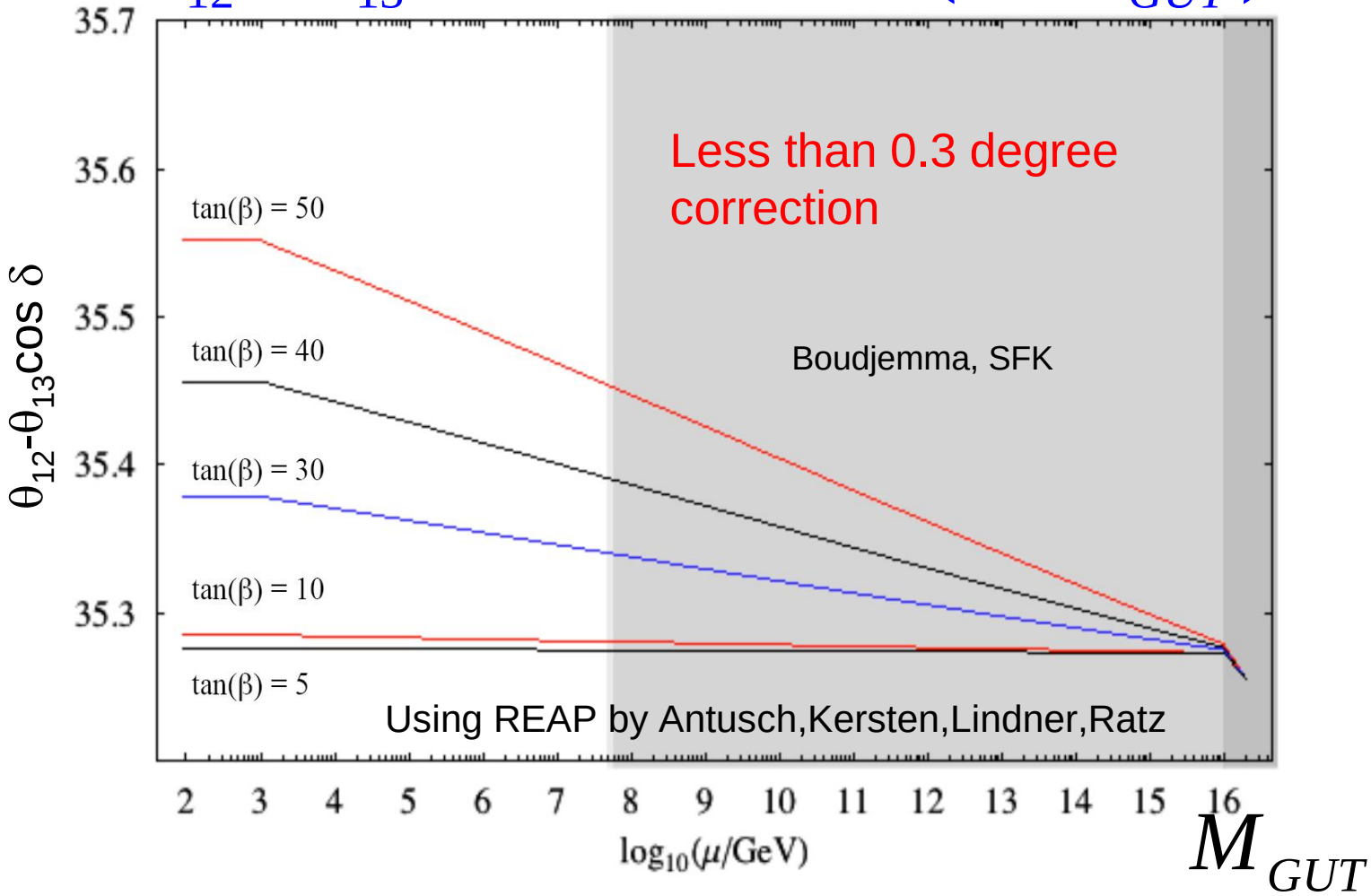
Bands show 3σ error for an optimized neutrino factory determination of $\theta_{13}\cos\delta$

$\theta_{12}=33.8^o \pm 1.4^o$
(current value)

Tri-bimaximal sum rule works incredibly well !!

RGE corrections to TBM sum rule

$$\theta_{12}^{o} - \theta_{13}^{o}\cos\delta \approx 35.3^{o} \quad (at\ M_{GUT})$$



Alternative Ideas

- Accidental family symmetry from messenger dominance Ferretti, SFK, Romanino; Barr
- SU(8) GUTs Barr
- Mass matrices from shift symmetry: $\nu_R \rightarrow \nu_R + \eta\,\theta$ Friedberg,Lee; Jarlskog
- Extremization of mass matrix Jarlskog invariants Harrison,Scott
- Theories of the Koide mass formula Koide,…
- Dirac screening in the double see-saw Lindner,Smirnov,Schmidt
- Low energy see-saw models with gauged B-L SFK,Yanagida
- Anarchy/Landscape (large θ_{13} only) Hall,Murayama,Weiner
- RH Neutrino masses in string theory Antusch,Ibanez; Nilles,Langacker
- Invariant classification of see-saw models SFK

Conclusion

- **Neutrino mass and mixing requires new physics BSM**
- **Many roads for model building, but answers to key experimental questions will provide the signposts**
- **One key question is how accurately is TBM realised?**
- **Goal of next generation of oscillation experiments is to show that the deviations from TBM r,s,a are non-zero and measure them and δ**
- **If TBM is accurately realised this may imply a new symmetry of nature: family symmetry**
- **GUTs $\times$ family symmetry with see-saw + CSD is very attractive framework for TBM → sum rule prediction**
- **Few realistic models, complicated vacuum alignment**
- **Status quo is not an option – neutrino physics demands a theory of flavour, and may provide further clues**