Can the $\gamma\gamma$ processes reveal the nature of the σ meson ?

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We reanalyse the $\gamma\gamma$ scattering data and conclude that in the mass region below 1 GeV the cross section for $\gamma\gamma \to \pi^0\pi^0$ can be largely explained by the one pion exchange process with $\pi\pi$ rescattering. The radiative width of the σ is estimated and a model dependent separation into contributions from direct $\gamma\gamma$ decay and decay through rescattering is obtained. We confront these findings with QCD spectral sum rule (QSSR) predictions and conclude that the σ can have a large gluonium component in its wave function.

1. Introduction

Understanding the nature of scalar mesons in terms of quark and gluon constituents is a long standing puzzle in QCD [1]. One might expect that the decay rate of these mesons into two photons could provide an important information about their intrinsic composite structure. The problem here is that some states , discussed intensively at present, are very broad (σ and κ mesons), others are close to an inelastic threshold ($f_0(980)$, $a_0(980)$), which makes their interpretation more difficult. Besides the interpretation within a $q\bar{q}$ model [1,2,3,4,5,6,7] or unitarized quark model [8,9], also the possibility of tetraquark states [10,11,12,13] (and some other related scenarios: meson-meson molecules [14,15], meson exchange [16]) is considered. In addition, a gluonic meson is expected in the scalar sector, according to QCD spectral sum rules (QSSR) [17,18,19,20,4,7] and lattice QCD [21,22]. Such a state could mix with the other $\bar{q}q$ mesons [3,4,6,23,24]. Among the light particles, the σ meson could be such a gluonic resonance, that can manifest itself in some effective linear sigma models [25,26] or contribute to the low-energy constants at $\mathcal{O}(p^4)$ of the QCD effective chiral Lagrangian [27].

The existence of glueballs/gluonia is a characteristic prediction of QCD and some scenarios have been developed already back in 1975 [28]. Today, there is agreement that such states exist in QCD and the lightest state has quantum numbers $J^{PC} = 0^{++}$. Lattice QCD calculations in the simplified world without quark pair creation (quenched approximation) find the lightest state at a mass around 1600 MeV [21]. These findings lead to the construction of models where the lightest glueball/gluonium mixes with other mesons in the nearby mass range at around (1300-1800) MeV (see, for example, [23]). However, recent results beyond this approximation [22] suggest that the lightest state with a large gluonic component is rather in the region around 1 GeV and, therefore, a scheme based on mixing involving only meson states with mass higher than 1300 MeV could be insufficient to represent the gluonic degrees of freedom in the meson spectrum.

The approach based on QSSR [29,30] has given quantitative estimates of the mass of glueballs/gluonia and also of some essential features of its branching ratios. The mass of the lightest scalar gluonium is [18,20,4,7] :

$$M_{0^{++}} \approx (750 \sim 1000)\ \text{MeV}\ , \tag{1}$$

with a corresponding total width ranging from 300 to 1000 MeV (see section 3). Some large mixing of this gluonium is expected with the nearby isoscalar $q\bar{q}$ states resulting in the physical states σ and $f_0(980)$[3,4].

In phenomenological studies of $\pi\pi$ interactions, a broad object has been identified. Because of the large width and the presence of other resonances overlapping the identification is not straightforward. This is also reflected in the record of the Particle Data Group refering previously to $f_0(400-1200)$ and now to $f_0(600)$ or σ. Definitive studies have been carried out in elastic $\pi\pi$ scattering where energy independent phase shift analyses allow the reconstruction of the unitary amplitude. The S-wave isoscalar scattering phase shift δ_0^0 rises slowly and moves through 90° around 850 MeV [31] and it continues its rise also above the inelastic $K\bar{K}$ threshold in the observed range up to 1800 MeV. Ambiguities occuring in the phase shift analysis [32,33] have been successively resolved [34,35] by including data from $\pi\pi$ charge exchange [36] and the resulting complex amplitude nearly completes a full circle in the Argand diagram. In between, two narrow states, $f_0(980)$ and $f_0(1500)$ are clearly established and are superimposed to the slow movement of the "background amplitude".

In the K-matrix fits to the $\pi\pi$ elastic scattering data up to the highest available energy [(500-1800) MeV], a pole is found in the S-wave amplitude with a large imaginary part which corresponds to a state of large width [31,37,38]:

$$\Gamma \approx M \approx 1 \text{ GeV} , \tag{2}$$

though this value may depend on the treatment of the other resonances [24,6,35,39].

The broad object, so defined as σ, has been identified and singled out as a "left over state" in a phenomenological classification of the low lying scalar meson spectrum into $q\bar{q}$ multiplets and has therefore been related to the lightest glueball [6]. The appearence of this state in gluon rich processes was considered in support of this hypothesis. In this analysis, results from elastic and inelastic $\pi\pi$ scattering as well as from D, B and J/ψ decays have been considered [6,40,41].

Because of its large width, which is of the order of its mass, the σ, as a particle, is standing out and predictions involving its mass and width are of a particular concern. The mass parameter in (2) is close to the value where the observed S-wave phase shift goes through 90° as in a simple Breit Wigner form without background. This parameter also depends on the mass range taken into account and analyses which do not include the high mass tale of the S wave spectrum above 1 GeV tend to yield smaller values for this mass.

In general, the complex resonance self-energy $\Sigma = M - (i/2)\Gamma$ is energy dependent and determines the zero's and poles of the S-matrix amplitude joining appropriate sheets in the cut s-plane. In recent determinations, where analyticity and unitarity properties are used to continue the amplitude into the deep imaginary region, values:

$$\begin{aligned} M_\sigma &= 441 - \text{i}272 \text{ MeV } [43] \\ &= 489 - \text{i}264 \text{ MeV } [44] \end{aligned} \tag{3}$$

have been obtained. We assume here that these poles refer to the σ as defined above. We use below the values in eq. (2) under the assumption that, in all directly observable experimental numbers, this is a good approximation, leaving aside the actual extrapolation to complex kinematic variables, which needs theoretical assumptions or approximations.

In the above mentioned prediction of the glueball/gluonium mass from QSSR, a narrow width approximation or a Breit Wigner form parametrization of the spectral function has been employed. So we compare it with the mass in (2), obtained in a similar approximation from the resonance amplitude, which is closer to the mean of the observed mass spectrum of the amplitude squared $|T_0^0|^2$. In this sense the prediction for the mass of ~ 1 GeV [4,20] agrees with the observed σ "Breit Wigner mass".

We are now coming back to the problems with the $\gamma\gamma$ width. A recent analysis of the $\gamma\gamma \to \pi\pi$ processes [42] has extracted:

$$\Gamma(\sigma \to \gamma\gamma) \simeq (4.1 \pm 0.3) \text{ keV} , \tag{4}$$

for the σ mass obtained in [43]. The $\gamma\gamma$ width looks fairly high compared with most of the available theoretical estimates based on QCD dynamics.
This result was interpreted in [42] as disfavouring a gluonic and tetraquark nature which is expected to have a small coupling to $\gamma\gamma$ [20,3,4,13,15,12]. In the following, we shall reconsider the analysis of the $\gamma\gamma \to \pi\pi$ process in the low energy region below 1 GeV, where we conclude that it is dominated by the coupling of the photons to charged pions and their rescattering, which therefore can hide any direct coupling of the photons to the scalar mesons.

2. Analysis of $\gamma\gamma$ scattering data

A striking feature of the low energy $\gamma\gamma$ scattering is the difference of cross sections for the charged and neutral $\pi\pi$ final states: the charged final state is produced with a rate about an order of magnitude larger than the neutral final state which can be due to the contribution from the one-pion-exchange Born term in the process $\gamma\gamma \to \pi^+\pi^-$. In the process $\gamma\gamma \to \pi^0\pi^0$, the photons cannot couple "directly" to $\pi^0\pi^0$ but through intermediate charged pions and subsequent rescattering with charge exchange. In fact, in specific field theory models, this is the dominant mechanism for the process with neutral pions and, in the following, we shall discuss two such examples.

2.1. Chiral perturbation theory

In the effective chiral Lagrangian approach based on $SU(2)_L \times SU(2)_R$ symmetry, the interactions between pions and photons are given in terms of parameters m_π, f_π and e. To one-loop accuracy, the cross-section can be written in the factorised form [45]:

$$\begin{aligned} \sigma(\gamma\gamma \to \pi^0\pi^0) = & \left(\frac{2\alpha^2 q}{8\pi^2\sqrt{s}}\right) [1 + \frac{m_\pi^2}{s} f(s)] \\ \times & \quad \sigma(\pi^+\pi^- \to \pi^0\pi^0), \end{aligned} \tag{5}$$

with $4q^2 = s - 4m_\pi^2$ and: $f(s) = 2[\ln^2(z_+/z_-) - \pi^2] + \frac{m_\pi^2}{s}[\ln^2(z_+/z_-) + \pi^2]^2$, where $z_\pm \equiv (1/2)\left(1 \pm (1 - 4m_\pi^2/s)^{1/2}\right)$. For large s or/and in the chiral limit $m_\pi^2 \to 0$, the chiral correction behaves as $(m_\pi^2/s)\ln^2(s/m_\pi^2)$, which is finite and tends to zero. The $\pi^+\pi^- \to \pi^0\pi^0$ cross-section reads:

$$\sigma(\pi^+\pi^- \to \pi^0\pi^0) = (8\pi/9q^2)|T|^2 , \tag{6}$$

and in terms of Isospin amplitudes and phase shifts $T_I = \sin\delta_I e^{i\delta_I}$

$$T = T_0 - T_2; \quad |T|^2 = \sin^2(\delta_0 - \delta_2) . \tag{7}$$

The 1-loop prediction meets the experimental data at around 500 MeV but is a bit below at lower energy and above at the higher energies. In this approximation, the amplitude is real and is strictly valid only in the threshold region. If one extrapolates Eq. (5) based on one-pion exchange towards higher energies, one obtains at 600 MeV with $|T|^2 \approx 0.75$ a cross section of ~ 9 nb which is near the experimental result of $\sim$ 12 nb [46]. At this energy, the correction term $(m_\pi^2/s)f(s)$ amounts to $\sim$10% and the two cross sections in (5) are almost proportional. An improved agreement with the data up to about 700 MeV has been obtained including 2-loop corrections [47]. The changes to the 1-loop results amount to about 30% and the important role of one pion exchange is confirmed.

2.2. Analytic K-matrix model

In [39], Mennessier introduced a model which describes the S-wave $I = 0$ $\pi\pi$ interaction using an analytic K-matrix approach with two poles σ and $f_0(980)$ resonances supplemented by contributions induced by the 4-point $\pi\pi$ and $K\bar{K}$ interaction vertices. The σ pole is found at $M_\sigma = 500 - i300$ MeV, not far away from (3)[1].

In this approach, a subclass of bubble pion loop diagrams including resonance poles in the s-channel are resummed (unitarized Born). Coupling to photons is introduced through their coupling to charged pions, kaons and vector-mesons.

Like in the case of ChPT, the $\gamma\gamma \to \pi^+\pi^-$ process is dominated by the Born term, whilst, in $\gamma\gamma \to \pi^0\pi^0$, the diagrams with the pion loop through one-pion exhange are most important below 1 GeV. To 1-loop order, the expression of the $\gamma\gamma \to \pi^0\pi^0$ is similar with the one given by ChPT, where, to this order, and, at higher energies, the proportionality between the $\gamma\gamma \to \pi^0\pi^0$ and the $\pi^+\pi^- \to \pi^0\pi^0$ cross-section continues to hold. The free coupling parameters are fixed by a fit to $\pi\pi$ data. However, in addition to ChPT, the model allows for a "direct" contribution of the $I = 0$ resonances (σ and f_0) from the vertices $\gamma\gamma\sigma$ and $\gamma\gamma f_0$ and of the $f_2(1270)$ resonance $\gamma\gamma f_2$:

- Comparing the predictions on the differential cross-section $d\sigma/d\Omega$ with the data on $\gamma\gamma \to \pi^+\pi^-$, Fig. 11 of [39] shows that the unitarised Born term alone, i.e. without any additional direct contribution, describes, within 20%, the data in [48]. For instance, at the peak near threshold, the prediction after angular integration is about 480 nb (data 420 nb) while at 800 MeV mass one predicts 150 nb (data 130 nb).

- Also the data on $d\sigma/d\Omega$ for the process $\gamma\gamma \to \pi^0\pi^0$ [46] are in rough agreement with the predictions of Fig. 14 of [39]. The data (after 20% acceptance correction) indicate $\sim$ 10 nb cross section in the range 400-800 MeV, which yields $d\sigma/d\Omega \sim 1.7$ nb/str. The predicted cross section reaches this value at around 500 MeV and rises to twice this value at 800 MeV. The analysis indicates, that one does not need a strong direct $\gamma\gamma$ coupling for describing the data.

- The situation looks different in the $f_2(1270)$ region. Here the differential cross section $d\sigma/d\Omega$ at 90° is observed in the interval 1200-1300 MeV at $\sim$ 50 nb/str while the prediction from the unitarized Born term is around 18 nb/str in Fig. 14 of [39]. Therefore, the "direct" contribution in $f_2 \to \gamma\gamma$ should be dominant. This is consistent with the observation that predictions for the radiative decays of tensor mesons involving the "direct" coupling of photons to the quark charges are rather successful.

2.3. Estimate of the $\gamma\gamma$ width

From the model of [39], we conclude that the major part of the $\gamma\gamma$ cross section below 1 GeV, besides the pion exchange Born cross section, is due to pion exchange with rescattering. The measured cross section of $\gamma\gamma \to \pi^0\pi^0$ in the low energy region is then related to the same cross section in $\pi\pi$ scattering and assumed to be dominated by the σ resonance.

To estimate the $\gamma\gamma$ width, we assume that the cross section for $\gamma\gamma \to \pi\pi$ is given by a Breit Wigner form and reads (see for example [49]):

$$\sigma_{\gamma\gamma\to\pi\pi} = \frac{(2J+1)}{4}\left(\frac{8\pi}{q^2}\right) \times \frac{B_{in}B_{out}(M\Gamma_{tot})^2}{(M^2-s)^2+(M\Gamma_{tot})^2} \ . \tag{8}$$

Here, the first ratio takes into account the spin states. B is the branching ratio for incoming and outgoing states $B_{in} = \Gamma_{\gamma\gamma}/\Gamma_{tot}$, $B_{out} = 1/3$ for $\pi^0\pi^0$, such that $\Gamma_{\gamma\gamma}$ can be obtained from the cross section at the peak position. In general, some background may be present.

To begin with, we extract for a confirmation of this procedure the $\gamma\gamma$ width of $f_2(1270)$. Using the Crystal Ball [46] $\pi^0\pi^0$ and MARK-II $\pi^+\pi^-$ data [48] we obtain from (8) $\Gamma_{f_2\to\gamma\gamma} \sim 3.6$ and ~ 2.6 keV respectively, a range which compares with 2.6 keV quoted by PDG [50].

- We use the same procedure to estimate the $\gamma\gamma$ width for the σ. To relate to the subsequent theoretical predictions, as explained in the introduction, we relate the mass in Eq. (8) to the "Breit Wigner mass" in Eq. (2). In the isoscalar S wave $\pi\pi$ scattering, where the phase shift goes through 90°, this mass is around 1 GeV if the effect from $f_0(980)$ is subtracted, and decreases to $\sim$850 MeV when including the $f_0(980)$ and f_2. In the following, we shall use here the range:

$$M_\sigma \simeq (750 \sim 1000) \text{ MeV} \ . \tag{9}$$

[1] A similar model has been also applied to the $f_2(1270)$ resonance for the D wave in the mass range up to 1400 MeV.

While we assume there is no background under the resonance in the $I = 0$ channel we include the slowly varying $I = 2$ amplitude as in $\pi^+\pi^- \to \pi^0\pi^0$ given in Eq. (7). At 900 MeV the $I = 2$ phase reaches $\delta_2 \approx -20°$. This "background" reduces the peak position by the factor 0.88, which is a small correction as, a priori, expected. We obtained a good description of the Crystal Ball $\gamma\gamma \to \pi^0\pi^0$ cross section data [46] in the energy range 350-800 MeV before D wave scattering becomes important by using the form (8) with $J = 0$ and a mass dependent width, including the phase space factor:

$$\Gamma_{tot}(s) = \Gamma_\sigma \sqrt{\frac{s - 4m_\pi^2}{M_\sigma^2 - 4m_\pi^2}} \ , \tag{10}$$

for the parameters M_σ considered. The branching ratio $B_{\gamma\gamma}$ is then obtained from the peak cross section σ_{peak} at $\sqrt{s} \to M_\sigma$ independently from Γ_{tot} and its mass dependence.

We see that, around 800 MeV, the cross section for the process is around 10 nb after acceptance correction. At higher masses, the S wave cross section is hidden under an increasing $f_2(1270)$ contribution, where we take again 10 nb by extrapolation. Then for the considered range of Breit Wigner masses M_σ in Eq. (9) and $\Gamma_{\sigma,tot} \simeq M_\sigma$, we find:

$$\Gamma(\sigma \to \gamma\gamma) \approx (1.4 \sim 3.2) \text{ keV} \ , \tag{11}$$

which is to be considered as an estimate of the full radiative width [2].

- Aiming at a further theoretical interpretation, we decompose the amplitude for $\gamma\gamma \to \sigma \to \pi\pi$ into two components $T_\sigma = T_{direct} + T_{resc}$ using the model of [39]. Theoretical models on glueball decays usually do not include T_{resc} but refer to T_{dir}. An upper limit of the direct coupling:

  $$\Gamma(\sigma \to \gamma\gamma)|_{direct} \leq 1.4 \text{ keV} \ , \tag{12}$$

  can be obtained in the case of a negative interference of the re-scattering amplitude shown in [39] for $\gamma\gamma \to \pi^0\pi^0$ with the direct amplitude. With our fit of the model to the $\pi^+\pi^-$ and $\pi^0\pi^0$ data involving both the $\pi\pi$ rescattering and direct meson coupling we obtain a small $\gamma\gamma$ width compatible with the previous bound:

  $$\Gamma(\sigma \to \gamma\gamma)|_{direct} \approx 0.3 \text{ keV} \ , \tag{13}$$

  within an accuracy of about 50%. This result corresponds to the value of the coupling $|f| \approx 0.1$ defined in [39]. In an earlier analysis, a value $f \simeq 0.65$ corresponding to a $\gamma\gamma$ width of 6 keV and for $M_\sigma \simeq 600$ MeV has been found [39], which would correspond to data two times bigger [51]. The accuracy of this estimate can be improved using more accurate data below 600 MeV.

- One can notice that, the highest value of the width in Eq. (11), corresponding to the highest mass M_σ, is comparable to the width in Eq. (4) [42]. However, no separation into different contributions has been considered in that analysis.

What this result indicates is, that using the wide and heavy σ along the physical region, yields a similar result than working with the complex pole at low energies in (3). Only, if one used a low energy mass around 440 MeV and took the corresponding Breit Wigner cross section, would one get the much smaller radiative width an order of magnitude smaller, but this would apparently be an inconsistent procedure.

3. Comparison with QSSR predictions

- The QSSR determinations of the $\bar{q}q$ and gluonium light scalar meson masses are performed in a narrow width approximation [3], i.e. with a real pole. The predicted value in Eq. (1) is compatible with the observed properties of the "visible" meson having a Breit-Wigner mass with parameters given in Eq. (9), which is closer to the theoretical calculation than the complex mass of the σ meson in Eq. (3) with large imaginary part.

- A radiative decay width of the size in Eq. (13) is expected from a direct bare (index B) unmixed gluonium σ_B decays obtained from QSSR [20,4] :

  $$\Gamma(\sigma_B \to \gamma\gamma) \simeq (0.2 \sim 0.3) \text{ keV}. \tag{14}$$

  A width of this size induces a tiny effect of about 3×10^{-11} to the muon anomalous magnetic moment [52].
  One can also notice that, within the QSSR approach, a four-quark state gives a much smaller width of the order of 10^{-3} keV [12], while a $\bar{q}q$ state leads to a larger width [12,3].

- A determination of the total hadronic width using, either a dispersion representation of the scalar-pion-pion vertex, or a Breit-Wigner

[2] In [39], a larger value of the full radiative width of 5 to 9 keV has been obtained in order to fit the DM2 and some older data [51] which were two times bigger than the one used in this paper.

[3] A Breit-Wigner parametrization of the spectral function leads to a tiny width correction on the mass prediction [20,4,30].

form of the σ_B in the two-point function sum rule, leads to a value [20,4,7]:

$$\Gamma_{\sigma_B \to \pi\pi} \approx \frac{3}{2}(200 \sim 700) \text{ MeV} , \tag{15}$$

in agreement with the ones from $\pi\pi \to \pi\pi$ and $\pi\pi \to KK$ scattering data [39]. It also follows that a σ having a Breit-Wigner mass below 750 MeV cannot be wide [20,4,7] (see also some papers in Ref. [19]) . Up to $SU(3)$ breaking corrections, one also gets the relation:

$$g_{\sigma_B \pi\pi} \approx \sqrt{\frac{3}{4}} g_{\sigma_B KK} . \tag{16}$$

One should remark that the strong coupling of the σ to $\pi\pi$ and $\bar{K}K$ is a characteristic gluonium ($\bar{q}q$ singlet) feature which is not present for a four-quark state or $\bar{K}K$ molecule model for the σ. A careful measurement of such couplings may select among the different scalar meson scenarios.

4. Summary and conclusions

- We have reanalyzed the $\gamma\gamma$ scattering data and concluded that in the mass region below 1 GeV the cross section for $\gamma\gamma \to \pi^0\pi^0$ can be largely explained by the one pion exchange process with $\pi\pi$ rescattering. An improvement of our estimates needs more accurate data below 600 MeV.

- The small direct coupling of the σ to $\gamma\gamma$ and its large hadronic width are consistent with a large gluonic component of the σ resonance, expected from QSSR calculations (see section 3).

- The large gluonic component of the σ has been exploited in some phenomenological models with glueball and $\bar{q}q$ nonet [6] and with a maximal $\bar{q}q$-gluonium mixing below 1 GeV [3,4,7]. We plan to come back to these different mixing models and to analyze the nature of some other scalar mesons in a future work.

- In addition to the present analysis of $\gamma\gamma$ and $\pi\pi$ scattering data, some tests of the gluon component of the σ have been proposed in the literature, like e.g. the one from D semileptonic decays [53] where in addition to $\pi\pi$ one equally also expects the one into $\bar{K}K$. $B \to K\bar{K}K$ is also expected to be a source of gluonic decay from the $b \to sg$ process [41].

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