Mixed convection of an axisymmetric flow of air with variable physical properties

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Abstract : The distribution speeds and temperatures in two-dimensional mixed convection around an ellipsoid of revolution in rotation, plunged in an ascending flow of air containing particles of metallic oxide, are given starting from purely numerical procedure, based on a finite difference method and using the method of Keller known as "Block elimination" to solve the linear equations. The variability of the physical properties of the air according to the temperature is taken into account and speed external of the boundary layer is calculated using the method of the singularities. We first studied the importance of the variability of the physical properties. Then, we conducted the survey of all possible combinations of three convection according to the parameters that characterise them which allowed us to calculate the Nusselt number and the coefficient of friction. The comparisons of the results for the three cases of the ellipsoids has been done. The importance of the consideration of the variability of physical properties of the air in high temperature has been demonstrated.

Key words : Thermal - boundary layer – ellipsoid - natural convection – forced convection – parietal coefficient.

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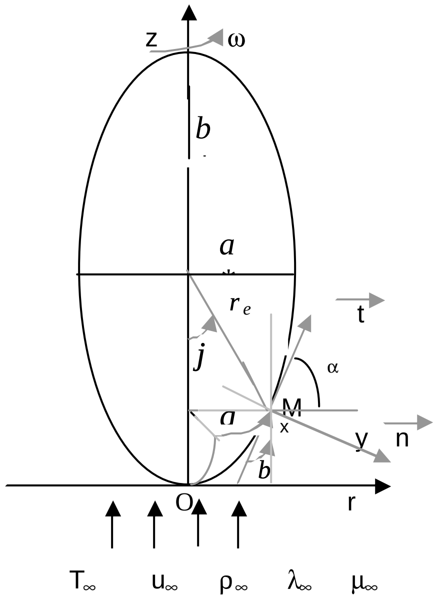
1. Introduction

Many works and publications (Ali Ch. A., et al., 1995), (Lepalec G., 1986), (Raminosoa Ch. R. R., et al., 1994) and (Rakotomalala M., et al., 1994) have already been carried out concerning the flows generated by the uniform rotation of body with symmetric of revolution since they intervene in many apparatuses and industrial processes. In particular, in the industrial operations of obtaining metal deposits in gas phase, if the wall of the body is at high temperature. Most of the authors consider that the physical properties of the fluid are constant. Some authors, (Ali Ch. A., et al., 1995) and (Raminosoa Ch. R. R., et al., 1994) already studied the problems of variable physical properties in the case of hydrogen gas. Currently, the techniques of deposits are very developed in the laboratory within the frame work of the acceleration of the deposit rate. In this work, we consider the flow of a gas mixture, of T_∞ = 25°C temperature, constituted by air containing particles of metallic oxide in small concentrations. This gas mixture is intended to give deposit on the hot wall, of T_p = 400°C temperatures after dissociation, with the result that in the vicinity of the wall, the physical properties of the air become variable. To standardize the thickness of deposits, it should be considered that the solid which will receive the deposit is in uniform rotation. To obtain an axisymmetric flow, it is considered that the body is an ellipsoid of revolution. The objective of this work consists in establishing the results ok to the transfers of impulse and heat in mixed convection around this body. As the transfers are coupled and two-dimensional, the difficulties which results from it are worth to be studied.

2. Studied system

Figure 1 represents the curvilinear frame of reference x, y and θ with local reference mark associated (M, $\vec{t}, \vec{n}$, $\vec{e_\theta}$). M is a point of the surface of the ellipsoid. We treat the problem with these hypothesis : the flows are laminar and permanent ; no heat source emerges in the system : chemical reaction caused at the time of the deposit does not have influence on the mechanism of transfer ; dissipation of energy by viscosity and transfers by radiations are negligible ; the effects Dufour and Sorret are negligible ; physical properties of the fluid with infinite are constant ; the fluid is encompassed to a perfect gas for the calculation of its voluminal dilation coefficient.

Figure 1: Schematic representation of the studied and referential systems



3. Equations of transfer

Under these conditions, the equations of transfer are written :

$$\left.\begin{aligned}
&\frac{\partial(?\,U)}{\partial x}+\frac{\partial(?\,V)}{\partial y}+\frac{?U}{r}\frac{dr}{dx}=0 \quad ; \quad ?\,Cp\,(U\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y})=\frac{\partial}{\partial y}(?\,\frac{\partial T}{\partial y})\\
&?\,(U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}-\frac{W^2}{r}\frac{dr}{dx})=?\,U_e\frac{\partial U_e}{\partial x}+\frac{\partial}{\partial y}(\mu\frac{\partial U}{\partial y})+g\ \mathrm{Cos}(ß)\,ß_t\,?_\infty\,(T-T_\infty)\\
&?\,(U\frac{\partial W}{\partial x}+V\frac{\partial W}{\partial y}+\frac{U\,W}{r}\frac{dr}{dx})=\frac{\partial}{\partial y}(\mu\frac{\partial W}{\partial y})
\end{aligned}\right\} \quad (1)$$

Conditions to the limits :

$$\left.\begin{aligned}
&\text{For } y=0 : U=V=0\,;\,W=r\,\omega\,;\,T=T_p\\
&\text{For } y\to\infty : U\to U_e\,;\,W\to 0\,;\,T\to T_\infty
\end{aligned}\right\} \quad (2)$$

The physical properties of the air in the boundary layer are calculated starting from the following empirical formulas (Midoux N., 1985) :

$$\rho(T) = 230.T^{-1}.Kg.m^{-3} \quad ; \quad Cp(T) = 1050 J.Kg^{-1}.K^{-1}$$
$$\mu(T) = 5,2.10^{-7}.T^{0,635}.Pa.s \quad ; \quad \lambda(T) = 2,66.10^{-4}.T^{0,805}.J.s^{-1}.m^{-1}.K^{-1} \qquad (3)$$

Where T is in Kelvin degree. According to KNUDSEN and KATZ (Midoux N., 1985), these relations apply correctly if $Re_\infty < 3.10^5$, $Pr > 0,6$ and if the properties of the fluid are evaluated with $0,58(T_p - T_\infty) + T_\infty$. For this case : $Re_\infty = 8.10^3$; $Pr = 0,71$. With the temperature reference $T_r = 25°C$:

$\rho(Tr) = 0,771, Kg.m^{-3}$: $\mu(Tr) = 19,376.10^{-6}$, Pa.s

$Cp(T_r) = 1050, J.Kg^{-1}.K^{-1}$; $\lambda(Tr) = 0,026, W.m^{-1}.K^{-1}$.

The speed at the external border of the boundary layer is calculated numerically using the method of singularities. Then, the results are directly introduced into the resolution of the equations of transfer.

4. Adimensionalisation of the equations

Choice of the adimensional sizes

$$Cp^* = Cp/Cp_\infty \; ; \; r^* = r/L \; ; \; T^* = (T - T_\infty)/(T - T_p) \; ; \; x^* = x/L$$
$$\lambda^* = \lambda/\lambda_\infty \; ; \; \mu^* = \mu/\mu_\infty \; ; \; \rho^* = \rho/\rho_\infty \; ; \; Pr = \nu_\infty/a_\infty$$
$$U_e^* = U_e/U_\infty \; ; \; Re_\omega = \omega (L)^2/\nu_\infty \; ; \; Re_\infty = U_\infty L/\nu_\infty \; ; \; B = (Re_\omega / Re_\infty)^2$$
$$\Omega = Gr/(Re_\infty)^2 \; ; \; \Omega' = Gr/(Re_\omega)^2 \; ; \; Gr = g_a\beta_t(L)^3(T_p - T_\infty)/(\nu_\infty)^2 \qquad (4)$$

Then

$$\frac{\partial(?*U*)}{\partial x*} + n_0\frac{\partial(?*V*)}{\partial y*} + \frac{?*U*}{r*}\frac{dr*}{dx*} = 0$$

$$?*(U*\frac{\partial U*}{\partial x*} + n_0 V*\frac{\partial U*}{\partial y*} - \frac{W*^2}{r*}\frac{dr*}{dx*}) = n_1 ?*U_e*\frac{\partial U_e*}{\partial x*} + n_0\frac{\partial}{\partial y*}(\mu*\frac{\partial U*}{\partial y*}) + n_2 Cos(ß)\,(T* - T_\infty*)$$

$$?*(U*\frac{\partial W*}{\partial x*} + n_0 V*\frac{\partial W*}{\partial y*} + \frac{U*W*}{r}\frac{dr*}{dx*}) = n_0\frac{\partial}{\partial y*}(\mu*\frac{\partial W*}{\partial y*}) \qquad (5)$$

$$?*Cp*(U*\frac{\partial T*}{\partial x*} + n_0 V*\frac{\partial T*}{\partial y*}) = n_3\frac{\partial}{\partial y*}(?*\frac{\partial T*}{\partial y*})$$

For $y = 0$: $U^* = V^* = 0$; $W^* = n_4 r^*$; $T^* = 1$

For $y^* \rightarrow \infty$: $U^* = n_5 U_e^*$; $W^* = T^* = 0$ $\qquad (6)$

With

$$y = \frac{(C_1 Re_\infty^{\frac{1}{2}} + C_2 Gr^{\frac{1}{4}} + C_3 Re_?^{\frac{1}{2}})\frac{y}{L}}{C_1 + C_2 + C_3} \quad ; \quad U = \frac{(\frac{C_1}{U_\infty} + C_2 Gr^{-\frac{1}{4}}\frac{L}{?_\infty} + \frac{C_3}{L\,?})U}{C_1 + C_2 + C_3}$$

$$V = \frac{(C_1\frac{Re_\infty^{\frac{1}{2}}}{U_\infty} + C_2\frac{L Gr^{\frac{1}{4}}}{?_\infty} + C_3\frac{Re_?^{\frac{1}{2}}}{L\,?})\,V}{C_1 + C_2 + C_3} \quad ; \quad W = \frac{(\frac{C_1}{U_\infty} + C_2\, Gr^{-\frac{1}{4}}\frac{L}{?_\infty} + \frac{C_3}{L\,?})W}{C_1 + C_2 + C_3}$$

$$n_0 = \frac{(C_1 + C_2 Gr^{\frac{1}{4}} + C_3)}{C_1 + C_2 + C_3} \quad ; \quad n_1 = \frac{(C_1 + C_2 O^{-1} + C_3 B^{-1})}{C_1 + C_2 + C_3} \quad ; \quad n_2 = \frac{(C_1 O + C_2 + C_3 O')}{C_1 + C_2 + C_3}$$

$$n_3 = \frac{(C_1 + C_2 G\,r^{\frac{1}{4}} + C_3)}{Pr(C_1 + C_2 + C_3)} \quad ; \quad n_4 = \frac{(C_1\sqrt{B} + C_2 O'^{\frac{1}{2}} + C_3)}{C_1 + C_2 + C_3} \quad ; \quad n_5 = \frac{(C_1 + C_2 O^{-\frac{1}{2}} + C_3 B^{-\frac{1}{2}})}{C_1 + C_2 + C_3}$$

$$n_6 = \frac{(C_1 Re_\infty^{\frac{1}{2}} + C_2 G\,r^{\frac{1}{4}} + C_3 Re_?^{\frac{1}{2}})}{C_1 + C_2 + C_3} \quad ; \quad n_7 = 2\frac{(C_1 Re_\infty^{-\frac{1}{2}} + C_2\frac{Gr^{\frac{3}{4}}}{Re_\infty^{2}} + C_3 Re_?^{-\frac{1}{2}})}{C_1 + C_2 + C_3} \qquad (7)$$

Coefficients C_i (i = 1 : axial convection ; i = 2 : natural convection ; i = 3 : rotatory convection) can take value one or null.

5. Local sizes

Numbers of Nusselt
$$\frac{hL}{l_\infty} = Nu = -L\frac{l}{l_\infty}\frac{\left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T - T_\infty)}$$

Coefficients of friction
$$CF_x = \frac{2\mu}{?(U_\infty)^2}\left(\frac{\partial U}{\partial y}\right)_{y=0} \quad CF_q = \frac{2\mu}{?(U_\infty)^2}\left(\frac{\partial W}{\partial y}\right)_{y=0} \qquad (8)$$

Then

$$Nu = -n_6\, ?^*\left(\frac{\partial T^*}{\partial y^*}\right)_{y=0} \quad CF_x = n_7\,\mu\left(\frac{\partial U^*}{\partial y^*}\right)_{y=0} \quad CF_q = n_7\,\mu\left(\frac{\partial W^*}{\partial y^*}\right)_{y=0} \qquad (9)$$

6. Method of resolution

We adopt the finite difference method to discretize the adimensionals equations and the method of Keller known as "Block elimination" was used to solve the linear system.

7. Design criteria

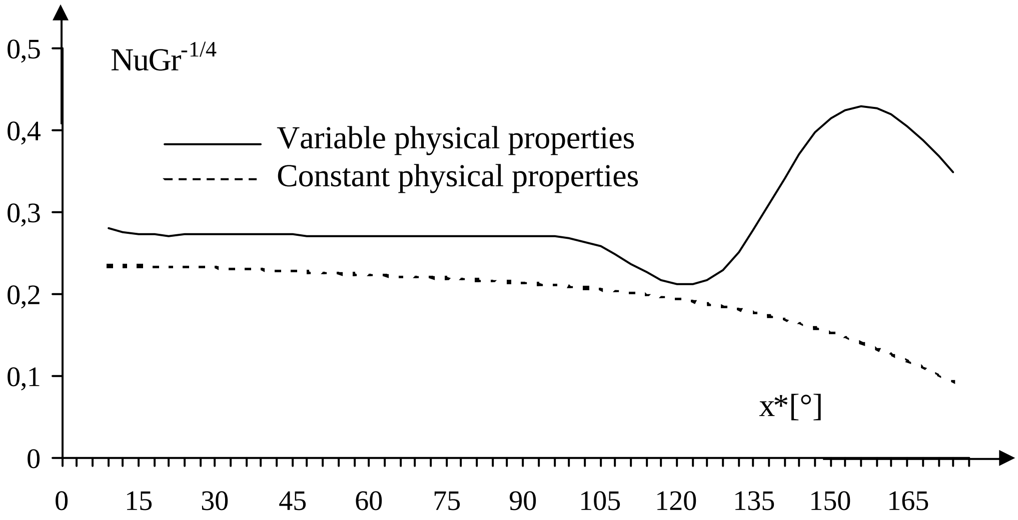
The horizontal semi-axis " a = 0,5m" of the ellipsoid is the characteristic length. For The number of Prandtl is considered in the T_r reference temperature. We take like the step of calculation $\Delta y^* = 0,03$.

8. Results

8.1 Pure natural convection($C_1 = C_3 = 0$ and $C_2 = 1$)

Figuire.2 shows the variations of $NuGr^{1/4}$ according to x* . It appears more significant, when the physical properties becomes function of the temperature. x* influence reduces this number, which means that the thermal transfer decreases. But the difference between the two stays more or less uniform around 15% to x* = 100° where the separation of thermal layer limit starts.

Figure.2 Variations of $NuGr^{-1/4}$ according to x* in pure natural convection.



8.1 Pure axial convection(C_1 = 1 and C_2 = C_3 = 0)

Figure.3 shows the variations of T*, according to y*. We note that the length of the thermal boundary layer increases and the mass of fluid becomes hotter when the physical properties prove to be variable.

Figure.4 illustrates the variations of U*, according to y*. We notice that depending only on the shape of the wall, speed outside the boundary layer remains the same one in both cases, and it is reached more quickly when the physical properties remain constant. This means that the length of the boundary layer increases when the physical properties become variable.

Figure.3 Variations of T* according to y* in pure axial convection.

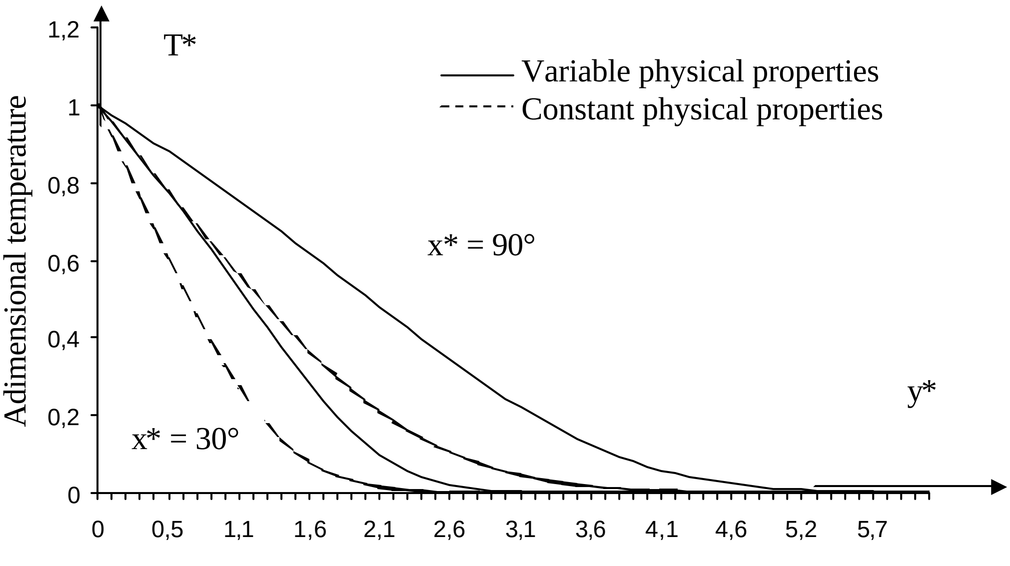
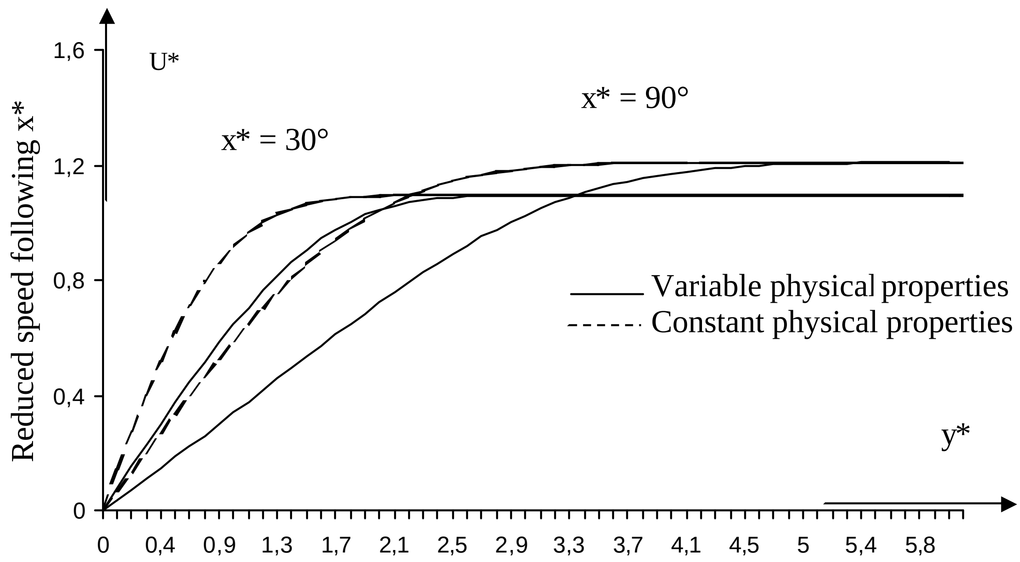


Figure.4 Variations of U* according to y* in pure axial convection.

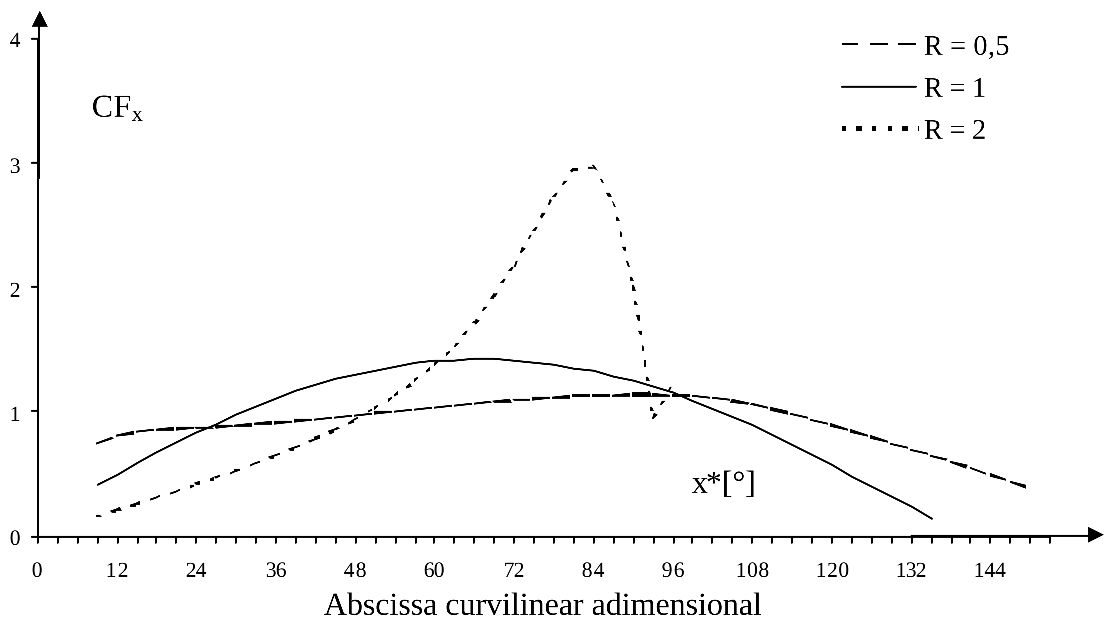


9 Influence of the geometrical configuration of the body(C_1 = C_2 = C_3 = 1)

We use the adimensional parameter R = a/b comparing the lengths of the vertical horizontal half axes.

Figure.5 illustrates the variations of CF_x according to x*. We notice that CF_x is an increasing function of x* and passes by a maximum then decrease rapidly. The maximum of CF_x and its sudden decrease beyond announces a separation of boundary layer. Besides, for the case of a sphere(R = 1), the maximum takes place at the neighbourhood of x * = 62°. It's worth noticing that (Lepalec G., 1986) attract the attention on the existence of a maximal friction around 60° for the case of a sphere. The same for (Raminosoa Ch. R. R., and al., 1994), when they studied the flow of gas hydrogen to variable physical properties around a sphere.

Figure.5 Variations of CF_x according to x*, for B = 1 and Ω = 1



Discussion

When physical properties fluid become variable, the transfer and the lengths of the boundary layers increases. In addition, if the temperature of the wall reaches 400°C, the results is only a more significant relative variation. Therefore, the insufficiency of constant physical properties hypothesis in this study is notorious when the temperature is high.

The sudden variation of the parietal sizes to a certain value of x* means that the separation of boundary layers starts. The results obtained show that the wall of a flattened ellipsoid can be made uniformly accessible from the thermal transfers point of view and that the prevalence of the natural convection is more significant in the fields of heat exchange. That means that the influence of those gives a great out put on the activity of the reaction in gas phase.

Conclusion

At the conclusion of this work, we have the data-processing calculation program FORTRAN. By respecting the preceding simplifying hypothesis, the distribution speeds and temperature are obtained from the resolution of the equations of transfers then, they make possible the termination of the local values of the number of Nusselt and the coefficients of friction. The consideration of the variability of the physical properties of the air, when the temperature is raised, is manifest for the survey of this phenomenon

Nomenclature

Latin Letters

a , b	:	lengths of the ellipsoid horizontal and vertical half axes, m
a_8	:	the thermal diffusivity of the fluid ad infinitum, $m^2.s^{-1}$
B	:	admensional parameter of rotation
CF_x , $CF_?$	:	Coefficients of friction following x and ?
Cp	:	mass heat- storage capacity, $J.Kg^{-1}.K^{-1}$
$\vec{e_r}$ $\vec{e_z}$, $\vec{n}$, $\vec{t}$	:	unit vectors
Gr	:	Grashof number
g	:	acceleration of gravity, $m.s^{-2}$
h	:	coefficient of transfer by convection, $W.m^{-2}.K^{-1}$
Nu	:	Nusselt number
L	:	caracteristic length (L = a), m
Pr	:	Prandtl number
r	:	radial distance, m
Re_8 , $Re_?$	:	Reynolds number relating to forced and rotatory convection
U, V, W	:	speeds according to x, y and ?, $m.s^{-1}$
T	:	temperature, K
U_e	:	external speed of the boundary layer, $m.s^{-1}$
x, y	:	curvilinear abscissa and normal local coordinates, m
z	:	side, m

Greek letters:

a, ß, φ	:	angles, rd
$ß_t$	:	voluminal expansion coefficient of the fluid $ß_t$ = 1/T, K^{-1}
ρ_e	:	radius vector, m
λ	:	thermal conductivity of the fluid, $W.m^{-1}.K^{-1}$

μ : dynamic viscosity of the fluid, $Kg.m^{-1}.s^{-1}$
ρ : voluminal mass of the fluid, $Kg.m^{-3}$
? : azimuth angle, rd
$?_8$: kinematics viscosity of the fluid ad infinitum , $m^2.s^{-1}$
? : angular velocity of rotation, $rd.s^{-1}$
O, O' : parameters of the natural and rotatory convection

Exposit

* : adimensional value

Indications

∞ : far from the wall

p : wall

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