MENU 2007
11th International Conference
on Meson-Nucleon Physics and
the Structure of the Nucleon
September10-14, 2007
IKP, Forschungzentrum Jülich, Germany

THE MASSLESS LINEAR SIGMA MODEL FOR FINITE NUCLEI AND AT FINITE TEMPERATURE

S. Tamenaga[1], *H. Toki, A. Haga, and Y. Ogawa*

RCNP, Osaka University, Ibaraki, Osaka 567-0047, Japan

Abstract

We study the effect of one-loop corrections from nucleon together with those from boson in the massless linear sigma model, where we perform the Coleman-Weinberg (CW) renormalization procedure [1, 2]. As a next step, we apply the massless linear sigma model to finite nuclei and finite temperature system.

1 Introduction

A model with the good properties of the chiral symmetry is the linear sigma model which has been used in the low energy phenomenology of hadron dynamics. This model has the renormalizabilities for both boson and fermion loop. However, there are long-standing problems for renormalization with chiral symmetry. It is well known that chirally symmetric renormalization for the nucleon loop in the conventional way gives unstable effective potential and too large non-linear interactions.

2 Lagrangian and renormalization procedure

We begin with the linear sigma model with fermion:

$$\begin{aligned}\mathcal{L}^{\rm LSM} &= \overline{\Psi}\left[i\gamma_\mu\partial^\mu - g_\sigma(\phi + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\right]\Psi \\ &+ \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi + \partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}\right) - \frac{\lambda}{4}\left(\phi^2+\boldsymbol{\pi}^2\right)^2 + \varepsilon\phi \\ &- \delta\mathcal{L}_{\rm CTC}, \end{aligned} \tag{1}$$

where Ψ, ϕ, and $\boldsymbol{\pi}$ are fermion field, sigma meson field, and pi meson field, respectively. $\varepsilon\phi$ is an explicit chiral symmetry breaking term. The last term $\delta\mathcal{L}_{\rm CTC}$ is the counterterm Lagrangian, which is necessary to renormalize the fermion and boson loops.

[1]E-mail address: stame@rcnp.osaka-u.ac.jp

2.1 Loop corrections with chiral symmetry

The effective action of Eq. (1) at the one boson-loop level can be written as

$$\begin{aligned}\Gamma_B^\chi &= \int d^4x \left[\frac{i}{2}\ln\det\left(-\frac{\delta^2\mathcal{L}}{\delta b\delta b}\right) - \mathrm{VEV} - \delta\mathcal{L}_{\mathrm{CTC}}^B\right] \\ &= \int d^4x \left[-V_B^R(\phi,\boldsymbol{\pi}) + \frac{1}{2}Z_{\sigma\pi}^B(\partial_\mu\phi\partial^\mu\phi + \partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}) + \cdots\right], \quad (2)\end{aligned}$$

where $\mathcal{L}$ includes the tree contribution and $\delta\mathcal{L}_{\mathrm{CTC}}^B$ is the counterterms for the boson loop. We take the renormalization conditions for mass, coupling constant, and derivative term to Eq. (2) as

$$\left.\frac{\partial^2 V_B^R}{\partial\phi^2}\right|_{\phi^2=0,\boldsymbol{\pi}^2=0} = 0, \quad \left.\frac{\partial^4 V_B^R}{\partial\phi^4}\right|_{\phi^2=m^2,\boldsymbol{\pi}^2=0} = 0, \quad Z_{\sigma\pi}^B\big|_{\phi^2=m^2,\boldsymbol{\pi}^2=0} = 0. \quad (3)$$

where we introduce the renormalization scale m to Eq. (3) in order to avoid the logarithmic singularity at the origin of the effective potential [3]. Finally the renormalized potential of boson with the chiral symmetry becomes

$$V_B^R = \frac{3\lambda^2}{16\pi^2}\left(\phi^2+\boldsymbol{\pi}^2\right)^2\left[\ln\left(\frac{\phi^2+\boldsymbol{\pi}^2}{m^2}\right) - \frac{25}{6}\right]. \quad (4)$$

In the same way as the boson loop, we obtain the renormalized potential of fermion loop as

$$V_F^R = -\frac{g_\sigma^4}{8\pi^2}(\phi^2+\boldsymbol{\pi}^2)^2\left[\ln\left(\frac{\phi^2+\boldsymbol{\pi}^2}{m^2}\right) - \frac{25}{6}\right]. \quad (5)$$

2.2 Effective potential and parameters

Here, we have defined a non-trivial local minimum away from the origin which determines the coupling constant λ which is dependent on the renormalization scale m,

$$\frac{3}{2\pi^2}\left[\ln\left(\frac{f_\pi}{m}\right) - \frac{11}{6}\right]\lambda^2 + \lambda - \frac{g_\sigma^4}{\pi^2}\left[\ln\left(\frac{f_\pi}{m}\right) - \frac{11}{6}\right] - \frac{\varepsilon}{f_\pi^3} = 0. \quad (6)$$

Equation (6) has two solutions as a function of m and we choose the positive coupling constant λ as the natural choice. Both the boson and the nucleon loops are too large as compared with the tree contributions. However, the total loop potential is a reasonable and negative one due to cancellation between the large positive potential from the nucleon loop and the

Table 1: **Parameter sets through the relationships using $m = f_\pi$.**

M[MeV]	m_π[MeV]	f_π[MeV]	g_σ	λ	m_σ[MeV]
939	139	93	10.09	85.0	669.7

large negative one from the boson loop. As a result, the total renormalized loop potential plays an important role as the negative mass term of the linear sigma model. Thus the spontaneous chiral symmetry breaking occurs through the loop corrections. The parameters can be determined from experimental values and chiral symmetry in Table 1. The spontaneous chiral symmetry breaking makes fermion and sigma meson massive. Only the pion mass is generated from the explicitly chiral symmetry breaking term. Using all of the parameters, then, the renormalized effective potential consistent with chiral symmetry becomes stable around the new origin by using the CW renormalization procedure for the first time [1, 2].

3 Finite system

In this section, we apply this model to finite nuclei and finite temperature. The massless chiral sigma model also has the same properties as those of the chiral sigma model [2] which includes the ω meson in order to generate an appropriate repulsive effect to obtain a stable nucleus. The difference between the chiral sigma model and the massless chiral sigma model is the loop corrections. In particular the renormalization of wave function is important for the finite system with the surface. Left hand side of Fig. 1 shows the baryon density distribution in ^{16}O with the massless chiral sigma model. The baryon density distribution becomes more smooth in spite of the large incompressibility. The effect of Dirac sea changes the density distribution in the interior region and surface region.

We also would like to see the hadron properties of the massless linear sigma model at finite temperature. In this calculation we include both Hartree and Fock terms. The stationary point, f_π, is obtained at any temperature by local minimum condition. Through this condensation, f_π, the masses of all the particles can be given. Right hand side of Fig. 1 shows the masses of mesons and nucleon as function of temperature. This behavior of the nucleon mass is almost the same as the quark-meson coupling model with the self-consistent approach. Moreover the differences between the mass parameters of σ and π mesons produce the crossover curve of the masses around the critical temperature.

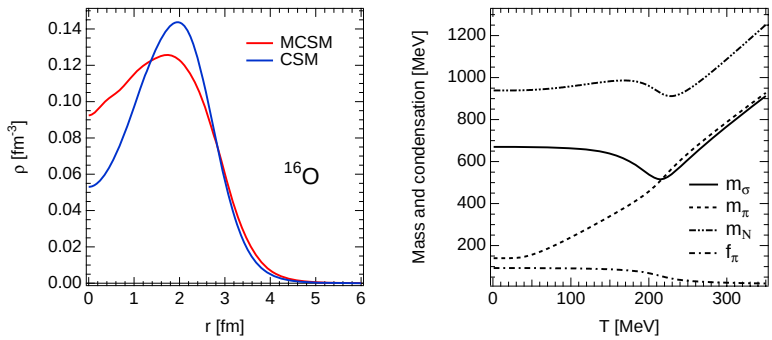

Figure 1: (LHS) The baryon density without pion mean field in ^{16}O with the chiral sigma model (CSM) in the mean field approximation and with the massless chiral sigma model (MCSM) in the relativistic Hartree approximation. (RHS) The behavior of mass and condensation dependent on temperature with the CW scheme in the case of broken chiral symmetry $\varepsilon \neq 0$.

4 Summary

We have studied the renormalization of the chiral model for the study of finite nuclei and hadron properties at finite temperature. By introducing the CW renormalization scheme we have been able to resolve the problem of the instability of the effective potential in the chiral model for the first time. We have found that the renormalization of wave function provide the smooth density distribution. We have also found the dependence of hadron masses on temperature with the boson and nucleon loops for the first time.

Acknowledgements

This work is supported by Research Fellowships of the Japan Society for the Promotion of Science (No. 18-09058).

References

[1] S. Tamenaga, H. Toki, A. Haga and Y. Ogawa, nucl-th/0604013.

[2] H. Toki, Y. Ogawa, S. Tamenaga and A. Haga, Phys. Part. Nucl. Phys. **59**, 209 (2007).

[3] S. R. Coleman and E. Weinberg, Phys. Rev. **D7**, 1888 (1973).