Supernova theory: simulation and neutrino fluxes

K G Budge[1], C L Fryer and A L Hungerford

CCS-2, Los Alamos National Laboratory[2]
M.S. D409, Los Alamos National Laboratory, Los Alamos, New Mexico, 87545, USA

E-mail: kgbudge@lanl.gov

Abstract. It is now generally accepted that Type II supernovae arise from the collapse of a massive stellar core that has reached the end point of silicon burning. The observation of a neutrino pulse from Supernova 1987A strongly supports the core collapse theory. However, 1-D simulations of core collapse fail to explode unless *ad hoc* mechanisms are invoked. Most recent multidimensional simulations explode without *ad hoc* mechanisms, but many simulations do not yet produce the high energies observed in SNe. We review the state of multidimensional core collapse simulations, with particular emphasis on neutrino transport algorithms and results. What are the predicted neutrino fluxes, and how can observations constrain theory and provide insights to computational physicists?

1. Introduction

Type II supernovae are extraordinarily bright stellar outbursts observed in star-forming regions of galaxies. It is generally believed that they result from the collapse of the iron core of a young, massive star at the completion of silicon burning. The gravitational energy released produces a powerful shock that ejects the stellar envelope, producing the optical signature and leaving behind a compact remnant (a neutron star or possibly a black hole). However, observations of the neutrino burst from Supernova 1987A confirm that the bulk of the gravitationally energy is released in the form of neutrinos.

Although the core collapse mechanism was proposed by Baade and Zwicky in 1938 [1], it was not until the 1960s that Colgate performed the first computational simulations of dynamic core collapse. These showed that the core collapse was reversed at nuclear density by the strong force, producing a "bounce" that drove a powerful shock into the surrounding mantle. However, this shock stalled and produced no explosion, mainly because dissociation of iron nuclei in the shock absorbed most of its energy. Colgate and White [2] proposed that heating by neutrinos emitted from the core and absorbed in the mantle was necessary to revive the shock and produce an explosion.

Neutrino heating remains important in computational simulations of core collapse supernovae, but this mechanism has proven inadequate to produce explosions in one-dimensional simulations. This has led

---

1. To whom any correspondence should be addressed.
2. Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.

researchers to investigate whether multidimensional effects are required for shock breakout following core collapse [3][4][5][6][7]. At present, the weight of evidence, both theoretical and observational, is that asymmetric effects are quite important. However, the jury is still out on whether simulations incorporating multidimensional hydrodynamics, neutrino transport, and standard neutrino physics reliably produce explosions.

It is not clear that the explosion mechanism needs to be robust. The observed frequency of Type II supernovae is roughly consistent with the estimated rate of formation of progenitor stars, but this rate is quite poorly constrained and it is possible that a large fraction of core collapses are "duds" or weak SNe. Nevertheless, the fraction of progenitor stars that actually explode must be significant.

1.1. Evidence for Asymmetry and Mixing

There is considerable observational evidence for strong asymmetries in Type II supernova. Many supernova remnants show distinct asymmetry, including non-uniform distributions of chemical elements (such as Cas A [8]) that suggest the asymmetry originates deep in the core of the supernova. In addition, most neutron stars show high space velocities, some in excess of 1000 km/s (such as the Guitar Pulsar [9].) These are far in excess of orbital velocities and show that the proto-neutron star received a strong asymmetric kick during the supernova explosion.

r-process nucleosynthesis is difficult to explain with symmetric supernova models. To produced the observed abundances of r-process nuclides, a significant amount of neutron-rich matter must follow tightly constrained trajectories in $\rho - T$ space. Symmetric models rarely produce these trajectories. Turbulent explosions produce a broader set of trajectories, with an increased likelihood that some of these will experience the necessary $\rho - T$ history [10].

However, the amount of turbulence is constrained by the modest abundance of tin in the universe, which demonstrates that only limited amounts of highly neutron-rich material are being dredged up during the explosion.

2. Simulations

Increases in computing power have made it possible to simulate the hydrodynamics of core collapse in two or three dimensions at useful resolutions. However, the additional degrees of freedom in the neutrino transport equations mean that two-dimensional transport calculations are just within the capability of the current generation of massively parallel computers, while the "computing iron" needed for three-dimensional transport calculations is unavailable even at the U.S. national laboratories. One must either restrict oneself to two-dimensional simulations, or adopt some severe approximations to the neutrino transport.

2.1. Transport Approximations

The neutrinos in a transport simulation are described by the invariant phase space distribution f, which is generally a function of position, time, ordinate $\hat{n}$, and energy, a total of seven independent variables. This dependence on seven variables makes the full Boltzmann equation

$$\frac{1}{c}\frac{\partial f}{\partial t} + n \bullet \nabla f + \sigma_t f = \eta \tag{1}$$

computationally challenging to solve, and most transport approximations focus on simplifying the energy or ordinate dependence.

2.1.1. Flux-limited diffusion. Many supernova calculations have used flux-limited diffusion because of its simplicity and robustness. The distribution is approximated as $f \approx J + 3\hat{n} \bullet \vec{F}$, yielding a closed pair of moment equations in the mean intensity J and flux $\vec{F}$:

$$\frac{1}{c}\frac{\partial J}{\partial t} + \nabla \bullet \vec{F} + \sigma_t J = \eta \tag{2}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\vec{F} + \frac{1}{3}\nabla J + \sigma_t \vec{F} = 0 \tag{3}$$

This is known as the P1 approximation in the transport community. Asymptotic analysis suggests dropping the explicit time derivative appearing in the flux equation, which allows the flux to be substituted into the mean intensity equation to yield a single diffusion equation [11]:

$$\frac{1}{c}\frac{\partial J}{\partial t} - \nabla \frac{1}{3\sigma_t}\nabla J + \sigma_t J = \eta \tag{4}$$

The diffusion approximation is quite accurate at large optical depth, where the radiation field is nearly isotropic. Under these conditions, one can usually make the approximation of gray diffusion as well, but this is less useful for neutrinos than photons due to their nonzero chemical potential.

At small optical depth, the diffusion approximation breaks down completely, but for a single compact source the streaming approximation $\vec{F} = -J(\nabla J/|\nabla J|)$ becomes applicable. Flux limiters are *ad hoc* interpolations between these two extreme transport regimes [12]. They may be expected to work fairly well when the transition zone is thin, as is the case with the neutrinosphere of a proto-neutron star.

2.1.2. The Pn approximation. One can expand the ordinate dependence of the radiation field in spherical harmonics and use their orthogonallity properties to derive a closed set of equations. This is known as the Pn approximation. However, the Pn equations are very complicated and difficult to solve in more than one dimension, and they are prone to "ringing" [13]. They have not been used for supernova simulations.

2.1.3. Sn Approximation. This is a brute-force approach to transport, in which one bites the bullet and accepts the necessity of solving the full Boltzmann equation on a discrete set of ordinate directions [14]. The method is relatively simple to formulate. However, while two-dimensional calculations with this method are within the reach of current computers, three-dimensional calculations are not.

To illustrate the scope of the problem, consider a rather coarsely resolved supernova core, with 100x100x100 or 10^6 cells. A sufficiently robust spatial discretization require eight values in each cell for each energy group, ordinate direction, and neutrino flavor. Assuming 40 energy groups and S8 discretization, with 80 ordinate directions, one has nearly 20 billion degrees of freeedom in the radiation field. These calculated a number of times at each time step to converge the scattering integral, and tens of thousands of time steps would be required. This is too heroic a calculation for today's computing hardware.

2.2. Results

A number of core-collapse simulations have been performed in 1-D using sophisticated transport methods. However, these have not reliably produced explosions. On the other hand, three-dimensional calculations using flux-limited diffusion have sometimes been successful in producing explosions.

Multidimensional hydrodynamics plus sectorial transport has been used in core collapse simulations, and this is arguably the state of the art for coupled simulations. The hydrodynamics is fully two- or three-dimensional, but the transport is solved as a set of 1-D problems on sectors along rays from the center of the protostar. One can use information from the previous time step to account for some of the coupling of radiation between sectors.

The state of the art for diagnostics is Monte Carlo postprocessing of a simulation performed using a less expensive transport approximation. Monte Carlo itself is simple, robust, capable of handling elaborate physics, and hideously expensive, which likely rules out its use in coupled calculations.

2.3. Do we need better radiation transport?

It is our opinion that poor approximations of radiation transport are probably not responsible for the failure of core-collapse simulations to reproduce Nature by producing successful explosions.
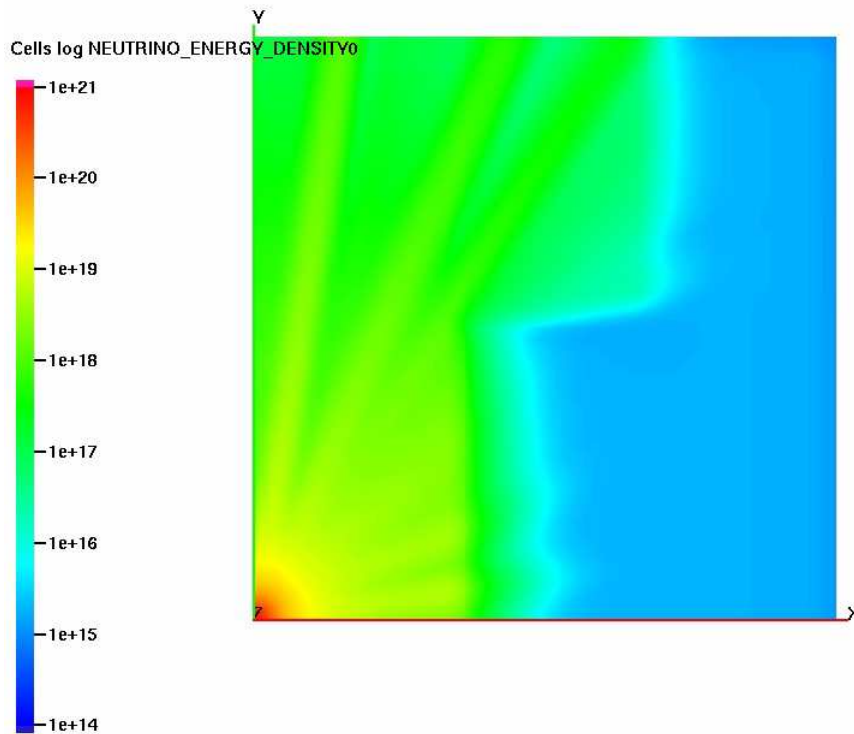

Figure 1. Compact source and stepped boundary, S16 approximation

Figure 1 shows a test problem with a compact source at the origin, a transparent region at the left, and an opaque region to the right. The boundary between the two regions is stepped to produce a sharp corner. The radiation field was calculated using the S16 approximation (36 ordinate directions per octant) and shows obvious ray effects. Figure 2 is the same problem where the radiation field was calculated using flux-limited diffusion. With this approximation, the most obvious problem is that the radiation "turns the corner" on the stepped boundary. The radiation field is also too intense close to the source. However, it is not clear that S16, with its ray effects, produces a more acceptable solution than flux-limited diffusion.

Figure 3 shows the spectrum from a supernova simulation postprocessed with different transport approximations. The S12 spectrum closely matches the Monte Carlo spectrum in spite of noticeable ray effects in the original calculation, demonstrating that integrated properties tend to wash out these effects. The flux-limited diffusion calculation shows about a 20% difference from the Sn calculation. However, this is within the bounds of other uncertainties in the simulation.

3. Conclusions

It is premature to conclude that the failure of most core-collapse simulations to produce an explosion points to missing neutrino physics. The simulation community has only recently begun computing three-dimensional core collapse simulations, and some of these have produced explosions. These computational experiments, together with observational data, suggest that asymmetry and mixing are essential to the Type II supernova process and may account for the failure of one-dimensional simulations to produce explosions.

Given the limits on resources, the reasonable accuracy of flux-limited diffusion, and the sometimes underappreciated liabilities with more sophisiticated transport apporoximations (e.g. ray effects in Sn), we
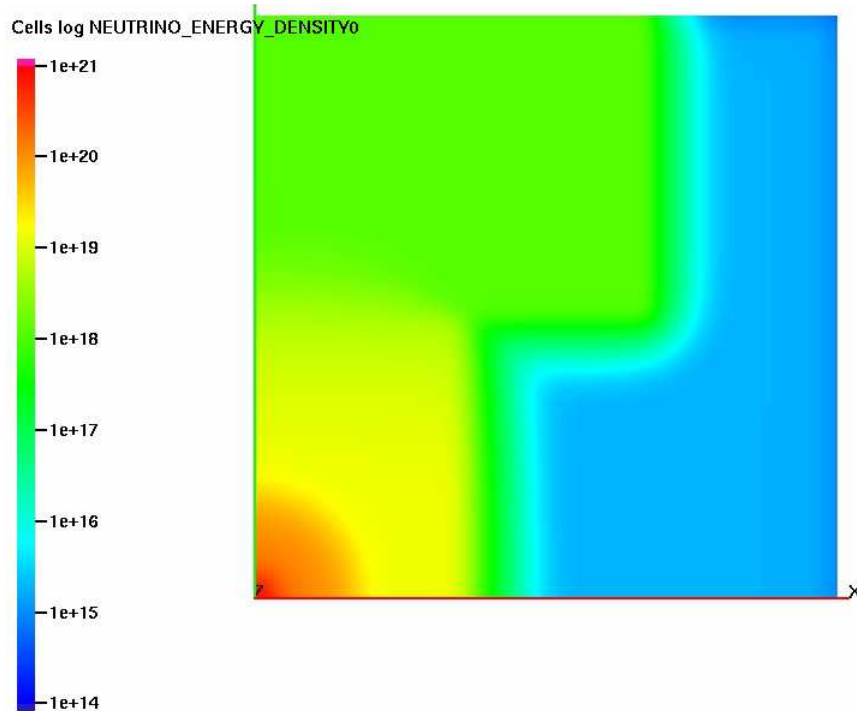
Figure 2. Compact source and stepped boundary, flux-limited diffusion

are skeptical that improved transport is a good investment for the core collapse simulation community at this time.

Supernova 1987A confirmed the general core-collapse scenario, but small-number statistics mean few serious constraints on models. A Type II supernova in the Milky Way would give us excellent statistics (albeit of one event) that could better constrain models. There is roughly a 20% chance of this happening during our careers.

[1] Baade W and Zwicky F 1938 *Ap. J.* **88** 411
[2] Colgate S A and White R H 1966 *Ap. J.* **143** 626
[3] Herant M, Benz W, Hix W R, Fryer C L and Colgate S A 1994 *Ap. J* **435** 339
[4] Blondin J M, Mezzacappa A and DeMarino C 2003 *Ap. J* **584** 971
[5] Burrows A, Livne E, Dessart L, Ott C D and Murphy J 2007 *Ap. J.* **655** 416
[6] Fryer C L and Young P A 2007 *Ap. J.* **659** 1438
[7] Scheck L, Janka H, Foglizzo T and Kifonidis K *2008*, *Astron. Ap.* **477** 931
[8] Hwang U et al. 2004 *Ap. J.* **615** L117
[9] Cordes J M, Romani R W and Lundgren S C 1993 *Nature* **362** 133
[10] Fryer C L, Herwig F, Hungerford A and Timees F X 2006 *Ap. J.* **646** L131
[11] Larson E W and Morel J E 1989 *J. Comput. Phys.* **83** 212
[12] Levermore C D and Pomraning G C 1981 *Ap. J.* **248** 321
[13] Lewis E E and Miller W F Jr 1993 *Computational Methods of Neutron Transport* (La Grange Park, IL: American Nuclear Society)
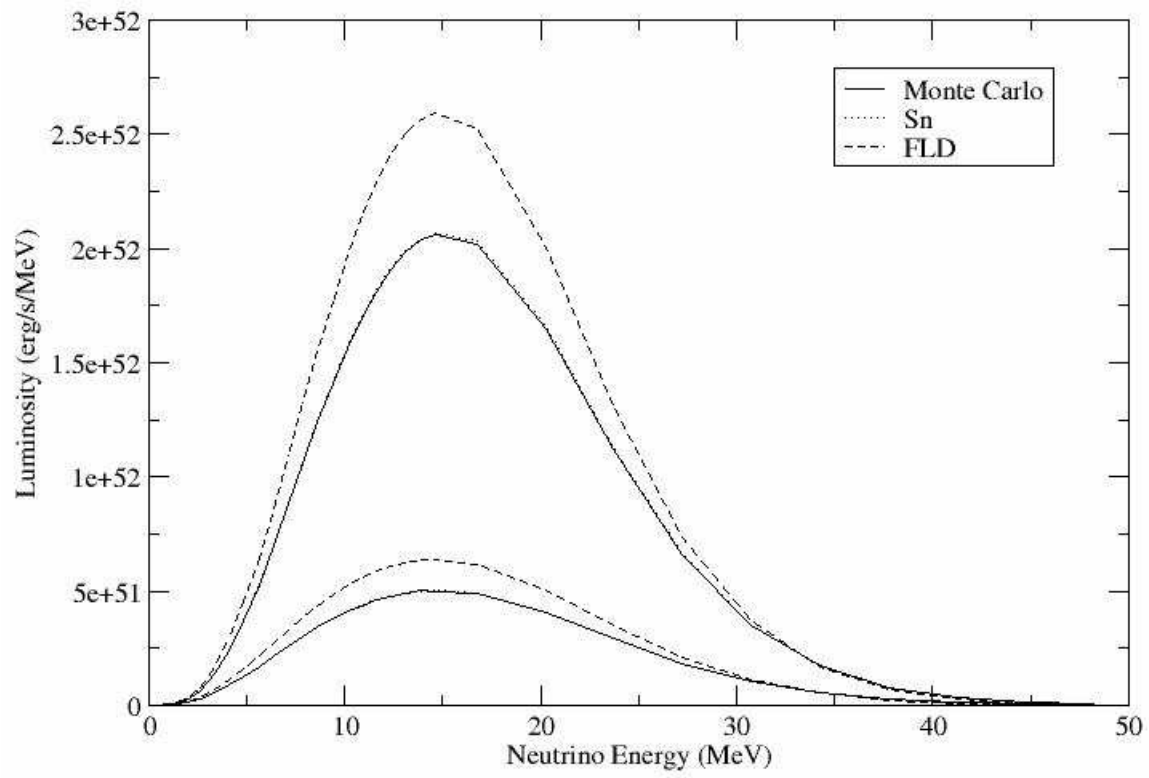[14] Brunner T A and Holloway J P 2005 *J. Comput. Phys.* **210** 386

Figure 3. Comparison of supernova spectra for various transport approximations