Supernova Theory:

Simulation and Neutrino Fluxes

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Abstract

It is now generally accepted that Type II supernovae arise from the collapse of a massive stellar core that has reached the end point of silicon burning. The observation of a neutrino pulse from Supernova 1987A strongly supports the core collapse theory.

However, 1-D simulations of core collapse fail to explode unless *ad hoc* mechanisms are invoked. Some recent multidimensional simulations explode without *ad hoc* mechanisms, but this result is not yet robust.

We review the state of multidimensional core collapse simulations, with particular emphasis on neutrino transport algorithms and results. What are the predicted neutrino fluxes, and how can observations constrain theory and provide insights to computational physicists?

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Neutrino Physics Relevant to Transport

- Neutrinos come in six flavors.
- Neutrinos are subject to oscillations between flavors.
- Neutrinos are subject to Fermi-Dirac statistics.
- The neutrino chemical potential is nontrivial.
- Neutrino scattering from leptons somewhat resembles Compton scattering of gamma rays.
- Neutrino opacities increase strongly with energy, yielding a low-energy "window."
- Neutrinos are subject to stimulated absorption.

Overview

- **Physics of core collapse**
  - The role of neutrinos
  - Energetics
  - Neutronization
  - Neutrino physics in core collapse
- **Simulations**
  - Is something missing from the models?
- **Algorithms for neutrino transport**
  - Flux-limited diffusion
  - Transport approximations
- **Observables**
  - Neutrino fluxes
  - Spectra
  - Time dependence

Core collapse and the role of neutrinos

Gravitationally bound systems display a negative heat capacity until short-range repulsive forces take over.

Temperatures at the end of silicon burning are enormous and the core radiates neutrinos freely. Photodissociation and neutronization reduce the adiabatic ratio.

This results in a thermal runaway that terminates in a “bounce” due to the short-range nuclear strong force.

The core becomes opaque to neutrinos.

About 1% of the neutrinos emitted by the core are thought to be reabsorbed in the mantle region, heating it sufficiently to drive a shock that breaks out as the visible Type II supernova.

Energetics and Neutronization

The core collapse converts about 1.5 solar masses of symmetric matter to a proto-neutron star about 30 km in radius. The final neutron star has a radius of about 10 km.

Energy of initial collapse: $U \sim \frac{3}{5}\frac{GM^2}{r} = 1.2\times10^{53}\ \text{erg} = 120\ \text{foes}$

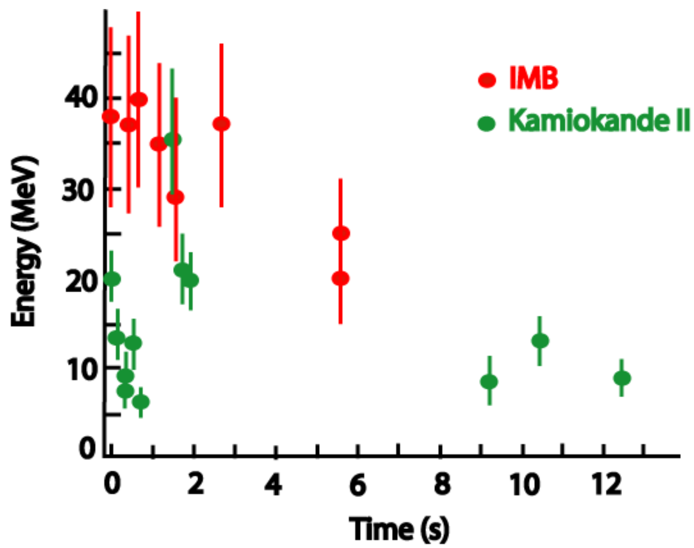
Energy of subsequent cooling: $U \sim 240\ \text{foes}$

Lepton number of collapse and cooling: $N \sim 0.4MN_A = 7.2\times10^{56}$

Collapse energy per net lepton: $\varepsilon = \frac{U}{N} \sim 100\ \text{MeV}$

The collapse temperature is $\sim \left(\frac{1.2\times10^{53}\ \text{erg}}{\left(\frac{4}{3}\pi r^3\right)(6a)}\right)^{1/4} = 35\ \text{MeV}$

SN 1987A



http://cosmos.colorado.edu/stem/courses/common/documents/chapter6/l6S6.htm

These detections are consistent with an antineutrino count of 10^{58} and an energy of 100 foes, corresponding to a spectrum considerably softer than the collapse temperature.

The visible energy release was 1.7 foes (Arnett 1996).

Neutrino Physics in Core Collapse

- During the early stages of collapse, neutrinos stream freely out of the core.
- A 1.5 solar mass core becomes opaque to 100 MeV neutrinos at a radius of about 5000 km.
- At bounce radius of 30 km, the mean free path in the core is on the order of a meter for all neutrinos.
- At core densities, neutrino oscillations are not thought to be significant.
- The proto-neutron star has a distinct neutrinosphere where the observable neutrino spectrum is formed.

Simulations

The core collapse model was first proposed in 1938 by Baade and Zwicky, but the first numerical simulations were performed in the early 1960s by Colgate.

It soon became clear that the bounce alone was not sufficient to produce an explosion. Neutrino heating behind the shock was then proposed as a coupling mechanism (Colgate and White 1962), and this remains a standard ingredient of models.

However, even with neutrino heating, one does not obtain explosions in 1-D calculations. What is still missing?

Convection within the proto-neutron star and mantle clearly helps, but there is disagreement on whether this is sufficient. Could the answer be better neutrino transport?

The Boltzmann Equation

- **The Boltzmann equation in curved space is**

$$p^{\alpha}\frac{\partial f}{\partial x^{\alpha}} - \Gamma^{\alpha}{}_{\beta\gamma}p^{\beta}p^{\gamma}\frac{\partial f}{\partial p^{\alpha}} = C[f]$$

where p^{α} **is the four-momentum,** $f \equiv \frac{I}{h\varepsilon^{3}/c^{2}}$ **is the invariant phase space distribution function, and** $\Gamma^{\alpha}{}_{\beta\gamma}$ **are connection coefficients.**

- **In flat space, this reduces to the familiar form**

$$\frac{1}{c}\frac{\partial I}{\partial t} + \hat{n} \bullet \nabla I + \sigma I = \eta$$

Diffusion Approximation

- Let $I \approx J + 3\hat{n} \bullet \vec{F}$. Then the moments of the Boltzmann equation are

$$\frac{1}{c}\frac{\partial J}{\partial t} + \nabla \bullet \vec{F} = \sigma_a (B - J)$$

$$\frac{1}{c}\frac{\partial}{\partial t}\vec{F} + \sigma_t \vec{F} = -\frac{1}{3}\nabla J$$

This is the P1 approximation. Asymptotic analysis suggests the further approximation

$$\vec{F} = -\frac{1}{3\sigma_t}\nabla J \qquad \frac{1}{c}\frac{\partial J}{\partial t} + \nabla \frac{1}{3\sigma_t}\nabla J = \sigma_a (B - J)$$

Flux-limited diffusion

- **The diffusion approximation is accurate at large optical depth.**
- **At small optical depth, the streaming approximation is often accurate:**

$$\vec{F} = -J\frac{\nabla J}{|\nabla J|}$$

- **Flux limiters are *ad hoc* interpolations:**

$$\vec{F} = -\frac{1}{3\sigma_t + \frac{|\nabla J|}{J}}\nabla J$$

- **Maximum entropy methods resemble flux limiters, but have a basis in information theory.**

Transport approximations

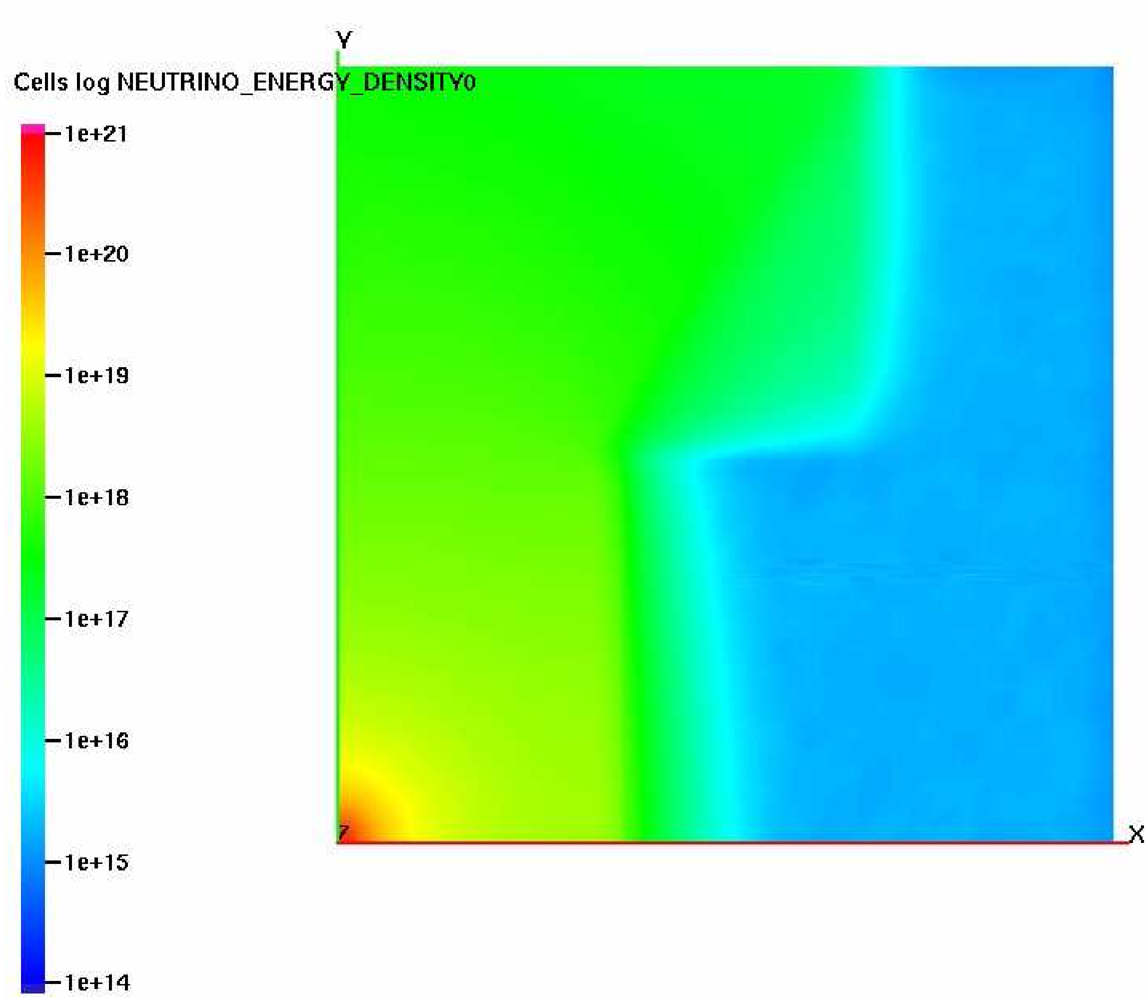
- Pn: Take moments beyond zeroth and first. Results in a very complicated method prone to ringing.
- Sn: Discretize on ordinates. Relatively straightforward method but subject to ray effects.
- Variable Eddington approximation:

$$\frac{1}{c}\frac{\partial J}{\partial t} + \nabla \bullet \vec{F} = \sigma_a (B - J)$$

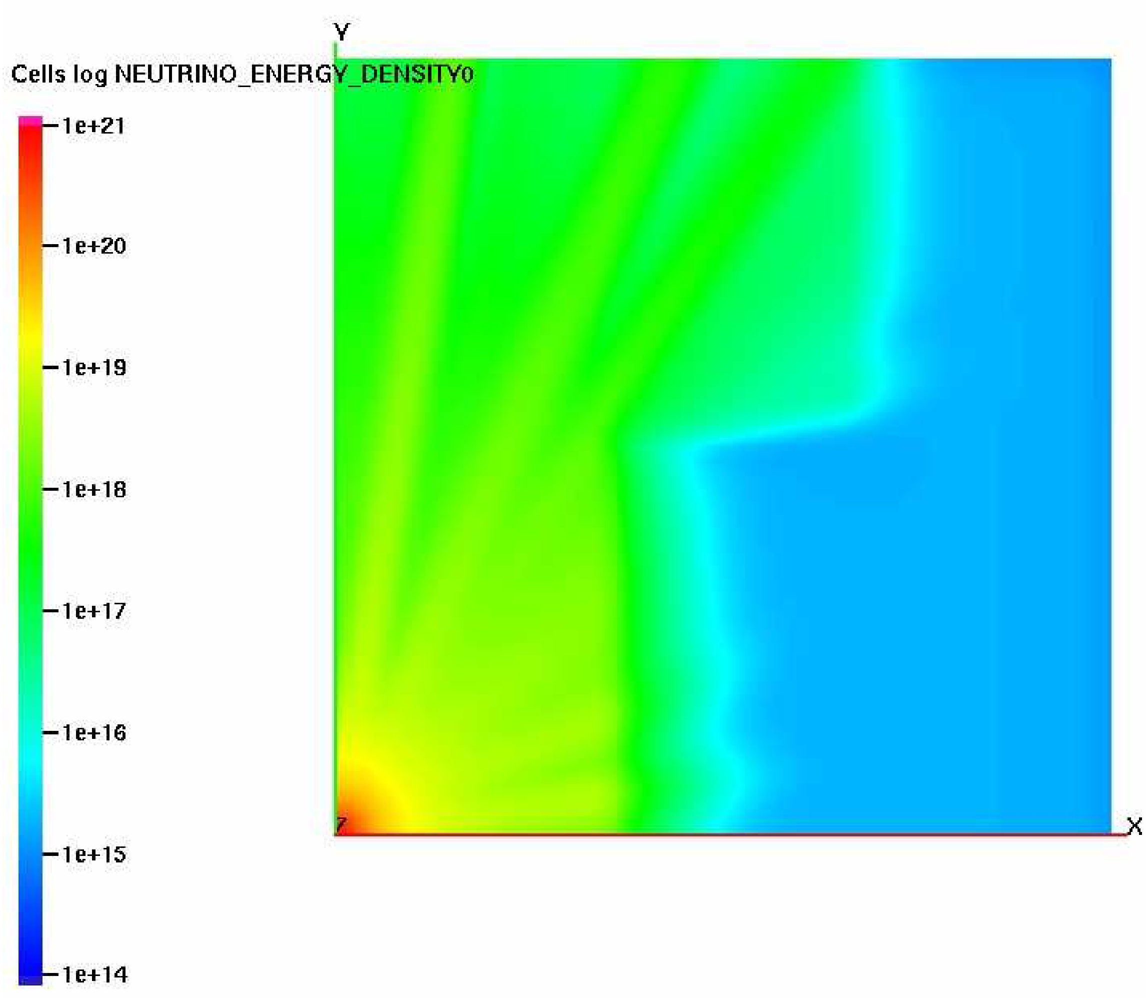
$$\frac{1}{c}\frac{\partial}{\partial t}\vec{F} + \sigma_t \vec{F} = -\nabla \bullet \underset{\sim}{f} J$$

$$\underset{\sim}{f} \equiv \frac{\oint I n \otimes n d\Omega}{\oint I d\Omega}$$

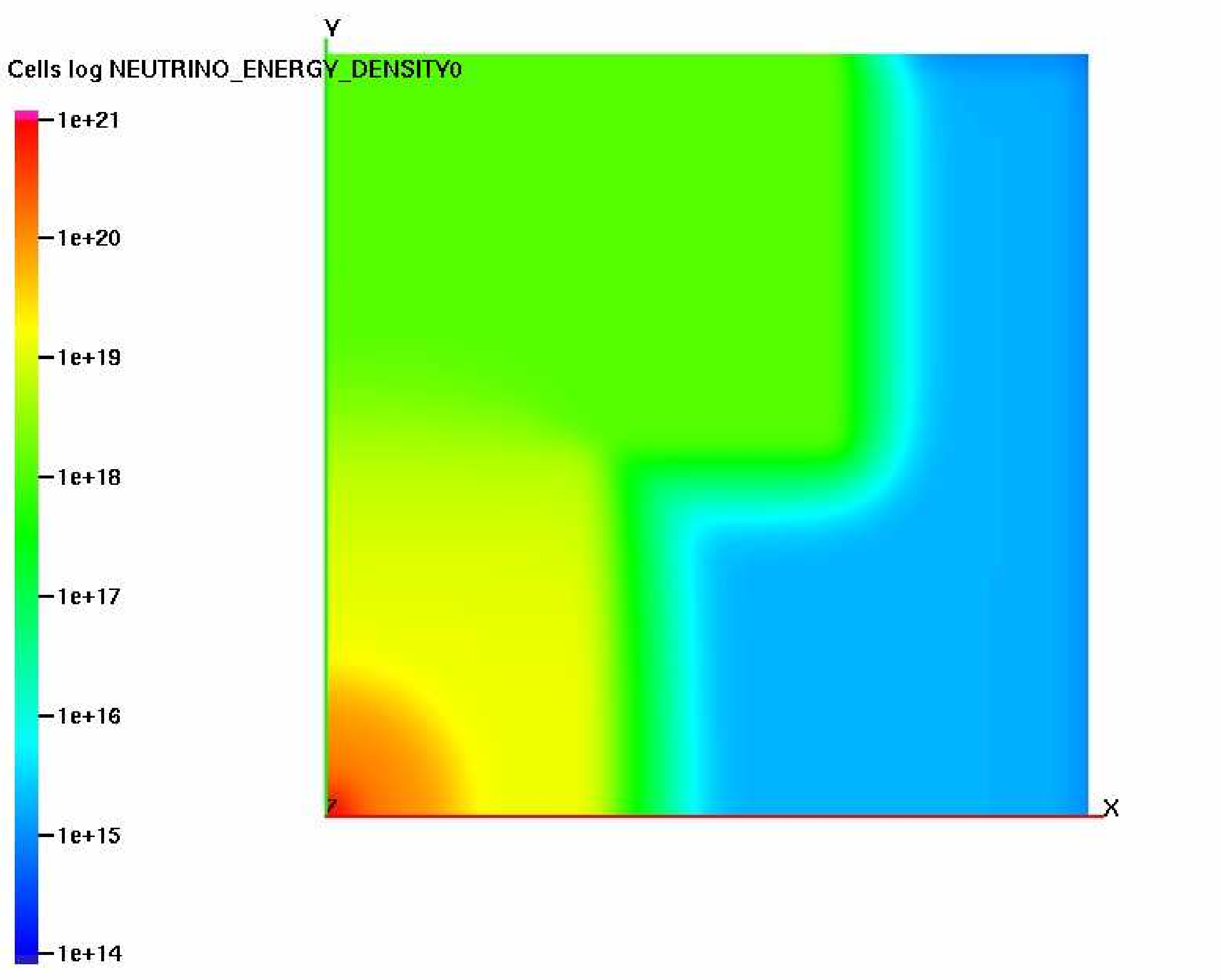
Toy Problem, Monte Carlo



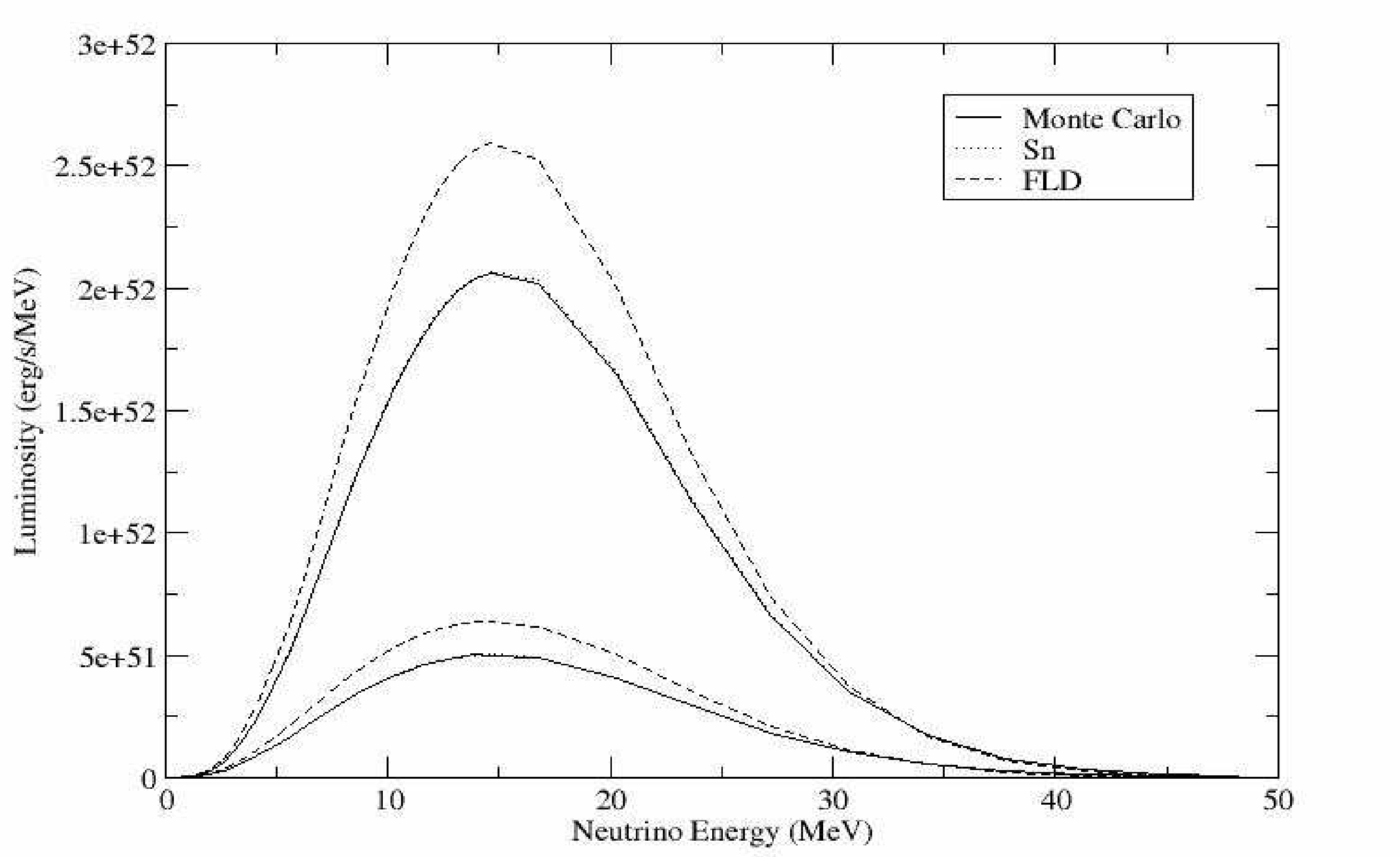
Toy Problem, S16



Toy Problem, Flux-Limited Diffusion



Collapse simulation: Spectrum



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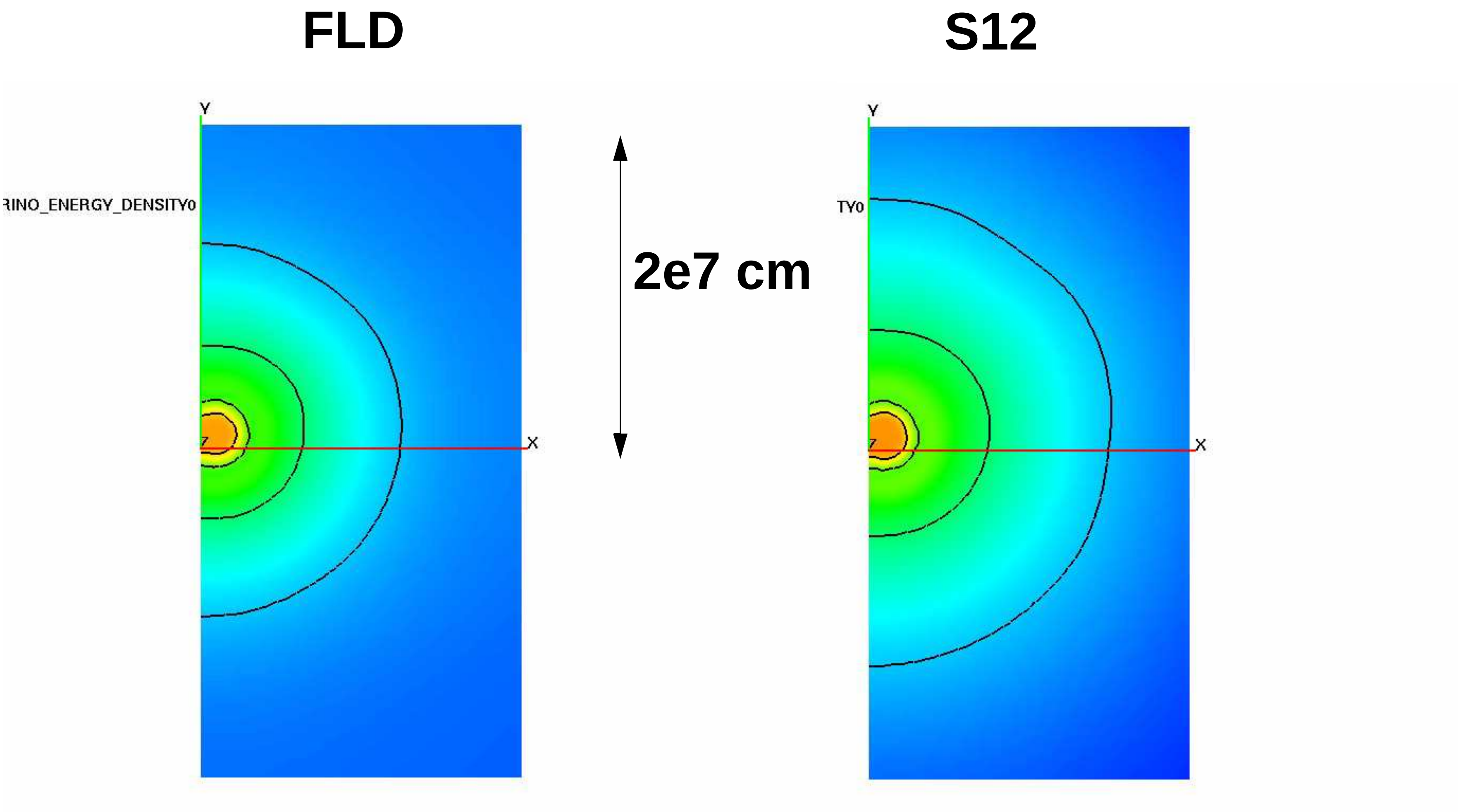
Collapse simulation, FLD versus S12



Asymmetry and Turbulence

- Type II supernova remnants show distinct asymmetry.
- Type II supernova remnants show nonuniform distributions of different elements.
- Most neutron stars show high space velocities (~1000 km/s) consistent with an asymmetric kick.
- R-process requires fairly constrained trajectories. Turbulent explosions seem necessary to place enough material on the right trajectories.
- However, the amount of turbulence is constrained by the modest abundance of tin in the universe: Highly neutron-rich material is not being dredged up in quantity.

Hypothetical Explosion Enhancers

- Softer nuclear equation of state: Unlikely
- Better neutrino transport or physics: Not likely
- Turbulent convection: Possibly
- Dominant low mode convection: Possibly
- “Singing stars” (Burrows): Not yet replicated and viewed with some skepticism.
- The Tooth Fairy: The One Tooth Fairy Limit from the days of the solar neutrino deficit applies.

Where the Simulation Community Is

- Most supernova calculations have used flux-limited diffusion because of its simplicity.
- Coupled transport calculations have been employed in 1-D simulations. They do not produce explosions.
- Multidimensional calculations using flux-limited diffusion have been carried out. These sometimes produce explosions.
- Multidimensional hydrodynamics plus sectorial 1-D transport has been employed in calculations. This is the state of the art for coupled simulations.
- Monte Carlo has been used to refine the output spectrum. This is the state of the art for diagnostics.

Where the Simulation Community Is Going

Building a large simulation code, like building a large neutrino detector, takes many years. It is cheaper only if you leave out the cost of supercomputers.

- Major efforts led by Janka, Mezacappa, and Burrows are in the works.
- 3-D hydrodynamics
- Boltzmann transport
- GR corrections
- Better neutrino physics

It is not clear to me that we have the computing iron, even at places like Los Alamos, to tackle the 3-D radiation hydrodynamics problem with Boltzmann transport.

Observables

- Total neutrino and antineutrino numbers.
- Spectrum
- Time evolution: The shorter the time resolution, the better. Statistics-limited, not instrumentation-limited.

Supernova 1987A gave us a glimpse at the high-energy tail of the antineutrino spectrum with poor time resolution.

The more interesting low-energy neutrinos are much harder to detect. Nature can be a real Mother sometimes.

Conclusions

Is something fundamentally wrong with our models? Probably not. Our guess is that incremental improvements in physics and better 3-D hydrodynamical models will close the gap.

Given the limits on resources, weam skeptical that vastly improved transport is a good investment at this time.

Supernova 1987A confirmed the general core-collapse scenario, but small-number statistics mean few serious constraints on models.

A Type II supernova in the Milky Way would give us excellent statistics (albeit of one event) that could better constrain models. There is roughly a 20% chance of this happening during my career. :(