

MENU 2007
11th International Conference
on Meson-Nucleon Physics and
the Structure of the Nucleon
September 10-14, 2007
IKP, Forschungszentrum Jülich, Germany

ELECTROMAGNETIC REACTIONS ON LIGHT NUCLEI USING CHIRAL EFFECTIVE THEORY

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Abstract

I describe the use of chiral effective theory (χ ET) to compute electromagnetic reactions in two- and three-nucleon systems. I first explain how chiral perturbation theory can be extended to the few-nucleon sector. I then explain the predictions of the resulting χ ET for electron-deuteron scattering, and how they will be tested by forthcoming data from BLAST. I conclude by displaying predictions for elastic Compton scattering from deuterium and Helium-3 nuclei. These computations, in concert with future data from MAX-Lab and HI γ S, should give significant new information on neutron polarizabilities, and hence yield insight into the structure of the nucleon.

1 Introduction

Chiral perturbation theory (χ PT) is the effective theory of the strong interaction at low energies. In χ PT quantum-mechanical amplitudes for the interaction of pions and photons with each other and with nucleons are expanded in powers of the small parameter P , where $P \equiv \frac{p, m_\pi}{\Lambda_{\chi SB}}$. The scale $\Lambda_{\chi SB} \sim m_\rho, 4\pi f_\pi$ in the meson sector, but is somewhat lower for reactions involving baryons unless additional degrees of freedom (in particular the Delta(1232)) are included explicitly in the theory.

The amplitudes we seek are computed using the technology of effective field theory (EFT), in which the field-theoretic Lagrangian is organized in an expansion in powers of P and loop calculations are then also organized via the same hierarchy. Computing the χ PT result for a given process at a fixed order in P is simply a matter of writing down the Lagrangian up to that order and computing all the pertinent diagrams. An introduction to,

and explicit examples of, this strategy was given in Prof. Gasser’s talk at this meeting [1].

This approach has had much success in treating $\pi\pi$ and πN interactions at energies below $\Lambda_{\chi\text{SB}}$ (see Ref. [2] for a recent review). However, an obvious problem arises when we attempt to extend it to light nuclei: a perturbative expansion of amplitudes is not adequate to describe bound states. In 1990 Weinberg proposed that the fact that the nucleon mass, $M \sim \Lambda_{\chi\text{SB}}$, mandates resummation of diagrams with NN intermediate states, and so, when computing $NN \rightarrow NN$, χ PT should be applied not to the NN amplitude, but to the NN potential V [3]. In such an expansion the leading-order (LO) NN potential, V , is:

$$\langle \mathbf{p}' | V | \mathbf{p} \rangle = -\frac{g_A^2}{4f_\pi^2} \tau_1 \cdot \tau_2 \frac{\sigma_1 \cdot (\mathbf{p}' - \mathbf{p}) \sigma_2 \cdot (\mathbf{p}' - \mathbf{p})}{(\mathbf{p}' - \mathbf{p})^2 + m_\pi^2} + C, \quad (1)$$

where g_A and f_π are the nucleon’s axial charge and the pion-decay constant, and the constant(s) C (there is actually one for spin-singlet and one for spin-triplet NN states) is not determined by chiral symmetry and must be obtained from NN data. V is then inserted into the Schrödinger equation

$$\left(\frac{\hat{\mathbf{p}}^2}{M} + V \right) |\psi\rangle = E|\psi\rangle, \quad (2)$$

to generate bound and scattering states of two, or more, nucleons. This strategy, which has come to be known as “chiral effective theory” (χ ET) produces a quantum-mechanical description of light nuclei, in which the potential V (and other operators too) have a systematic chiral expansion and a rigorous connection with the chiral symmetry of QCD and the pattern of its breaking.

Since the potential V is singular a cutoff, Λ , must be introduced. The constant C is then a function of Λ . The question arises as to whether this will be sufficient to renormalize the NN amplitude obtained by iterating V , i.e. whether there is significant residual Λ -dependence in NN observables after the value of C is adjusted to reproduce the very-low-energy NN data.

There has been much debate on this point over the past 10 years, but it has now been shown that a single constant C is sufficient to renormalize the NN problem in the ${}^3\text{S}_1$ - ${}^3\text{D}_1$ channel at LO [4–6]. Moreover, these papers argue that it is necessary to solve the Schrödinger equation with the LO chiral potential precisely because that potential is not weak. In contrast to the $A = 0$ and $A = 1$ sectors a perturbative expansion for the NN interaction mediated by pions only converges for $p \lesssim m_\pi$: the one-pion exchange part of V is strongly attractive—singular even—in the ${}^3\text{S}_1$ - ${}^3\text{D}_1$ channel.

More recently it has been pointed out that there are channels of higher angular momentum where the LO potential (1) is also singular and attractive,

but where the constant C is not operative [7]. Consequently it is impossible to generate Λ -independent predictions in those channels (e.g. ${}^3\text{P}_0$) over a wide cutoff range. How wide a Λ range should be employed is still debated [8]. Ultimately renormalization-group techniques would seem the best way to determine what operators must be included to renormalize χ ET to a given level of accuracy [6]. This is an ongoing discussion.

But its ultimate resolution should not have a significant impact on the results I present here, which are predominantly for deuterium, where this is a solved problem (at least at LO). In Sec. 2 I show results for deuteron electromagnetic form factors as a function of the cutoff Λ and demonstrate that cutoff artifacts indeed disappear as $\Lambda \rightarrow \infty$. I also describe how a chiral expansion for the deuteron charge operator generates precision predictions for the ratio G_C/G_Q that was recently measured at BLAST. In Sec. 3 I summarize χ ET calculations of elastic photon scattering from deuterium and Helium-3 nuclei. And in Sec. 4 I provide a brief summary of other reactions involving light nuclei that have been successfully described in χ ET.

2 Electron-deuteron scattering in χ ET

In Ref. [7] Eq. (2) was solved for the potential (1) in momentum space for $\Lambda = 0.4\text{--}4$ GeV. In Ref. [5] the same problem was solved in co-ordinate space by converting C into a boundary condition on the wave function as $r \rightarrow 0$ [5]. We now present results for deuteron electromagnetic form factors that show that the latter wave function can be regarded as the $\Lambda \rightarrow \infty$ limit of the Fourier transform of the momentum-space wave functions [9].

The deuteron charge and quadrupole form factors G_C and G_Q involve matrix elements of the (Breit-frame) deuteron charge operator J_0 between these wave functions (for formulae see, e.g. Refs. [10, 11]). Here we will compare χ ET predictions for G_C with extractions from data for the deuteron structure function A and the tensor-polarization observable T_{20} [12]. For this purpose we use the deuteron current operator

$$\langle \mathbf{p}' | J_0(\mathbf{q}) | \mathbf{p} \rangle = |e| \delta^{(3)}(p' - p - q/2) G_E^{(s)}(Q^2), \quad (3)$$

with $G_E^{(s)}$ the nucleon's isoscalar electric form factor. This is the result for J_0 up to corrections suppressed by P^3 (apart from some small effects that have coefficients $\sim 1/M^2$). The strict LO χ ET result for deuteron form factors is found by taking $G_E^{(s)} = 1$. However, here we wish to test χ ET's predictions for deuteron structure, so we adopt ‘‘experimental’’ data for $G_E^{(s)}$ (the parameterization of Belushkin *et al.* [13]) and compute G_C . Up to the order we work to this is equivalent to computing the ratio $G_C/G_E^{(s)}$ in χ ET.

The results are shown in the left panel of Fig. 1. Here the value of C is adjusted to ensure that the deuteron binding energy is reproduced, but the calculation contains no other free parameters. We observe that as $\Lambda \rightarrow \infty$ the momentum-space wave functions produce a G_C that converges to a definite result (although asymptopia is not reached in G_C until $\Lambda \approx 10 \text{ fm}^{-1}$). This result is consistent with that found using the co-ordinate space approach of Ref. [5]. The agreement with experimental data at low- $|\mathbf{q}|$ is quite good, but the LO wave functions predict a minimum in G_C at too large a Q^2 . These trends are even more pronounced in G_Q (not shown), where we are within a few per cent of the asymptotic result at $\Lambda = 4 \text{ fm}^{-1}$, and the agreement with experimental data is excellent to a surprisingly large value of $|\mathbf{q}|$.

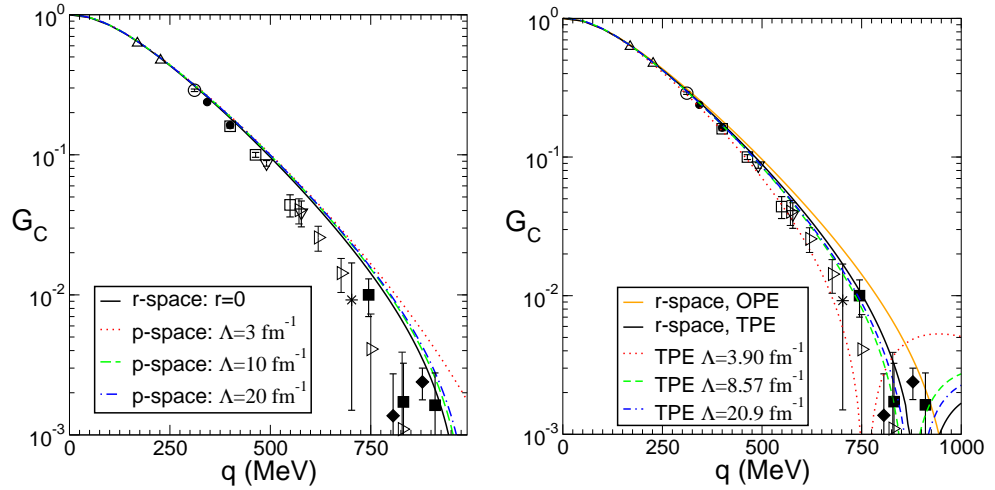


Figure 1: Predictions for G_C with LO wave functions (left panel) and wave functions including two-pion exchange (right panel). In each case results for four different regulators are shown. Data are taken from Ref. [12].

To go beyond LO we must consider corrections to the NN potential V , and to the charge operator J_0 . When the two-pion-exchange mechanisms that define V up to $\mathcal{O}(P^3)$ [14] are included the renormalization becomes a little more complicated since there are two undetermined parameters summarizing deuteron physics at scales $> \Lambda_{\chi SB}$. Here these constants are adjusted to reproduce the deuteron properties $B = 2.22457 \text{ MeV}$, $A_S = 0.885 \text{ fm}^{-1/2}$ and $\eta = 0.0256$ [9]. Once again we see that convergence to the $\Lambda \rightarrow \infty$ result is somewhat slow, but a smooth $\Lambda \rightarrow \infty$ limit does exist. We also see that the two-pion-exchange corrections to V result in only a small shift in G_C in the range $|\mathbf{q}| < 800 \text{ MeV}$. This lends credence to the idea that these corrections could be treated in perturbation theory—at least in the 3S_1 – 3D_1 channel. It is also significant that these corrections shift the G_C minimum to

the left as compared to the LO result, thereby improving the agreement with experiment. This suggests that electron-deuteron data provide evidence for the presence of two-pion-exchange pieces in the χ ET potential V .

The existence of the minimum in G_C provides an opportunity to examine the impact on G_C of the meson-nucleon dynamics that enters the chiral NN potential. In particular, different choices of the πN LECs c_i that enter V produce minima in somewhat different locations [9]. However, it is difficult to draw any real conclusion as regards the preference of existing G_C data for a particular set of c_i 's, since the $\mathcal{O}(P^3)$ corrections to J_0 that were not included in Eq. (3) have an impact on the position of the minimum of G_C that is at least as large as the effect of choosing different sets of c_i 's.

We close this section by pointing out that this type of analysis, when carried out to higher orders in χ ET, results in a precise prediction for the ratio of deuteron form factors G_C/G_Q in the kinematic range of forthcoming data from BLAST. In Ref. [11] all contributions to J_0 (including two-body effects) up to order P^3 relative to leading were computed. A variety of wave functions that included all the two-pion-exchange effects up to $\mathcal{O}(P^3)$ were also employed. Significant sensitivity to short-distance NN physics was found in the resulting deuteron quadrupole moment Q_d : it varied by 2% when the cutoff in the NN system was changed by $\sim 100\%$. Intriguingly, this is roughly the magnitude of the discrepancy between the Q_d predicted at $\mathcal{O}(eP^3)$ and the experimental value $Q_d = 0.2859(3) \text{ fm}^2$. This encouraged us to include in our analysis a short-distance operator that represents the contribution of modes above $\Lambda_{\chi\text{SB}}$ to G_Q . This operator has much slower Q^2 -dependence than the one-body mechanisms that give the LO contribution to G_Q , so we can constrain its impact by demanding that its coefficient is such that the experimental Q_d is reproduced. This vitiates our ability to predict G_Q at $Q^2 = 0$, but we can still predict the Q^2 -dependence of G_Q . The prediction's remaining theoretical uncertainty—which comes from the Q^2 -dependence of short-distance NN physics—is small, being only 3% at $|\mathbf{q}| = 2 \text{ fm}^{-1}$. It is important to note that the χ ET predictions for G_C/G_Q [11] have a rather different Q^2 -dependence to those obtained in potential models (see, e. g., Fig. 11 of Ref. [10]), and so the BLAST data will provide a significant test of this approach to deuteron electromagnetic structure.

3 Compton scattering on $A = 2$ and 3 nuclei

Now we turn our attention to Compton scattering from the deuteron. The first calculation of this process in χ ET computed the γd amplitude

$$\mathcal{A} \equiv \langle \psi_d | \hat{O} | \psi_d \rangle \quad (4)$$

by considering the operator \hat{O} up to $\mathcal{O}(e^2P)$ [NLO] and using a variety of phenomenological deuteron wave functions [15]. At this order χ ET makes a prediction for \mathcal{A} , and hence for the γd differential cross section (dcs) [15], as well as single- and double-polarization observables [16].

For photon energies ω such that $\frac{m_\pi^2}{M} \ll \omega \sim m_\pi$, \hat{O} begins at $\mathcal{O}(e^2)$ with the proton Thomson amplitude $\sim \frac{-e^2}{M}$. At $\mathcal{O}(e^2P)$ \hat{O} includes ‘‘pion-cloud’’ mechanisms that generate the leading contribution to the nucleon’s electric and magnetic polarizabilities, α and β . It also includes analogous two-nucleon mechanisms where the incoming and outgoing photons couple to what can be thought of as the deuteron’s pion cloud, i.e. the exchanged pions that generate the LO χ PT potential. These exchange currents are large, providing about 50% of the γd dcs at $\omega = 65$ MeV. At these energies the χ ET prediction is in good agreement with data (see left panel of Fig. 2). However, the agreement with data at 95 MeV is not good (see right panel of Fig. 2). Later χ ET calculations extended the calculation of \hat{O} to $\mathcal{O}(e^2P^2)$ and used χ ET wave functions [17]. However, this leads to very little improvement in the description of the 95 MeV data.

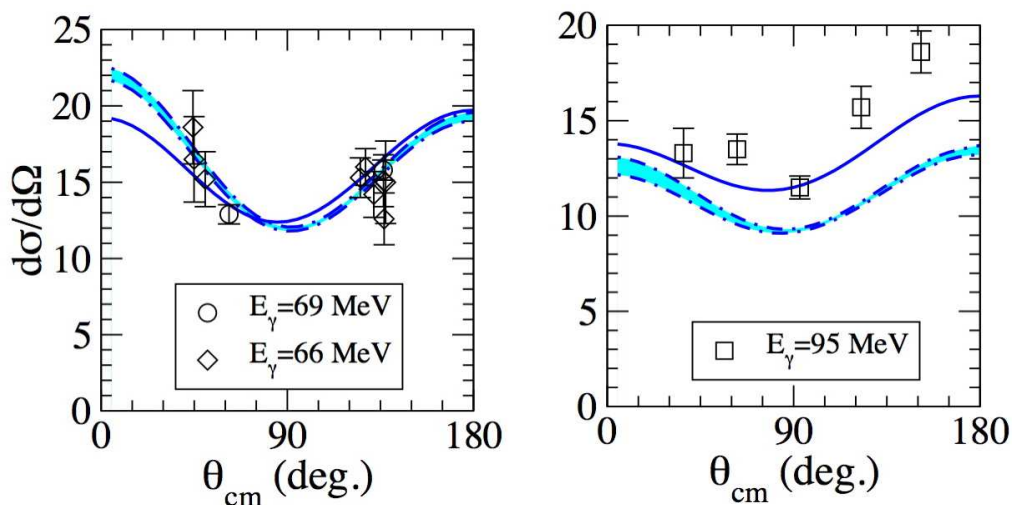


Figure 2: Centre-of-mass frame γd dcs at $\omega = 67$ and 94.5 MeV respectively. The dot-dashed line is the prediction at $\mathcal{O}(e^2P)$, with the band an estimate of the uncertainty due to short-distance NN physics. The solid line is a fit at $\mathcal{O}(e^2P^2)$. Adapted from Ref. [17], which includes references to data.

The rapid rise in the γd dcs at backward angles is now understood to be due to M1 excitation of the Delta(1232) resonance [18]. Chiral EFTs that include this resonance as an explicit degree of freedom describe the backward-angle 95 MeV data well [19]. At the same time the power counting

in χ ET for $\omega \sim \frac{m_\pi^2}{M}$ has been worked out. In this domain additional diagrams that ensure that the low-energy theorem for γ d at $\omega = 0$ is obeyed must be included, and these have now been computed [20]. But these diagrams are formally and numerically sub-leading for $\omega \sim m_\pi$. If they are included in the computation of \hat{O} for $\omega \approx 90$ MeV the variation in the cross section due to short-distance physics in the NN system is reduced to 1–2% [19, 20].

Therefore the elements needed for a χ ET calculation of the γ d dcs in the range $\omega = 50$ – 100 MeV with an accuracy $\sim 3\%$ are now understood. This ability of χ ET to calculate the NN dynamics in such a controlled way motivates experimental efforts that aim to use new γ d data to extract the isoscalar combinations of nucleon electric and magnetic polarizabilities $\alpha_N \equiv (\alpha_p + \alpha_n)/2$ and $\beta_N \equiv (\beta_p + \beta_n)/2$ [19]. One such experiment is underway at the MAX-Lab facility at Lund, and will significantly increase the world data-base on the γ d reaction [21]. When this new data is used in concert with a new, precision χ ET calculation of γ d it should yield an extraction of $\alpha_N - \beta_N$ with an accuracy comparable to that with which $\alpha_p - \beta_p$ is presently known. This will provide important constraints on the interplay between the pion-cloud mechanisms that generate the dominant piece of the nucleon's electric polarizability and other mechanisms that contribute to α_N and β_N .

Recently it has been pointed out that elastic Compton scattering from the Helium-3 nucleus also provides access to information on neutron polarizabilities [22]. In this case the presence of two protons in the nucleus significantly enhances the Compton cross section. It also enhances (in absolute terms) the impact of α_n and β_n on observables, because in coherent $\gamma^3\text{He}$ scattering the polarizability effects in the single-nucleon Compton amplitude interfere with *two* proton Thomson terms.

We have performed χ ET calculations of $\gamma^3\text{He}$ scattering at $\mathcal{O}(e^2P)$ [NLO] [22]. These calculations employ the same operator \hat{O} as was used for γ d scattering in Ref. [15], as well as a variety of χ ET three-nucleon wave functions that are consistent with \hat{O} at this order in χ ET. This is the first consistent χ ET calculation of an electromagnetic reaction on the three-body system (but see also Ref. [23] for static electromagnetic properties of the trinucleons) and—so far as I am aware—the first calculation of $\gamma^3\text{He}$ scattering. Once again, χ ET makes a prediction for Compton observables at this order, with the impact of α_n and β_n on the dcs shown in Fig. 3.

We can also predict the asymmetries that would be obtained were circularly polarized photons to scatter from Helium-3 nuclei polarized parallel (Σ_z) or perpendicular (Σ_x) to the incoming beam. In the Helium-3 nucleus, the two protons are predominantly in a 1S_0 state, and so the double-polarization observables are dominated by the contribution from the neutron

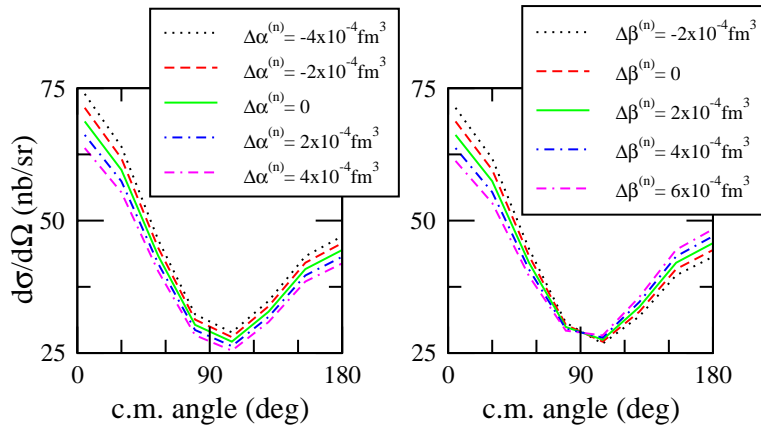


Figure 3: Centre-of-mass frame differential cross section for $\gamma^3\text{He}$ scattering at 80 MeV as predicted at $\mathcal{O}(e^2P)$, with the addition of shifts in α_n (left panel) and β_n (right panel). Taken from Ref. [22].

that is (mostly) carrying the spin of the polarized Helium-3. For photon energies above 100 MeV we find significant sensitivity in Σ_z and Σ_x to neutron spin polarizabilities [24]. Experiments that will measure these asymmetries are planned for the HI γ S facility at the Triangle Universities Nuclear Laboratory [25], and will provide important new constraints on low-energy neutron spin structure.

4 Other reactions on light nuclei in χ ET: briefly

The dynamics of mesons and nucleons that is the focus of these meetings has significant consequences for nuclear physics. In particular, the πN interactions that are encoded in χ PT are now being used as the basis for a quantitative understanding of few-nucleon systems. In this talk I have discussed only the portion of this understanding that pertains to electromagnetic reactions. But the χ ET approach to nuclear dynamics has also had significant success in describing both elastic scattering and breakup reactions in neutron-deuteron and proton-deuteron experiments [26]. For comprehensive reviews of the application of χ ET to few-nucleon systems see Refs. [27].

Meanwhile, χ ET has also been used with significant success in understanding low-energy weak processes [28]. The fact that the chiral Lagrangian provides a connection between these reactions and pionic processes such as $\pi^- d \rightarrow nn\gamma$ [29] is now being exploited to yield new, more precise calculations of the latter process [30]. Furthermore, several contributions to this conference described progress in pionic reactions on deuterium [31]. The re-

sulting calculations are evidence of the power of a description which allows us to trace the consequences of QCD's chiral symmetry and the pattern of its breaking through into predictions for experiments involving light nuclei.

Acknowledgments

I thank the organizers of MENU2007 for a very stimulating and efficiently run meeting. I am also grateful for enjoyable and educative collaborations with those who worked with me on the research described here. Responsibility for any aberrant views and/or omissions in this paper is, however, entirely mine. This work was supported by DOE grant DE-FG02-93ER40756.

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