

NUMERICAL RESOLUTION OF THE DYNAMICAL EQUATION OF COMETARY DUST

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Abstract: Large amount of particles ejected from the nucleus surface are present in the neighbourhood of the cometary nuclei when comets are near the Sun (at heliocentric distances ≤ 2 AU). We tried to obtain the bounded orbits of those particles and to investigate their stability along several orbital periods. The model includes the solar and the cometary gravitational forces and the solar radiation pressure force. The nucleus is assumed to be spherical. The dust grains are also assumed to be spherical. We include the effects of centrifugal forces owing to the comet rotation. An expression for the most heavy particles that can be lifted is proposed.

Keywords: Comets: general – Comets: individual – Inteplanetary medium – Meteoroids.

1 Introduction

Particles of different sizes (in the millimetric to decimetric size range) around asteroids and comets have been reported for a long time. These particles may constitute a hazard for spacecrafts visiting the neighbourhood of the comets and asteroids. Although it might be thought that this risk does not exist if the comet-to-sun direction is avoided, the danger remains if large grains are orbiting around the nucleus. Suisei and Giotto were hit by millimetric grains when both spacecraft visited the Comet Halley in 1986. In the case of Suisei, that hit was over 150.000 km distance and the two particles were several milligrams and probably nearly millimetric in size, and in the Giotto impact the grains were larger, about forty milligrams (see, for example, Campbell et al. [1] and references therein). The danger is much larger if the spatial vehicle is planned to make a close approach to the nucleus of the comet, as planned for

Rosetta mission, which is intended to visit the Comet 67P/Churyumov-Gerasimenko in 2014. The interest has increased after the detection of large grains (> 2 cm) in the coma of comet LINEAR from radar observations [6].

The purpose of this work is to show the appropriated expressions to study the dynamical properties of the ejected cometary nuclei dust particles, including those terms coming from the comet rotation. Specifically, a determination of the largest size grain that can be lifted from the nucleus surface is made. The numerical integration of the differential equations of the motion will allow us to obtain the orbits of the dust particle around the nuclei of the comets.

2 Equation of motion

We write $\sum \vec{F}_i = m_d \vec{a}$, where $\sum \vec{F}_i$ is each force term applied over the mass m_d of the dust particle which suffers an acceleration \vec{a} . We consider the following forces (all units are in the International System of units):

- Drag force, \vec{F}_D . This force is due to the gas drag, and it can be derived from Navier-Stokes equations after suitable assumptions. We consider radially-symmetric outgassing. If the gas velocity is assumed to be constant and the dust velocity is considered much lower than the gas velocity, drag force is $\vec{F}_D = (1/32)C_D d^2 \dot{m}_g v_g r_d^{-3} \vec{r}_d$, which is a particular case of that one given by Wallis [8], where C_D is the drag coefficient, d the diameter of the grain, \dot{m}_g the gas loss rate, v_g the gas velocity, and r_d the dust grain to the nucleus centre distance.
- Gravitational force, \vec{F}_G . This force can be written as $\vec{F}_G = -m_d [M_c G r_d^{-3} \vec{r}_d + M_S G r_S^{-3} \vec{r}_S]$, being m_d the dust grain mass, M_c the comet mass, M_S the Solar mass, \vec{r}_S the vector joining the centre of Sun to the dust grain and G the gravitational constant.
- Solar radiation pressure force, \vec{F}_{rad} . Introducing the dimensionless parameter β as the ratio of the radiation pressure to the Sun gravitational force, the radiation pressure becomes $\vec{F}_{rad} = m_d M_S G \beta r_S^{-3} \vec{r}_S$.
- Inertial forces, \vec{F}_I . In this work, we used a nucleus-attached reference system with origin at the centre of the comet. Obviously, this frame is non inertial and then we must consider two inertial forces. One of them is due to the gravitational comet attraction by the Sun and the other is due to the rotation of the comet (spin). The final expression of this force is $\vec{F}_I = G M_S m_d r_c^{-3} \vec{r}_c + m_d \{ \vec{\Omega} \times (\vec{r}_d \times \vec{\Omega}) + 2 \vec{v}_d \times \vec{\Omega} \}$, where $\vec{\Omega}$ is the nucleus angular speed and \vec{v}_d is the

dust particle speed. Here, $\vec{\Omega}$ is considered constant with time and so no angular acceleration term is included.

Now we have every \vec{F}_i to include in $\sum \vec{F}_i = m_d \vec{a}$. Due to $\vec{r}_S = \vec{r}_c + \vec{r}_d$, we can rewrite the previous expression with only \vec{r}_c and \vec{r}_d , expanding r_S^{-3} in a Taylor series and assuming that $(r_d/r_c)^n = 0$ for $n > 1$ [7]. Then, we divide every term by $m_d = (4/3)\pi(d/2)^3\rho_d$ and use β ($\beta = 1 - \mu$) to obtain:

$$\begin{aligned} \frac{d^2\vec{r}_d}{dt^2} = & \left(\frac{3C_D\dot{m}_g v_g}{16\pi C_p Q_p}\beta - GM_c\right)\frac{\vec{r}_d}{r_d^3} + \beta M_S G\frac{\vec{r}_c}{r_c^3} + \frac{\mu M_S G}{r_c^3}\left(\frac{3\vec{r}_d\cdot\vec{r}_c}{r_c^2}\vec{r}_c - \vec{r}_d\right) + \\ & \Omega^2\vec{r}_d + (\vec{\Omega}\cdot\vec{r}_d)\vec{\Omega} + 2\vec{v}_d\times\vec{\Omega} \end{aligned} \quad (1)$$

which is similar to that by Richter and Keller [7] and that by Fulle [3], but including new terms due to the rotation of the comet. These new terms, which only have to be taking into account before the particles are ejected, are not negligible and are important when the equation of motion is resolved.

3 Dust grain ejection

As noted by Crifo et al. [2], dust grains will lift from the nucleus surface if $\sum \vec{F}_i\cdot\hat{n} > 0$, being \hat{n} an unitary normal vector to the surface. In the limit, that scalar product will be equal to 0 for the largest grain that can be lifted from the nucleus surface. Thus, multiplying both sides of the equation (1) by \hat{n} , rearranging and simplifying, and considering that the second and the third terms of the equation (1) do not contribute to the equation (2) because $R \ll r_c$, we obtain:

$$d_{max} = \frac{1}{\rho_d g_{eff}} \frac{3C_D\dot{m}_g v_g}{16\pi R^2} \quad (2)$$

where $g_{eff} = g - \Omega^2 R \sin^2(\phi) + 2\vec{v}_d \times \vec{\Omega}$, g is the gravity of the comet, ϕ is the angle between $\vec{\Omega}$ and \hat{n} , R is the radius of the cometary nucleus, and d_{max} is the maximum diameter of the grain that can be lifted. Since Whipple [9] similar formulae have been reported (see, for example, Yuehua Ma et al. [5]). Assuming $C_D = 2$ (sphere), $\rho_d = 10^3 \text{ kg m}^{-3}$ to the density, $1.4 \times 10^3 \text{ m}$ for R and $2.5 \times 10^5 \text{ kg m s}^{-1}$ for \dot{m}_g , appropriated values for comet Halley [4], we obtain a result of 5 cm for the diameter of the largest grain lifted from the nucleus surface neglecting the nucleus rotation. When the rotation is included, the maximum size increases to 20 cm at the nucleus equator due to spin force. We have assumed isotropic outgassing, but if the gas ejection is confined to certain active areas, the maximum diameter could be much larger.

4 Orbit simulations

As stated before, our purpose is to obtain the orbits of dust particles released from the nucleus surface, and to investigate the circumstances under which the orbits become bounded for a considerable fraction of the comet orbital period. In figure 1, the two orbits shown are typical cases of orbital stability, which are obtained for very specific values of the physical parameters involved, as detailed in the caption.

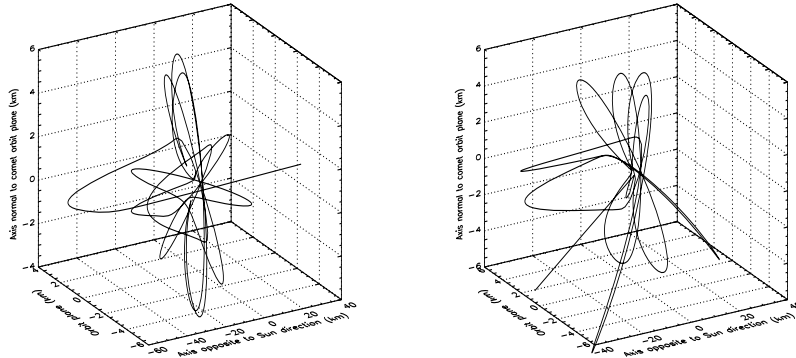


Figure 1: Left: Dust particle with $\beta = 5.94 \cdot 10^{-2}$, released from the nucleus surface of 49P/Wirtanen at latitude $+28^\circ$, longitude $+13.1947^\circ$ and obliquity 0° . Right: Dust particle with $\beta = 10^{-2}$, released from the nucleus surface of 49P/Wirtanen at latitude $+11^\circ$, longitude $+1.9852^\circ$ and obliquity 30° . The rotation period is 7.133 h.

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