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RENORMALIZATION GROUP ANALYSIS OF THE CHIRAL PION PRODUCTION OPERATOR FOR $NN \to d\pi$

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Abstract

We are interested in the consistency between the cutoff, chiral symmetry, and the power counting. For this purpose, we apply the Wilsonian renormalization group (RG) to an operator and then decrease the Wilsonian cutoff. As an example, we study the s-wave pion production operator for $NN \to d\pi$, derived in chiral perturbation theory. We find that the renormalized part of the RG effective operator is accurately absorbed by chiral counter terms of higher order with natural coefficients. Thus, the use of the (sharp) cutoff regularization does not require us to introduce chiral-symmetry-breaking counter terms, at least in the case of the $NN \to d\pi$ reaction.

1 Introduction

Chiral symmetry and its spontaneous breaking are important low-energy properties of QCD, and therefore it is natural to hope to incorporate them into the description of nuclear systems. A promising solution is to employ the chiral perturbation theory (χPT), which has been successfully used in nucleonic systems since the beginning of the 1990's. The nuclear operators are derived from a chiral Lagrangian following a counting rule. The transition matrix element is evaluated by convoluting the operator with nuclear wave functions. In the evaluation, a cutoff is usually applied to the operator, suppressing high momentum modes. The question then arises whether this regulated operator is still consistent with chiral symmetry. If not, we may need to introduce a chiral-symmetry-violating operator to recover the symmetry, or find another regularization scheme, which is consistent with the symmetry. Although it is conventionally supposed that effects of the high momentum modes are captured by higher order chiral counter terms, it is important to confirm this. We consider a renormalization group (RG) analysis to be useful for this purpose.

In this work, we apply the Wilsonian Renormalization group (WRG) equation to the s-wave pion production in $NN \to d\pi$ as described by χPT [1]. The cutoff is reduced using the WRG equation, and the operator runs as a result. In the next step, we try to reproduce the RG low-momentum operator using the starting operator plus chiral counter terms of higher order. If the chiral symmetry is not violated by the cutoff, the renormalized part of the operator should be accurately captured by the chiral counter terms.

2 Wilsonian RG equation for operator

At first, we discuss why we use the WRG equation to reduce the cutoff. This is based on a consistency with the construction of an effective Lagrangian. An effective Lagrangian can be obtained formally via a path integral formulation based on the Lagrangian of the underlying, more fundamental theory. One then integrates out the high energy degrees of freedom. When integrating out the high momentum states of the nucleon in the heavy-baryon χPT Lagrangian, we can also use the path integral. This procedure is equivalent to solving the WRG equation derived below. One of the present authors derived the WRG equation for the NN interaction in this way in Ref. [2]. The WRG equation for a transition operator (π production operator in our case) can also be derived in essentially the same way.

Here we outline a simpler derivation of the WRG equation following Ref. [3] in which a detailed account is given (Appendix A). We start with a matrix element in which the transition operator is defined in a model space spanned by plane wave states of the two-nucleon system. The maximum magnitude of the relative momentum in this model space is given by the cutoff Λ. We differentiate the matrix element with respect to Λ and impose the renormalization condition that the matrix element is invariant under cutoff changes. This gives the WRG equation for the low-momentum transition operator. (For explicit expressions, see Refs. [3,4].) This differential equation is solved, and the solution in integral form is actually the same as the effective operator in the Bloch-Horowitz formalism.

3 Results

We start with the chiral NLO s-wave pion production operator for the $NN \to d\pi$ reaction [1]. Using the integral form of the WRG equation, we calculate the RG low-momentum operator for $\Lambda = 500$ MeV, using the CD-Bonn NN-potential. We employ near threshold kinematics, *i.e.*, $\eta = 0.1$, where $\eta = q/m_\pi$ is the emitted pion momentum divided by the pion mass. We use the chiral counter terms of higher order {see Eq. (7) of Ref. [4] for expressions} to simulate the renormalized part of the low-momentum operator; the lowest chiral counter terms are NNLO.

The running of the radial part of the ${}^3P_1 \to {}^3S_1$ transition operator (diagonal matrix elements) is shown in Fig. 1. The solid line is the starting NLO chiral operator (before RG running). After the RG running, we obtain the RG low-momentum operator shown by the dashed line ($\Lambda = 1000$ MeV), dotted line ($\Lambda = 750$ MeV), and dash-dotted line ($\Lambda = 500$ MeV). We parameterize this low-momentum operator using the NLO operator plus the counter terms, omitting the kink part when fitting the counter terms. In Fig. 2 we show the simulation of the low-momentum operator using the NLO operator plus the counter terms. The dash-dotted line is the starting NLO chiral operator. The dashed line contains the NLO operator plus the lowest counter term. The dotted line contains the additional terms of one higher order. The renormalized part (the difference between the solid and dash-dotted lines) is thus accurately captured by the higher order counter terms of natural strength. As for the off-diagonal matrix elements, we find a similar trend in the simulation using the counter terms. Our result allows us to conclude that, to the level of precision (and order) we are working, the $NN \to d\pi$ calculation is renormalized without including a chiral-symmetry-violating counter term. However, this outcome does not necessarily mean that the (sharp) cutoff regularization does not generate *any* chiral-symmetry-violating operator. Nevertheless, we could still claim that there is no *practical* ground for introducing a chiral-symmetry-violating operator for this process.

4 Summary

We have studied what happens to the pion production operator when a cutoff is reduced. In particular, we are interested in the possibility that the running with cutoff requires us to introduce a chiral-symmetry-violating interaction. We started with the chiral NLO operator [1] for s-wave pion production in $NN \to d\pi$ and reduced a cutoff by using the WRG equation. After the RG running, we parameterized the RG low-momentum operator by a

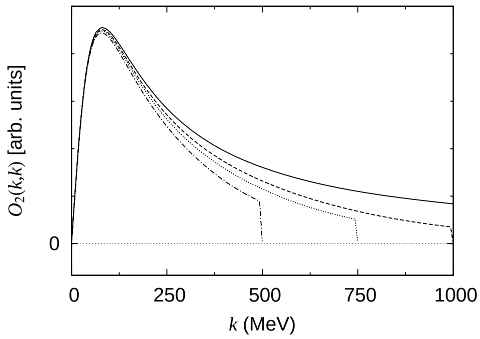

Figure 1: Running of the π production operator for $NN \to d\pi$ ($\eta = 0.1$, ${}^3P_1 \to {}^3S_1$). Solid line indicates the starting chiral NLO operator. RG running results in the low-momentum operators for $\Lambda = 1000$ MeV (dashed line), 750 MeV (dotted line), and 500 MeV (dash-dotted line).

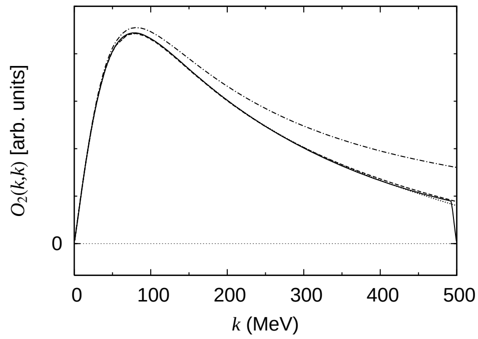

Figure 2: The RG low-momentum pion production operator ($\Lambda = 500$ MeV, $\eta = 0.1$, solid line) is simulated by the original NLO operator (dash-dotted line) plus a contact term with one nucleon derivative (dashed line). The dotted line has additional contact terms with three derivatives.

chiral expansion including higher-order counter terms, which are expected to absorb contributions from the high momentum states that were integrated out. We found that the expansion is indeed accurate, with the LECs of natural strength. Therefore, we see no evidence that the running with the cutoff reduction generates chiral-symmetry-violating interactions.

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