

STUDYING PION EFFECTS IN THE QUARK PROPAGATOR

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Abstract

Within the framework of Schwinger-Dyson and Bethe-Salpeter equations we investigate the importance of pions for the quark-gluon interaction. To this end we choose a truncation for the quark-gluon vertex that includes intermediate pion degrees of freedom and adjust the interaction such that unquenched lattice results for various current quark masses are reproduced. The corresponding Bethe-Salpeter kernel is constructed from constraints by chiral symmetry. After extrapolation to the physical point we find a considerable contribution of the pion back reaction to the quark mass function as well as to the chiral condensate. The quark wave function is less affected.

1 Introduction and framework

Dynamical chiral symmetry breaking is one of the most striking properties of low-energy QCD. The resulting appearance of massless pions in the chiral limit is the basis of a systematic expansion for hadronic observables in terms of chiral perturbation theory. The difference between quenched and unquenched simulations in lattice QCD is also attributed to the importance of the pions' dynamics. Here we study the pion contribution and back reaction on the quark degrees of freedom in the Green's function approach to Landau gauge QCD using Schwinger-Dyson and Bethe-Salpeter equations (SDE/BSE) [?, 1]. We consider this as an extension of previous NJL-model studies [3] to full QCD and to be complementary to previous studies within the SDE/BSE approach neglecting the back reaction of pions [?, 4].

Our starting point is the SDE of the quark propagator

$$S^{-1}(p) = Z_2 S_0^{-1}(p) + \Sigma(p), \quad (1)$$

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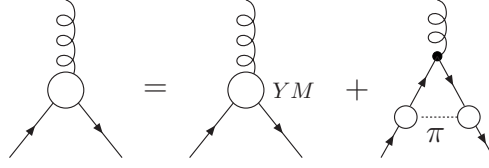


Figure 1: The approximated Schwinger-Dyson equation for the quark-gluon vertex. All internal propagators are fully dressed.

where $S_0^{-1}(p) = ip \cdot \gamma + m$ denotes the inverse bare quark-propagator, while $S^{-1}(p) = (ip \cdot \gamma + M(p^2))/Z_f(p^2)$ is the dressed propagator being parameterized by the quark mass $M(p^2)$ and the quark wave function $Z_f(p^2)$. Z_2 is the renormalization factor of the quark field. The quark self energy in Landau gauge is given by

$$\Sigma(p) = g^2 C_F Z_{1F} \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu S(q) \Gamma_\nu(q, k) \frac{Z(k^2)}{k^2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right), \quad (2)$$

with $k = p - q$, the Casimir $C_F = (N_c^2 - 1)/(2N_c)$ and the renormalization factor Z_{1F} of the quark gluon vertex. The self energy depends on the fully dressed quark-gluon vertex $\Gamma_\nu(q, k)$ and the gluon dressing function $Z(k^2)$.

The widely used rainbow-ladder approximation amounts to the replacement

$$\gamma_\mu Z(k^2) \Gamma_\nu(q, k) \rightarrow \gamma_\mu \Gamma(k^2) \gamma_\nu, \quad (3)$$

where $\Gamma(k^2)$ can be viewed as a combination of the gluon dressing function and a purely k^2 -dependent dressing of the γ_ν -part of the quark-gluon vertex. Aiming at an extension of this approximation which includes explicit pion degrees of freedom, we can motivate the quark-gluon vertex diagrammatically shown in Fig. 1 by its SDE (see Ref. [6] for details). The main idea is to single out the leading contribution involving pions and approximate the remaining part as the vertex used in the rainbow-ladder approximation. Using this ansatz in the SDE of the quark propagator we can motivate - after intermediate steps - the truncation diagrammatically shown in Fig. 2. Guided by chiral symmetry constraints we construct a Bethe-Salpeter kernel that guarantees the pion to be massless in the chiral limit (see again Ref. [6] for details).

2 Results

Given the unquenched lattice QCD results for the quark propagator in Landau gauge at relatively large quark masses [7] and a parameterization for the

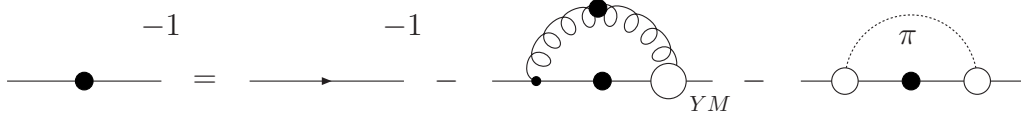


Figure 2: The approximated Schwinger-Dyson equation for the quark propagator with effective one-gluon exchange and one-pion exchange.

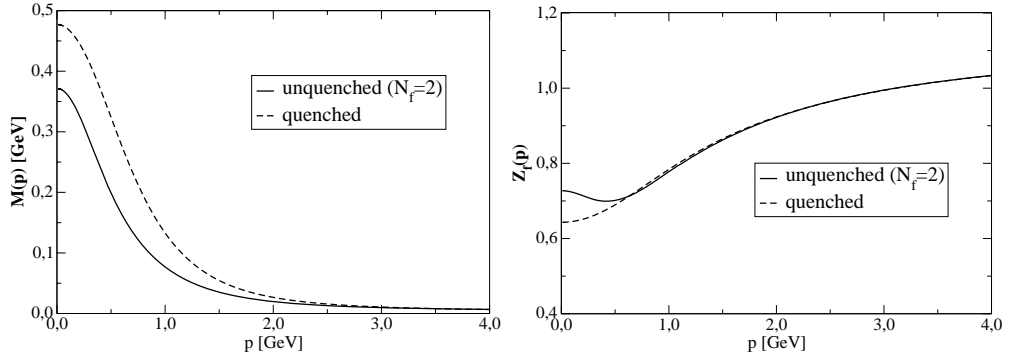


Figure 3: The quenched and unquenched ($N_f=2$) quark mass (left) and wave function (right) for physical up/down quarks with $M(2.9 \text{ GeV}) = 10 \text{ MeV}$.

dressings of the quark-gluon vertex in rainbow-ladder approximation adopted from similar investigations of quenched lattice QCD results [8], we can adjust the parameters of the rainbow-ladder contribution to our truncation of the quark-gluon vertex. We get nice agreement with the lattice QCD results for the quark mass function as well as for the wave function (see Ref. [6]). Switching off the pion contribution to the quark self-energy and furthermore comparing to unquenched lattice QCD results, we find the pion contribution to be overestimated. This may be due to a further numerical approximation and is detailed in Ref. [6].

The general tendency is however in line with the lattice QCD results and we can perform an extrapolation to physical value of the pion mass, *i.e.* current quark mass, and to the chiral limit. In Fig. 3 we also show the quark mass function and the wave function at the physical point. At small momenta the current quark mass dependence is most pronounced for the wave function, whereas the dependence of the mass function is strongest at intermediate momenta. To illustrate this we present the chiral extrapolation of the mass function at some momenta accessed in the lattice QCD simulations in Fig. 4. In contrast to the rainbow-ladder truncation, which gives an almost linear extrapolation, we find a sizeable non-linear behavior at small quark masses and momenta due to the pion contribution.

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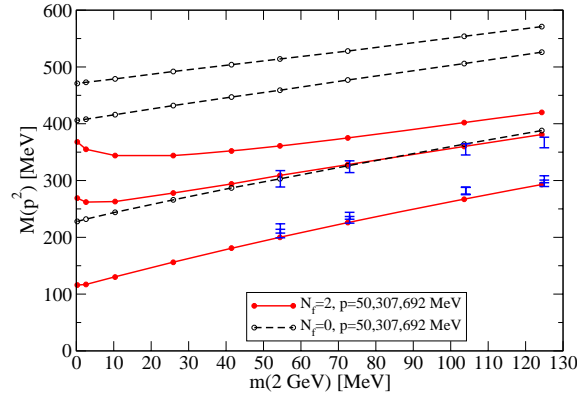


Figure 4: Chiral extrapolation of the mass function at stated momenta from bottom to top including the pion effects in the quark-gluon vertex (red) and neglecting it (black). Also given are lattice QCD results for the momenta corresponding to the touching red lines.

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