# Use of Matrix Calculation in Arithmetic and Numeration Theory for Scientific Calculator Design

RAOELINA ANDRIAMBOLOLONA, R. Hanitriarivo
*Madagascar-INSTN, Antananarivo 101, P.O.Box 4150, MADAGASCAR.*
*e-mail: raoelinasp@yahoo.fr ; rakotosonhanitriarivo@yahoo.fr*

Abctract: The use of matrix calculation in arithmetic is made- The general writing and the writing of a number, the using of matrix calculation in numerical basis, are given- The left hand side (L) – right hand side (R) and increasing (i) order i.e LRi writing of number is exploited to build a computer programmation for a scientific calculator- The later is shown and is operational.

Key words: arithmetic, matrix calculation, numeration, scientific calculator.

## 1. INTRODUCTION

In many languages (english, french, german, malagasy…), there are incoherencies, illogical and irrational rules in writing and reading numbers.

For instance, the number 2879 in 10-basis means:

$2879 = 10^3 \times \mathbf{2} + 10^2 \times \mathbf{8} + 10^1 \times \mathbf{7} + 10^0 \times \mathbf{9}$

and is read:

- in french language as <u>deux</u> mille <u>huit</u> cent <u>soixante</u> <u>dix</u> <u>neuf</u> i.e:

$\mathbf{2} \times 10^3 + \mathbf{8} \times 10^2 + \mathbf{6} \times 10^1 + \mathbf{1} \times 10^1 + \mathbf{9} \times 10^0$

- in english language as <u>two</u> thousand <u>eight</u> hundred <u>seven</u>ty <u>nine</u> i.e :

$\mathbf{2} \times 10^3 + \mathbf{8} \times 10^2 + \mathbf{7} \times 10^1 + \mathbf{9} \times 10^0$

- in german language as <u>zwei</u> tausend <u>acht</u> hundert <u>neun</u> und <u>siebzig i.e</u>:

$\mathbf{2} \times 10^3 + \mathbf{8} \times 10^2 + \mathbf{9} \times 10^0 + \mathbf{7} \times 10^1$

- in malagasy language as <u>sivy</u> amby <u>fito</u>polo sy <u>valo</u>njato sy <u>roa</u> arivo as [1] :

$\mathbf{9} \times 10^0 + \mathbf{7} \times 10^1 + \mathbf{8} \times 10^2 + \mathbf{2} \times 10^3$

The inconsistency from left hand side (L) to right hand side (R) i.e **LR** writing and **RL** reading may be complete as in malagasy language or partial.

The main idea is to consider the number 2879 as a row matrix, to introduce the 10-numeral basis as a column matrix and to use row-column matrix product:

$$2879 = \begin{bmatrix} 2 & 8 & 7 & 8 \end{bmatrix} \bullet \begin{bmatrix} 10^3 \\ 10^2 \\ 10^1 \\ 10^0 \end{bmatrix}$$

This point of view was introduced by RAOELINA ANDRIAMBOLOLONA in 1986 ["I. Application: une nouvelle proposition pour l'énoncé des nombres en malgaches", Bull. Acad. Malg, t. LXiV /1-2] and developed by him to study the problem of numeral basis change using matrix calculation. [2]

## 2. UTILISATION OF MATRIX CALCULATION IN ARITHMETICS

The basic idea as already said is then to consider the number N written in b numeral basis as

$$N = a_n b^n \pm a_{n-1} b^{n-1} + \dots + a_1 b^1 + a_0 b^0$$
$$= \underline{N}_b . \overline{B}_b$$

$\underline{N}_b = [\, a_n\ a_{n-1} \dots a_1\, a_0]$ is the usual writing of the number N in b numeral basis. $\underline{N}_b$ is the row-matrix basis

$\overline{B}_b = \begin{bmatrix} b^n \\ b^{n-1} \\ ... \\ b^1 \\ b^0 \end{bmatrix}$ is the column matrix represent the b-numeral basis. [2]

N is the intrinsic value (i.e. independent on the basis). $\overline{B}$ of the number

The extension to negative power of n is straight forward; the separation between positive and negative power of n is marked by a dot in english and a comma in french in arithmetics.

This new point of view is very fruit full for many reasons. It shows that it is possible

a) the four arithmetic operations (addition, substraction, multiplication, division) using the matrix calculation rules to study.

b) to study the problem of numeral basis change by using the basis change in a linear space through a passage matrix. The result thus obtained is much simplifier and more direct than the usual rule using the euclidian successive division.

c) to have new rules to write and to read numbers rules these are logical, rational and coherent with the writing of numbers.

## 3. GENERAL WRITING

Let us suppose that we write on a plane surface. Then there are four possibilities: from left hand side (L) to right hand side (R) denominated by LR, from right hand side to left (RL), from up (U) to down (D) denominated by UD and from down to up denominated by DU.

Some civilisations in the world have adopted one of these four possibilities. But in the history of humanity, we may have the bostrophader combination, for instance LR and RL. But we exclude the other possibilities.

## 4. GENERAL WRITING OF A NUMBER

For a number, there are 2 possibilities increasing (i) order or in decreasing (d) order.

Then to write a number, we have 8 and only 8 possibilities: LRi, LRd, RLi, RLd, UDi, UDd, DUi, DUd.

The international system (IS) i.e the most used is LRd. But let us consider also LRi.

We have studied these 2 possibilities. We have shown that the system LRi is the only one which gives a consisting rule to the 4 arithmetics operation (sum, substraction, multiplication and division).

The reading of a number in malagasy language is LRi, the writing is LRd imitating the international system (IS), which is of course completely illogical with reading. As we said at the beginning, this is the case of many languages in the world.

Example of basis change:

Let us look for the decimal writing of the octal number (in 8-basis) 177606

The matrix representing the basis change with no null elements:

$$\begin{bmatrix} \overline{8}^5 \\ \overline{8}^4 \\ \overline{8}^3 \\ \overline{8}^2 \\ \overline{8}^1 \\ \overline{8}^0 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 & 7 & 6 & 8 \\ 0 & 0 & 4 & 0 & 9 & 6 \\ 0 & 0 & 0 & 5 & 1 & 2 \\ 0 & 0 & 0 & 0 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \overline{10}^5 \\ \overline{10}^4 \\ \overline{10}^3 \\ \overline{10}^2 \\ \overline{10}^1 \\ \overline{10}^0 \end{bmatrix}$$

The column matrix on the left hand side is $\overline{B}_8$. The column matrix on the right hand side is $\overline{B}_{10}$. The row-column matrix M representing the change

$$\overline{B}_8 = [M].\overline{B}_{10}$$

whose elements are not null is given in the right hand side. It is worth while to not that the matrix M is not necessarily a square matrix.

**Insert PSN Here**

Let us take now the problem of the arithmetic basis change from of a number N.
We have successively

$$N = \underline{N}_8.\overline{B}_8 = \underline{N}_{10}.\overline{B}_{10}$$

or $\overline{B}_8 = [M].\overline{B}_{10}$

Then as we have basis, we obtain $\underline{N}_{10} = \underline{N}_8.[M]$

Suppose that $\underline{N}_8 = [\ 1\ 7\ 7\ 6\ 0\ 6]$. Its writing $\underline{N}_{10}$ in decimal basis is given by

$\underline{N}_{10} = [\ 1\ 7\ 7\ 6\ 0\ 6].[\mathrm{M}]$

We calculate the following matricial product to obtain the writing in decimal basis of the octal number:

$$= [\ 1\ \ 7\ \ 7\ \ 6\ \ 0\ \ 6\ ] \bullet \begin{bmatrix} 0 & 3 & 2 & 7 & 6 & 8 \\ 0 & 0 & 4 & 0 & 9 & 6 \\ 0 & 0 & 0 & 5 & 1 & 2 \\ 0 & 0 & 0 & 0 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \left\{ \begin{array}{cccccc} & & & & (9 & 4) \\ & & (1 & 1 & 2) & \\ & & (4 & 2) & & \\ & (3 & 0) & & & \\ & 3 & & & & \\ 0 & & & & & \end{array} \right\}$$

$$= [\,0 \quad 6 \quad 5 \quad 4 \quad 1 \quad 4\,]$$

We add the figures column by column from right hand side to left hand side in bringing the carried over if necessary and to use the addition in octal basis we obtain 65414.

We have

$\underline{N}_8 = 177606$

$\underline{N}_{10} = 65414$

One may check:

$8^5$ x **1** + $8^4$ x **7** + $8^3$ x **7** + $8^2$ x **6** + $8^1$ x **0** + $8^0$ x **6** = **65414**

N = $8^5$ x **1** + $8^4$ x **7** + $8^3$ x **7** + $8^2$ x **6** + $8^1$ x **0** + $8^0$ x **6** = $10^4$ x **6** + $10^3$ x **5** + $10^2$ x **4** + $10^1$ x **1** + $10^0$ x **4**

The number N is independent on the basis. $\underline{N}_8$ is its components on the octal basis and $\underline{N}_{10}$ is its comonents in decimal basis.

Remarks: In many applications the matrix [M] appears not to be a square matrix but we write put as null terms if necessary in order to obtain an inversible (regular) matrix. The matrix [M] representing a basis change in matrix calculation is necessarely inversible (regular).

Remark: let (J) the following set

We have shown that the LRi system is the only one that makes the set (J) coherent. This is not the case of LRd because of the carried other.

We note that writing LRi is logical with the arithmetic operations rules, which can not be the case of habitual writing LRd of numbers with the usual operations rules.

The reading of numeration should be in coherence with the writing of numeration. So we should have to write and read numbers in increasing order (LRi), which can not be the case in almost languages (french, english, german, malagasy …). It entails that we propose to change programmation of all computers [4], [5].

**Insert PSN Here**

## 5. APPLICATION OF LRi SYSTEM TO SCIENTIFIC CALCULATOR DESIGN

Let us give a computer program of a scientific calculator using the algorithm is based on LRi system.

Let us look for the addition representation in n-basis:

In LRi method, let A and C two intrinsic numbers

$A = a_0\ n^0 + a_1\ n^1 + \ldots + a_p\ n^p$

$C = c_0\ n^0 + c_1\ n^1 \ldots + c_q\ n^q$

In arithmetic n-basis they are respectively written :

$\underline{A}_n = a_0\ a_1 \ldots a_p$

$\underline{C}_n = c_0\ c_1 \ldots c_q$

To calculate the component $\underline{(A+C)}_k = f_n\ (a_k\ ,c_k\ )$ we associate the carried over $r_k$ the value of which is 1or 0 depending the case we cross from n-1 to 0 or not.

We notice that $f_n(a_k, .) = a_k$ and $f_n(., c_k) = c_k$ with the carried over $r_k$ is equal to 0.

$\underline{A}_n + \underline{C}_n = f_n\ (a_o, c_o)\ \ f_n(r_o, f_n(a_1,c_1)) \ldots f_n(r_{k-1},f(a_k, c_k)) \ldots$

The multiplication is obtained as a succession of additions and the division is obtained as a succession of substractions.

The trigonometric, hyperbolic, logarithmic, and exponential functions… are obtained in programming their Taylor developpement.

The new algorithm is run in n - basis:

Let us represent the n elements of the set $F_n = \{0, 1, 2, \ldots, n-1\}$ as a cycle

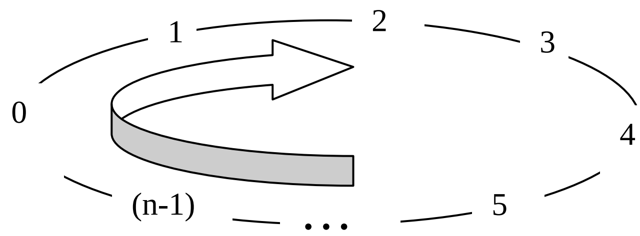


Figure 1: Cycle with n elements in clockwise direction

To represent the addition in n-basis, we define a totally symmetric function $f_n$ to which we associate a variable r with r $\in \{0, 1\}$

$f_n : F_n \times F_n \longrightarrow F_n$

$(a, c) \longmapsto f_n(a, c)$

$\forall (a, c) \in F_n \times F_n$ , $f_n(a, c) = f_n(c, a)$

$f_n(a, c)$ is the a $^{-ieth}$ next element to c in clock wise direction (increasing order), when we pass from (n-1) to 0 the carried over r is equal to 1.

To represent the substraction in n-basis, we define a totally anti-symmetric function $g_n$ to which we associate the variable r with r $\in \{0, 1\}$

$g_n : F_n \times F_n \longrightarrow F_n$

$(a, c) \longmapsto g_n(a, c)$

$\forall (a, c) \in F_n \times F_n$ , $g_n(a, c) = - g_n(c, a)$

where $F_n=\{0, 1, 2, \ldots, n-1\}$the corresponding cycle may be represented as

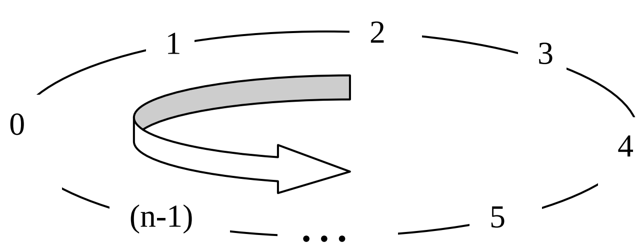


Figure 2: Cycle with n elements

$g_n(a, c)$ is the a$^{-ieth}$ next element to c in anticlockwise direction (decreasing order) when we pass from o to (n-1). The r carried over is equal to -1.

For I,stance, let us consider addition in decimal (n = 10) and octal (n = 8) basis:

In decimal basis for $F_{10} = \{0, 1, 2, \ldots, 9\}$ the cycle is

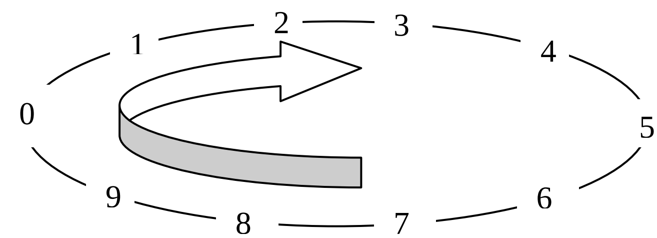


Figure 3: Cycle with 10 elements

$f_{10}$ (3,4) is the third element after 4 that means $f_{10}$ (3,4) = 7 and the carried over r is 0.

$f_{10}$ (5, 7) is the fifth element after 7 that means 2 and the carried over r is equal to 1 because we must pass from 9 to 0 before getting 2.

In octal basis we have n = 8

For $F_8$ = {0, 1, 2… 7} the cycle is

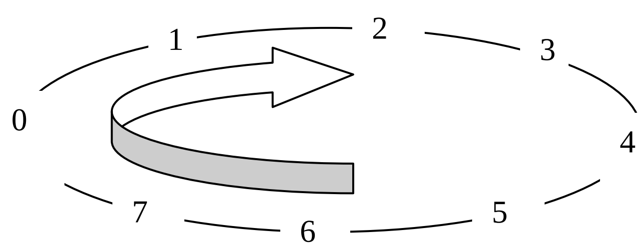


Figure 4: Cycle with 8 elements

$f_8$ (3, 4) the third element after 4 that means $f_8$ (3, 4) = 7 and the carried over r is 0

$f_8$ (5,7) is the fifth next element after 7 that means $f_8$ (5,7) gives 4 and the carried over r is equal to 1 because we pass from 7 to 0 before having 4.

## 6. SCIENTIFIC CALCULATOR DESIGN

The scientific calculator built with the LRi system is shown in the fig.5 in malagasy language. The english translation of addition, substraction, division, multiplication, clear, equal are respectively ampiana, analana, zaraina, mamafa, mira. The button "IZA?" indicates our names.

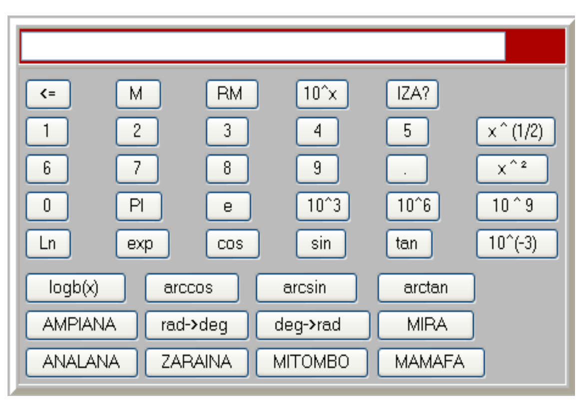

Figure 5: Scientific calculator

## 7. CONCLUSION

We point out that the LRi system of writing number is better than the LRd one, better in the sense that it makes the set (J) completely coherent. The latter is the international system which is widely utilized. We write down an algorithm for a computer programmation in the LRi system and we give a scientific calculator designed in LRi system.

## References

[1] RAOELINA ANDRIAMBOLOLONA, "Remarques sur la malgachisation des termes scientifiques", Séances de l'académie malgache, 1974.

[2] RAOELINA ANDRIAMBOLOLONA, "Théorie générale des numerations écrites et parlées utilisant le calcul matriciel", 1986.

[3] RAOELINA ANDRIAMBOLOLONA, Algèbre linéaire et multilinéaire tome I et II collection lira, 1998.

[4] M. Jean-Michel, "Arithmétique des ordinateurs", Masson, 1989.

[5] J. V. zur Gathen, Jurgen Gerhard, "modern computer algebra", Cambridge University press, 1984.