The physics case of the Neutrino Factory

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Abstract. I discuss the physics case of the standard Neutrino Factory facility coupled to an iron detector to exploit the so-called ‘Golden-Channel’. The performance of the facility is impressive, although it is not free from degeneracies arising from a combination of physics and instrumental limitations. Nevertheless, one could explore at great depth the parameter of the leptonic mixing matrix as well as the mass hierarchy. Best performance is obtained with two baselines (one of them very long) and an improved magnetic detector with low energy detection threshold.

1. Introduction
The observation of neutrino oscillations in both the atmospheric and solar sectors imply that neutrinos have mass and mix[1]. This is the first beyond-the-standar-model result in many decades.

Ongoing and approved experiments make use of intense pion beams to generate neutrinos. They are designed to seek and measure the third mixing angle $\theta_{13}$ of neutrino mixing matrix (the ‘PMNS’ matrix), but will have little or no sensitivity to matter-antimatter symmetry violation. To measure $\theta_{13}$ (if too small for current generation of experiments) and to explore CP violation in the leptonic sector, as well as determining the mass hierarchy ($\text{sgn}\Delta(m^2_{32})$), new facilities are needed.

I discuss in this paper some aspects of the physics potential of the Neutrino Factory, one of the most ambitious proposed future facilities. For reasons of space I only cover a few topics, mainly quoting results and using figures from the ISS Physics Report [2] to which the interested reader is referred for a comprehensive discussion.

2. The Neutrino Factory
In a Neutrino Factory [3, 4] muons are accelerated from an intense source to energies of several tens of GeV and injected into a storage ring with long straight sections. The muon decays $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ and $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ provide a very well known flux of neutrinos with energies up to the muon energy itself. Neutrino Factory designs have defined a ‘standard’ facility in which a 50 GeV stored-muon beam delivers a luminosity of $1 \times 10^{21}$ muon decays per year.

Twelve oscillation processes can be studied using the Neutrino Factory which produces and store beams of both positive and negative muons (see table 1). In order to take full advantage of this flavour-richness, the optimal detector should be able to perform both appearance and disappearance experiments, providing lepton identification and charge discrimination.

The search for $\nu_e \to \nu_\mu$ transitions (the ‘golden channel’) [5] appears to be particularly attractive at the Neutrino Factory. It can be studied in appearance mode, by looking for muons
\[ \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \quad \mu^- \rightarrow e^- \bar{\nu}_e \]

- \( \nu_\mu \rightarrow \nu_\mu \)
- \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)
- \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)
- \( \nu_\mu \rightarrow \nu_\tau \)
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Table 1. Oscillation processes in a Neutrino Factory.

with charge opposite to that of the stored muon beam (‘wrong-sign muons’), thus strongly reducing the dominant background (‘right-sign muons’). The wrong-sign-muon channel yields an impressive sensitivity to \( \sin^2 \theta_{13} \) and sensitivity to the leptonic CP-violating phase, \( \delta \), down to very small values of \( \theta_{13} \) [5, 6, 7]. For example, with two 40 Kton MINOS-like magnetised-iron detectors at two different baselines, exposed to beams of both polarities and \( 10^{21} \) muon decays, it will be possible to explore \( \theta_{13} \) down to \( \sin^2 2\theta_{13} \geq 1 \times 10^{-5} \left( \theta_{13} \geq 0.1^\circ \right) \) and to measure \( \delta \) for most of the parameter space [6]. The relatively high energy of the neutrinos produced through the decay of high-energy stored muons implies that baselines of several thousand kilometers are needed for Neutrino Factory experiments. For such baselines, CP asymmetries are dominated by matter effects [8, 9, 10] that can be used to determine unambiguously \( \text{sign} \left( \Delta m^2_{31} \right) \) for large enough \( \theta_{13} \).

The determination of \( (\theta_{13}, \delta) \) at the Neutrino Factory is not free of ambiguities. In order to solve them a single experimental measurement for a single neutrino beam is not enough. One possible solution to this problem is to combine detectors looking for ‘golden’ muons at different baselines (i.e., different \( L/E \)). A second possibility is to make use of the rich flavour content of the Neutrino Factory beam. The \( \tau \) appearance channel (‘silver channel’) [11, 12] has also been advocated as a powerful means of resolving ambiguities, if a detector capable of \( \tau \) identification can be used. This can readily be understood since the \( \delta \)-dependence of the silver and the golden channel are different, while the dependence of the two channels on matter effects and \( \theta_{13} \) is similar. On the other hand, the \( \nu_\mu \)-disappearance channel is rather effective for large values of \( \theta_{13} \) in measuring the \( \theta_{23} \) octant [13]. Last but not least one can reduce drastically the ambiguities by using an improved detector (with a much lower muon energy threshold) to look for ‘golden muons’ solving at the same time all the degeneracies.

The most important signal at the Neutrino Factory is the ‘golden channel’, i.e. the appearance channel \( \nu_e \rightarrow \nu_\mu \). The signal is tagged by ‘wrong-sign muons’, detected with charge opposite to that of those stores in the ring. In order to extract the signal from the dominant source of background, i.e. non-oscillated \( \bar{\nu}_e \) (giving rise in the detector to a huge number of ‘right-sign muons’), a magnetised detector is required. This requirement represents the most important difference between the detectors adopted for super-beam and beta-beam facilities and those needed to take full advantage of the Neutrino Factory. The reference detector for many studies, a 50 Kton magnetised iron calorimeter of the MINOS type, was optimised in reference [5] for the study of \( \nu_e \rightarrow \nu_\mu \) oscillations. Tight kinematic cuts were applied to decrease the dominant and sub-dominant backgrounds (right-sign muons and charmed-meson decays). Such cuts, although strongly reducing the background, have the dis-advantage that a significant proportion of the signal with neutrino energy below 10 GeV is removed. Measurements of the energy spectrum below 10 GeV, however, have been shown to be extremely important; the first oscillation peak for \( L \sim 3000 \) Km lies precisely in this energy range. For this reason, the physics of the Neutrino Factory is somewhat compromised by degeneracies. The measurement of the spectrum both below and above the oscillation maximum has been shown to be crucial in the solution of many
of the parametric degeneracies that distort the \((\theta_{13}, \delta)\)-measurement.

In the remaining of this paper I will only discuss the physics potential of the standard facility and golden-channel detector (see [2] for a much wider and deeper discussion).

3. Sensitivity to \(\theta_{13}\)

Figure 1 shows the \(\sin^2 2\theta_{13}\) sensitivity at 5\(\sigma\) as a function of the baseline \(L\) and the parent muon energy \(E_\mu\). The different panels correspond to taking into account, successively, statistical uncertainties, systematic uncertainties, correlations, and degeneracies. The different contours represent the region within a factor of 0.5, 1, 2, 5, and 10 above the maximum sensitivity in each plot. The maximum sensitivities are \(\sin^2 2\theta_{13} < 1.4 \cdot 10^{-5}\) (statistics), \(2.8 \cdot 10^{-5}\) (systematics), \(2.4 \cdot 10^{-4}\) (correlations), and \(5.0 \cdot 10^{-4}\) (degeneracies), obtained at the energies and baselines marked by the diamonds.

**Figure 1.** Sensitivity to \(\sin^2 2\theta_{13}\) (5\(\sigma\)) relative to the optimum (white) within each plot. The different panels correspond to successively taking into account statistics, systematics, correlations, and degeneracies. The different contours represent the region within a factor of 0.5, 1, 2, 5, and 10 above the maximal sensitivity in each plot. The maximal sensitivities are \(\sin^2 2\theta_{13} < 1.4 \cdot 10^{-5}\) (statistics), \(2.8 \cdot 10^{-5}\) (systematics), \(2.4 \cdot 10^{-4}\) (correlations), and \(5.0 \cdot 10^{-4}\) (degeneracies), obtained at the energies and baselines marked by the diamonds.
When statistical and systematic uncertainties only are considered (i.e., $\delta$ is fixed to the value for which we get maximum sensitivity), figure 1 (upper row), baselines from 1000 to 4000 km with as much muon energy as possible give the best sensitivity. However, when correlations and degeneracies are taken into account, the benefit of the ‘magic baseline’ becomes more apparent. At the magic baseline all dependence on $\delta$ cancels and many of the degeneracies disappear ‘by magic’, thus improving the $\sin^2 2\theta_{13}$ sensitivity. This happens for $V = \sqrt{2}G_FN_e = 2\pi/L$, or, in terms of the constant matter density $\rho$, for approximately two nucleons per electron, equivalent to:

$$L_{\text{magic}} [\text{km}] \simeq 32726 \frac{1}{\rho [\text{g/cm}^3]}.$$  \hspace{1cm} (1)

The magic baseline, however, has two obvious drawbacks: the event rate is reduced by the large distance; and it does not allow for a CP measurement. Also, the engineering issues associated to the tilt of the ring are difficult to solve.

4. CP-discovery potential

Figure 2 (left panel) shows the CP-discovery potential for the standard Neutrino Factory defined above for a baseline of $L = 4000$ km (the best compromise between sensitivity to CP violation and to matter effects, where the effect of degeneracies, however, is important). No CP-discovery potential has been evaluated for the Neutrino Factory and a baseline of 7000 km; due to matter effects and the choice of the baseline (close to the magic baseline), the sensitivity to $\delta$ vanishes. The Neutrino Factory with a baseline of 4000 km is not as good as one would expect from its $\theta_{13}$-sensitivity. This may be explained as follows: as a general rule, for small values of $\theta_{13}$ the degeneracies flow toward $\delta = 0^\circ$ and $|\delta| = 180^\circ$ (see references [16] and [15]), thus mimicking a non-CP violating phase.
Figure 3. 3σ sensitivity to the sign($\Delta m^2_{31}$) for normal ‘true’ mass hierarchy. Solid (dashed) lines refer to the $L = 4000$ km (7500 km) Neutrino Factory.

5. Sensitivity to the mass hierarchy
The $\nu_e \rightarrow \nu_\mu$ oscillation probability in matter depends on the sign of $\Delta m^2_{31}$. A change of this sign is equivalent to a CP transformation, that is, interchanging the probability of neutrinos and anti-neutrinos. Thus, matter effects themselves induce a non-vanishing CP-odd asymmetry. The maximum sensitivity to the sign of $\Delta m^2_{31}$, is expected at a baseline $O(7000)$ km. The asymmetries from different energy bins, however, peak at slightly different baselines. Therefore, spectral information can be used to improve the measurement of the sign of $\Delta m^2_{31}$.

The discovery potential for the normal ‘true’ mass hierarchy is shown at the 3σ confidence level in figure 3, evaluated for baselines of $L = 4000$ km and $L = 7500$ km. The sensitivity of the short and the long baselines are identical for $\delta \simeq -110^\circ$. For this particular parameter set it is also possible to lift the degeneracies at the short baseline. For all other values of $\delta$, the longer baseline has a better sensitivity.

6. Measurement of the atmospheric parameters
Except for any suppressed three-flavour effects, a Neutrino Factory will be useful for the precision measurement of the leading atmospheric parameters $\Delta m^2_{31}$ and $\theta_{23}$. For simplicity, the case in which the true $\sin^2 2\theta_{13} = 0$ is considered in this section. Also, only the normal hierarchy is shown, as an illustration.

The $\nu_\mu$ dis-appearence channel is extremely useful for the determination of the atmospheric-neutrino parameters $\Delta m^2_{31}$ and $\sin^2 \theta_{23}$. An impressive accuracy can be attained, even with the standard setup. However, a better precision can be achieved with a lower muon identification threshold. This can be achieved by loosening the kinematic cuts needed for a good muon charge identification. Figure 4 shows the relative precision on $\Delta m^2_{31}$ as a function of $L$ and $E_\mu$ (at 1σ CL for 1 degree of freedom), including all parameter correlations, for a normal ‘true’ mass hierarchy.
Figure 4. Relative precision on $\Delta m^2_{31}$ (at $1\sigma$) as a function of $L$ and $E_\mu$, including all parameter correlations for a normal mass hierarchy and $\sin^2 2\theta_{13} = 0$. The upper end (left panel) and lower end (right panel) of the allowed region are given separately because the $\Delta \chi^2$ is quite asymmetric. The minima, marked by the diamonds, occur at 0.14% (left panel) and 0.18% (right panel).

7. Sensitivity to maximal $\theta_{23}$ and the octant-discovery potential
A natural explanation for maximal mixing ($\theta_{23} = \pi/4$) might involve a new symmetry between $\nu_\mu$ and $\nu_\tau$. Therefore, the degree to which $\theta_{23}$ differs from $\pi/4$ is a powerful tool to discriminate between different neutrino-mass models [17].

Deviations as small as 10% of $\sin^2 \theta_{23}$ from maximal mixing could be established at the $L = 4000$ km baseline for certain values of $\Delta m^2_{31}$. A better sensitivity, however, may be obtained for $L = 7500$ km, since the energy and baseline match the first oscillation peak in matter.

8. The optimal Neutrino Factory
The optimization of the Neutrino Factory is a complicated business and I can only discuss a few points here.

- For the optimal baseline, CP-violation measurements favour a baseline around 4000 km (but baselines between 3000 km and 5000 km do not affect the sensitivity too much).
- As far as baseline upgrades are concerned, a degeneracy-solving baseline is necessary to improve the $\sin^2 2\theta_{13}$ sensitivity, the $\sin^2 2\theta_{13}$ discovery reach, and the mass-hierarchy discovery reach. A baseline in the range $L \sim 7000 - 7500$ km (i.e., the magic baseline) can play this role, since the appearance probability does not depend on $\delta$ at this distance and the intrinsic-degeneracy can be solved unambiguously independent of the oscillation parameters, possibly over-estimated luminosities, confidence level, etc. (see reference [6]). Furthermore, matter effects are stronger than for the shorter baseline, which means that the magic baseline is sensitive to different physics, rather than being simply a luminosity upgrade. Moreover, it helps CP-violation measurements at large $\sin^2 2\theta_{13}$, and can establish the MSW effect in the Earth even for $\sin^2 2\theta_{13} = 0$ [18]. Since this baseline is useful in all physics scenarios, one may want to choose a Neutrino Factory setup with two such baselines from the very beginning. The second baseline will be a major challenge from the engineering
Figure 5. Left panel: CP-fraction of the sensitivity to the mass hierarchy at 3\(\sigma\). The different shaded areas correspond to successively taking into account: 1) the magic baseline (yellow) and 2) an improved detector at \(E_\mu = 20\) GeV (green). Right panel: CP-discovery potential at 3\(\sigma\). The different lines correspond to successively taking into account additional optimisations as given in the legend. Solid (dashed) stands for a 5\% (2\%) matter density uncertainty. Shaded areas represent the improvement potential with respect to the unknown matter density profile. Notice that in going from Golden to Golden* the muon energy goes down from \(E_\mu = 50\) GeV to \(E_\mu = 20\) GeV.

point of view. However, the physics potential of this baseline is well established and the technical feasibility should be rather predictable.

- For detector upgrades, an improvement of the golden-channel detector is certainly the main objective. In particular, lowering the detection threshold will greatly improve the physics potential in all physics scenarios and for both the mass-hierarchy and the CP-violating-phase measurements. It has been demonstrated that an improved detector would allow the use of a lower parent-muon energy, \(E_\mu \sim 20\) GeV instead of \(E_\mu \sim 50\) GeV, thus reducing the effort on the accelerator side. The improvement of the detector with respect to energy resolution and threshold should be possible.

- An improvement of the detection threshold could reduce the muon energy to 20 GeV while achieving excellent physics sensitivities, and the physics scenario ‘large \(\sin^2 2\theta_{13}\)’ may even allow for lower energies. Note that the use of the silver channel disfavours low muon energies, i.e., \(E_\mu\) should be \(\sim 25\) GeV or greater.

The left panel of figure 5 summarises the outcome of this optimisation discussion by presenting the CP-fraction for the sensitivity to the mass hierarchy, successively switching on the magic baseline and the golden* improved detector. One can easily read off the excellent combined potential for mass hierarchy and CP-violation of the Neutrino Factory below \(\sin^2 2\theta_{13} < 10^{-2}\). It is important to remark that none of the suggested improvements could be achieved with a simple luminosity upgrade, i.e., adding mass to the golden-channel detector.
In conclusion, the optimal Neutrino Factory setup for oscillation parameter measurements has two baselines (at $L \sim 1500 - 4000$ km and one at $L \sim 7500$ km, respectively), a ‘better’ golden channel detector (with lower threshold and higher energy resolution) and a muon energy of $E_\mu \sim 25$ GeV. This set of improvements exhausts the optimisation potential in most of the parameter space.

As far as future Neutrino Factory R&D is concerned, the ability to operate two baselines as well as the lower detection threshold of the golden detector are the most critical components to the optimised physics potential. Furthermore, a better energy resolution of the golden-channel detector would improve the physics potential further.

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References