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Neutrino energy quantization in rotating media: new mechanism for neutrino trapping inside dense rotating stars

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1. Introduction: "method of exact solutions"

In a series of our papers, starting from [1] (see also [2,3] and references therein) we have developed quite a powerful method in investigations of different phenomena that can appear when neutrinos are moving in the background matter. In addition, we also have demonstrated how the developed method can be applied to electrons moving in matter [2-5] . The developed approach establishes a basis for investigation of different phenomena arising when neutrinos and electrons move in dense media, including those peculiar for astrophysical and cosmological environments.

The method discussed is based on use of exact solutions of the modified Dirac equations for particles wave functions, in which the correspondent effective potentials accounting for matter influence on particles are included. It is similar to the Furry [6] representation in quantum electrodynamics, widely used for description of particles interactions in the presence of external electromagnetic fields. In this technique, for instance, the evolution operator $U_F(t_1,t_2)$ which determines the matrix element of the synchrotron radiation process, is represented in the usual form

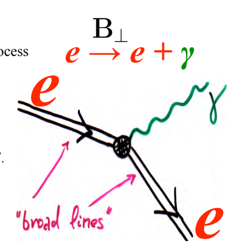
$$U_F(t_1,t_2) = Texp\Big[-i\int_{t_1}^{t_2} j^{\mu}(x)A^q_{\mu}(x)dx\Big] \quad , \qquad (1)$$

where $A^q_{\mu}(x)$ is the quantized part of the potential $A_{\mu}(x) = A^q_{\mu}(x) + A^{ext}_{\mu}(x)$ corresponding to the radiation field, which is accounted within the perturbation-series techniques. At the same time, the electron current is represented in the form

$$j_{\mu}(x) = \frac{e}{2}\big[\overline{\Psi}_e\gamma_{\mu}, \Psi_e\big], \qquad (2)$$

where Ψ_e are the exact solutions of the Dirac equation for the electron in the presence of external magnetic field given by the classical non-quantized potential $A^{ext}_{\mu}(x)$,

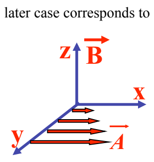
$$\Big\{\gamma^{\mu}\Big(i\partial_{\mu} - eA^{ext}_{\mu}(x)\Big) - m_e\Big\}\Psi_e(x) = 0 \quad . \qquad (3)$$



In the correspondent Feynman diagramme of the synchrotron radiation process the electron initial and final states are displayed by "broad lines" that indicates use of the Dirac equation exact solutions in the presence of magnetic field. Note that the discussed approach based on Furry representation is often termed in the literature "method of exact solutions".

Here below within the "method of exact solutions" we consider the problem of a neutrino propagation in rotating dense medium and, in particular, show that neutrino energy spectrum is quantized [3,7] very much like an electron spectrum is quantized in the case of the particle motion in a constant magnetic field. The explicit form of exact solutions of the Dirac equation for an electron in a constant magnetic field can be found in [8] . For the purpose of the subsequent discussion of the existed analogy between a neutrino motion in a rotating medium and an electron motion in a constant magnetic field we remind that the electron energy spectrum in the later case corresponds to a set of Landau levels

$$p_0^{(e)} = \sqrt{m_e^2 + p_3^2 + 2\gamma N}, \quad \gamma = eB, \quad N = 0,1,2,\dots, \qquad (4)$$



where $p_{\perp}^{(e)} = \sqrt{2\gamma N}$ is the electron transversal in respect to the magnetic field quantized momentum component (the vector potential is indicated by $\vec{A}$ in the figure attached here). In Section 5 we introduce [3] an effective "matter-induced Lorentz force" acting on a neutrino (and also on an electron) that can be done within the quasiclassical approach to description of particles propagation in media. We give the quasiclassical interpretation of the neutrino circular orbits in a rotating medium and also discuss possible new mechanism of electromagnetic radiation emitted by an electron moving in the inhomogeneous neutrino background.

2. Modified Dirac equation for neutrino in matter

In [1] we derived the modified Dirac equation for a Dirac neutrino wave function exactly accounting for the neutrino interaction with the background matter composed of electrons, neutrons and protons, in the most general form,

$$\Big\{i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5)f^{\mu} - m\Big\}\Psi(x) = 0\,, \qquad (5)$$

where for unpolarized media

$$f^{\mu} = \frac{G_F}{\sqrt{2}}\Big\{[\rho + 4\sin^2\theta_W]n_e(1,\mathbf{v}_e) + [1 - 4\sin^2\theta_W]n_p(1,\mathbf{v}_p) - n_n(1,\mathbf{v}_n)\Big\}, \qquad (6)$$

$\rho = 1$ for the electron neutrino and $\rho = 0$ for the muon and tau neutrinos, θ_W is the Weinberg angle, n_f and $\mathbf{v}_f$ are the initial number density and average speeds of matter components $(f = e,n,p)$. Other modifications of the Dirac equation were previously also used [9] for studies of the neutrino dispersion relations, neutrino mass generation and neutrino oscillations in the presence of matter. It should be mentioned here that different Majorana neutrino interactions with hypothetical scalar majorons (like the process of a neutrino decay into majoron, $\nu \to \tilde{\nu} + \chi$ the presence of matter were considered in [10] . Note that the corresponding quantum wave equation for a Majorana neutrino can be obtained from (5) via the $1+\gamma_5 \to 2\gamma_5$. (see, for instance, [11]).

The exact solutions of the modified Dirac equation (5) in nonmoving matter of constant density were used [1] for evaluation of the quantum theory of the spin light of neutrino (***SLv***) in matter, a new type of electromagnetic radiation that can be emitted by Dirac neutrino in the background matter. The ***SLv*** is produced by the neutrino magnetic moment when the particle moves in matter. Within the quasiclassical treatment the existence of the ***SLv*** was first proposed and then studied in [12] . Recently, on the basis of the exact solutions of the correspondent Dirac equation in matter for an electron we also predicted the existence of the spin light of electron (***SLe***) [2-5] . The ***SLe*** in matter also originates from the particle magnetic moment procession in matter. It should be stressed that ***SLv*** and ***SLe*** in matter are really new mechanisms of electromagnetic radiation of quite a different nature than ones considered before including the Cherenkov radiation of particles in medium. For the rate of ***SLv*** in the case of ultra-relativistic neutrinos ($p \gg m$) we obtained [1]

$$\Gamma_{SL\nu} = 4\mu^2\alpha^2 m_{\nu}^2 p, \qquad m_{\nu}/p \ll \alpha \ll p/m_{\nu}, \qquad (7)$$

where the matter density parameter $\alpha = f^0/(2m_{\nu})$. For the rate of ***SLe*** in neutron matter we have found [4]

$$\Gamma_{SLe} = e^2m_e^2/(2p)\,\big[\ln\big(4\alpha_n p/m_e\big) - 3/2\big]\,, \qquad m_e/p \ll \alpha_n \ll p/m_e, \qquad (8)$$

where it is supposed that $\ln\frac{4\alpha_n p}{m_e} \gg 1$. A detailed discussion on the ***SLv*** and ***SLe*** properties can be found in [1-5,11] . The figure attached below contains the correspondent Feynman diagramme and display the key features of the discussed spin light radiation mechanisms. Note that a reasonable part of the particles initial energies can be radiated by this mechanism, therefore the ***SLv*** and ***SLe*** can be important in case when ultra-relativistic particles propagate in different astrophysical and cosmological environments of high density.

$SL\nu$ $\quad$ SLe

Spin light of neutrino and electron in matter originate from two subdivided phenomena:

- **shift** of particle's **energy levels** in presence of background matter, which is different for two opposite **helicity states**

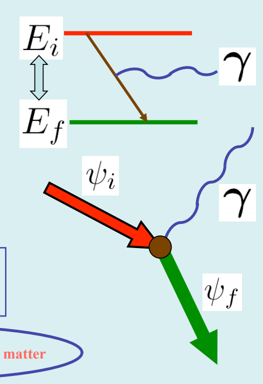
$$E^{(l)} = \sqrt{\mathbf{p}^2\Big(1 - s\alpha^{(l)}\frac{m_l}{p}\Big)^2 + m_l{}^2} + c_l\alpha^{(l)}m_l$$

$$\alpha^{(l)} = \frac{1}{2\sqrt{2}}G_F\frac{n^{(l)}}{m_l} \qquad c_l = \begin{cases} 1, & l = \nu_e \\ 1 - 4\sin^2\theta_W, & l = e \end{cases}$$

$$n^{(l)} = \begin{cases} n_e, & l = \nu_e \\ n_n, & l = e \end{cases}$$

- radiation of **photon** in process of particle transition from **"excited" helicity state** to the **"low-lying" helicity state** in matter

spin self-polarizatin effect in matter



3. Neutrino quantum states in rotating medium

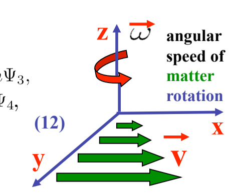
We apply the "method of exact solutions" to the case when a neutrino is propagating in a rotating medium of constant density [7,3] . Suppose that a neutrino is propagating in uniformly rotating medium of constant density composed of neutrons. This can be considered for modeling of neutrino propagation inside a rotating neutron star. The corresponding matter potential in the modified Dirac equation (12) in this case is

$$f^{\mu} = -\,G\,n\,(1\,,\mathbf{V}\,), \quad \mathbf{V} = (\omega y, 0, 0\,), \quad G = \frac{G_F}{\sqrt{2}}. \qquad (10)$$

Here ω is the angular frequency of matter rotation around the OZ axis.

For the neutrino wave function components we get from (5) a set of equations (the chiral representation for Dirac matrixes is used)

$$\begin{aligned}
&\big[i\,(\partial_0 - \partial_3) + Gn\big]\Psi_1 + \big[-(i\partial_1 + \partial_2) + Gn\omega y\big]\Psi_2 = m\Psi_3, \\
&\big[(-i\partial_1 + \partial_2) + Gn\omega y\big]\Psi_1 + \big[i\,(\partial_0 + \partial_3) + Gn\big]\Psi_2 = m\Psi_4, \\
&i\,(\partial_0 + \partial_3)\,\Psi_3 + (i\partial_1 + \partial_2)\,\Psi_4 = m\Psi_1, \\
&(i\partial_1 - \partial_2)\,\Psi_3 + i\,(\partial_0 - \partial_3)\,\Psi_4 = m\Psi_2.
\end{aligned} \qquad (12)$$



For the relativistic neutrino energies this set of equations can be solved exactly. In this case the mass terms can be ignored in the right-hand sides of (12) with respect to the kinetic and interaction terms in the left-hand sides of these equations. Thus, two pairs of the neutrino wave functions components decouple from each other and four equations (12) disintegrate to two independent sets of two equations, that couple together the neutrino wave function components in pairs, $(\Psi_3,\ \Psi_4)$ and $(\Psi_1,\ \Psi_2)$.

The second pair of equations (12) does not contain the matter term and is attributed to the sterile right-handed chiral neutrino Ψ_R . The correspondent solution can be taken in the plain-wave form,

$$\Psi_R \sim L^{-\frac{3}{2}}\exp\{i(-p_0t + p_1x + p_2y + p_3z)\}\psi, \qquad (12)$$

where p_{μ} is the neutrino momentum. Then for the components Ψ_3 and Ψ_4 we obtain the following equations

$$\begin{aligned}
(p_0 - p_3)\,\Psi_3 - (p_1 - ip_2)\,\Psi_4 &= 0, \\
-\,(p_1 + ip_2)\,\Psi_3 + (p_0 + p_3)\,\Psi_4 &= 0.
\end{aligned} \qquad (13)$$

Finally, for the sterile right-handed neutrino we get

$$\Psi_R = \frac{\mathrm{e}^{-ipx}}{L^{3/2}\sqrt{2p_0(p_0 - p_3)}}\begin{pmatrix} 0 \\ 0 \\ -p_1 + ip_2 \\ p_3 - p_0 \end{pmatrix}, \qquad (14)$$

where $px = p_{\mu}x^{\mu},\ p_{\mu} = (p_0,p_1,p_2,p_3)$ and $x_{\mu} = (t,x,y,z)$. This solution, as it should be, has the vacuum dispersion relation.

In the relativistic neutrino energy limit the first pair of equations (12) corresponds to the active left-handed neutrino Ψ_L. The form of these equations is similar to the correspondent equations for an electron moving in a constant magnetic field $\vec{B}$ given by the potential $\vec{A} = (\,By, 0, 0\,)$ (see, for instance, [8]). Thus, there is an analogy between neutrino dynamics in rotating medium and electron dynamics in constant magnetic field. To display this analogy, we note that in our case components of the matter current $n\vec{\mathbf{V}}$ play the role of the vector potential, $\vec{A} = n\,(\,\omega y, 0, 0\,)$. The solution of the first pair of equations (12) can be taken in the form

$$\Psi_L \sim \frac{1}{L}\exp\{i(-p_0t + p_1x + p_3z)\}\psi(y), \qquad (15)$$

and for the components Ψ_1 and Ψ_2 of the neutrino wave function we obtain

$$\begin{aligned}
\Big(p_0 + p_3 + Gn\Big)\Psi_1 - \sqrt{\rho}\,\Big(\tfrac{\partial}{\partial\eta} - \eta\Big)\,\Psi_2 &= 0, \\
\sqrt{\rho}\,\Big(\tfrac{\partial}{\partial\eta} + \eta\Big)\,\Psi_1 + \Big(p_0 - p_3 + Gn\Big)\Psi_2 &= 0,
\end{aligned} \qquad (16)$$

where $\eta = \sqrt{\rho}\Big(x_2 + \dfrac{p_1}{\rho}\Big)$, $\quad \rho = Gn\omega$. For the wave function we finally get

$$\Psi_L = \frac{\rho^{\frac{1}{4}}\mathrm{e}^{-ip_0t + ip_1x + ip_3z}}{L\sqrt{(p_0 - p_3 + Gn)^2 + 2\rho N}}\begin{pmatrix} (p_0 - p_3 + Gn)\,u_N(\eta) \\ -\sqrt{2\rho N}u_{N-1}(\eta) \\ 0 \\ 0 \end{pmatrix}, \qquad (17)$$

where $u_N(\eta)$ are Hermite functions of order N. For the active left-handed neutrino energy we get

$$p_0 = \sqrt{p_3^2 + 2\rho N} - Gn, \quad N = 0,1,2,\dots\ . \qquad (18)$$

The energy depends on the neutrino momentum component p_3 along the rotation axis of matter and the quantum number N that determines the magnitude of the neutrino momentum in the orthogonal plane. For description of antineutrinos one has to consider the 'negative sign" energy eigenvalues. Therefore, the energy of an electron antineutrino in the rotating matter composed of neutrons is given by

$$\tilde{p}_0 = \sqrt{p_3^2 + 2\rho N} + Gn, \quad N = 0,1,2,\dots\ . \qquad (19)$$

Obviously, the performed consideration can be easily generalized for different other neutrino flavours and matter composition.

4. Radius of neutrino orbits in matter

Thus, it is shown [7,3] that the transverse motion of an active neutrino and antineutrino is quantized in moving matter very much like an electron energy is quantized (Landau energy levels) in a constant magnetic field. Consider again antineutrino. The transversal momentum is given by

$$\tilde{p}_{\perp} = \sqrt{2\rho N}. \qquad (20)$$

The quantum number N determines also the radius of the antineutrino quasi-classical orbit in matter (it is supposed that $N \gg 1$ and $p_3 = 0$)

$$R = \sqrt{\frac{2N}{Gn\omega}}. \qquad (21)$$

Thus, due to the energy quantization antineutrinos can have bound orbits inside a rotating star. As it follows from (21) the radius R depends on the neutrino transversal momentum. Let us estimate the energy scale of neutrinos that can be bound inside the star. Consider a model of a rotating star with radius $R_{NS} = 10\ km$, matter density $n = 10^{37}cm^{-3}$ and angular rotation frequency $\omega = 2\pi\times10^3\ s^{-1}$. For this set of parameters, the radius of orbits is less than the typical star radius R_{NS} if the quantum number $N \le 10^{10}$. Therefore, antineutrinos that occupy orbits with $N \le N_{max} = 10^{10}$n be bound inside the star. The scale of the bound antineutrino energy is given by (20) and is of the order $\tilde{p}_0 \sim 1\ eV$should be underlined that within the quasiclassical approach the neutrino binding on circular orbits is due to an effective force that is orthogonal to the particle speed. Note that there is another mechanism of neutrino binding inside a neutron star when the effect is produced by a gradient of the matter density [13] .

5. Conclusion: "matter-induced Lorentz force"

We can further develop the established above analogy between neutrino motion in a rotating matter and electron motion in a magnetic field. It is possible to explain the neutrino quasiclassical circular orbits as a result of action on the neutrino of the attractive central force,

$$\mathbf{F}_m^{(\nu)} = q_m^{(\nu)}\boldsymbol{\beta}\times\mathbf{B}_m, \ \ \mathbf{B}_m = \boldsymbol{\nabla}\times\mathbf{A}_m, \ \ \mathbf{A}_m = n\mathbf{v}, \qquad (22)$$

where the effective neutrino "charge" in matter (composed of neutrons in the discussed case) is $q_m^{(\nu)} = -G$, whereas $\mathbf{B}_m$ and $\mathbf{A}_m$ play the roles of effective "magnetic" field and the correspondent "vector potential". Like the magnetic part of the Lorentz force, $\mathbf{F}_m^{(\nu)}$, is orthogonal to the neutrino speed $\boldsymbol{\beta}$.

It is possible to generalize the above-discussed description of the matter effect on neutrinos for the case when the matter density n is not constant. For the most general case the "matter-induced Lorentz force" is given by

$$\mathbf{F}_m^{(\nu)} = q_m^{(\nu)}\mathbf{E}_m + q_m^{(\nu)}\boldsymbol{\beta}\times\mathbf{B}_m, \qquad (23)$$

where the effective "electric" and "magnetic" fields are, respectively,

$$\mathbf{E}_m = -\boldsymbol{\nabla}n - \mathbf{v}\frac{\partial n}{\partial t} - n\frac{\partial\mathbf{v}}{\partial t}, \qquad (24)$$

and

$$\mathbf{B}_m = n\boldsymbol{\nabla}\times\mathbf{v} - \mathbf{v}\times\boldsymbol{\nabla}n. \qquad (25)$$

These expressions can be generalized for the background composed of different matter species. Note that the force acting on a neutrino, produced by the first term of the effective "electric" field in the neutron matter, was considered in [13] . Similar to the developed above quasiclassical treatment of a neutrino motion in the electron plasma was considered in [14] .

To conclude, we argue that it is also possible to introduce the "matter-induced Lorentz force" for an electron moving in the background matter. The (electro)weak forces acting on a neutrino and electron in matter are identical. Therefore, similar to the case of neutrino, we can write for the force acting on an electron $\mathbf{F}_m^{(e)}$ in background matter

$$\mathbf{F}_m^{(e)} = q_m^{(e)}\mathbf{E}_m + q_m^{(e)}\boldsymbol{\beta}\times\mathbf{B}_m, \qquad (26)$$

where an appropriate magnitude for the effective electron "charge" in matter $q_m^{(e)}$ should be used. As it follows from (24) and (26) , an accelerating force acts on an electron when it moves in matter with nonvanishing gradient of density. Using this observation, we should like to discuss existence of an electromagnetic radiation mechanism of an electron moving in the neutrino background $(m = \nu)$onzero gradient of its density. This situation can be realized in different astrophysical and cosmology settings. For instance, this phenomenon can exist when an electron propagates in the radial direction from a compact object inside a dense environment composed predominantly of neutrinos moving in the radial direction after they were emitted from a central part of the object. In this case, the total power of the radiation (in quasiclassical limit) is given by

$$I = \frac{2}{3}q_{\nu}^{(e)}\Big[\frac{\mathbf{a}^2}{(1-\beta^2)^2} + \frac{(\mathbf{a}\boldsymbol{\beta})^2}{(1-\beta^2)^3}\Big], \qquad (27)$$

where $\boldsymbol{\beta}$ is the electron speed and $\mathbf{a}$ is the electron acceleration induced by the gradient of the neutrino background density. We expect that the proposed mechanism of the electromagnetic radiation can be important in other astrophysical settings like one that can be realized in neutron stars, gamma-ray bursts and black holes.

The developed quasiclassical approach to description of the matter effect, driven by (electro)weak forces, is valid as long as interaction of particles with the background is coherent. This condition is satisfied when a macroscopic amount of the background particles are confined within the scale of a neutrino or electron de Broglie wavelength. For the relativistic particles the following condition should be satisfied $\frac{n}{\gamma_l m_l^3} \gg 1$, where n is the number density of matter, $\gamma_l = \frac{E_l}{m_l}$ $(l = \nu \text{ or } e)$. For applicability of our approach in case of varying density of the background matter, there is an additional condition, $\big|\frac{\boldsymbol{\nabla}n}{np}\big| \ll 1$ɜe, for instance, [2], [13] and [14]), stating that the variation scale of matter density should be much larger than the de Broglie wavelength.

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