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# BRIDGING OVER $p$-WAVE $\pi$-PRODUCTION AND WEAK PROCESSES IN FEW-NUCLEON SYSTEMS WITH CHIRAL PERTURBATION THEORY

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**Abstract**

We focus on a powerful aspect of chiral perturbation theory ($\chi$PT) which provides us with a "bridge" over different reactions. For this purpose, we study a contact operator which contributes $p$-wave $\pi$-production and several weak processes. We fix the unknown coupling of the contact operator using a matrix element of a low-energy weak process ($pp \to de^+\nu_e$), and then calculate a partial wave amplitude ($a_0$) for the $p$-wave $\pi$-production ($pp \to pn\pi^+$). We find that the chiral operator including the contact term does not reproduce $a_0$ extracted from data, showing that the bridging over reactions with significantly different kinematics is not necessarily successful. We argue the importance of a specific higher order calculation. In order to gain an insight into a higher order calculation, we consider a higher order counter term, and find that the energy dependence of $a_0$ is then consistent with the data.

# 1 Introduction

Since the beginning of 1990's, chiral perturbation theory ($\chi$PT) has been extensively applied to few-nucleon system. An advantageous point inherent in $\chi$PT is that it bridges over different reactions in a model-independent manner. Once couplings (the so-called low-energy constant, LEC) included in a chiral operator are fixed using data for one of the reactions, then the other reactions are predicted using the same operator. An interesting interaction in this context is, $\mathcal{L} = \tilde{d} N^\dagger S \cdot u N N^\dagger N$ , with $f_\pi u_\mu = -\tau_a \partial_\mu \pi_a - \epsilon_{3ab} V_\mu \pi_a \tau_b + f_\pi A_\mu + \cdots$. The spin operator is $S$, and the external vector (axial) current is

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$V_\mu$ ($A_\mu$). The constant $\tilde{d}$ is a LEC. This contact interaction contributes to the three-nucleon force, and several processes such as the $p$-wave $\pi$-production ($pp \to pn\pi^+$ [1]), and weak processes (tritium $\beta$-decay, $pp \to de^+\nu_e$ [2]). We can fix $\tilde{d}$ using one of the above reactions, and predict the others.

Such a "bridging program" has been done in several works. One of them was done by Park *et al.* [2], where they fixed $\tilde{d}$ using the experimental tritium $\beta$-decay rate, and performed a parameter-free calculation of the weak proton capture by a proton (or $^3$He). Another work was due to Hanhart *et al.* [1]. The authors calculated a partial wave amplitude ($a_0$) for the $p$-wave $\pi$-production ($pp \to pn\pi^+$). Even though this work was not a fully consistent bridging program, they showed that the use of $\tilde{d}$ fixed by three-nucleon observables consistently reproduces $a_0$ extracted from data [3].

In this work (see Ref. [4] for a full account), we investigate more seriously how reliably the bridging program, an important aspect of $\chi$PT, works. We believe that our investigation is important because we have sometimes seen an argument which supposes, without a serious test, that the bridging program works. For this purpose, we calculate a partial wave amplitude ($a_0$) for the $p$-wave $\pi$-production ($pp \to pn\pi^+$), with $\tilde{d}$ fixed by a low-energy weak process. This obviously provides a stringent test of $\chi$PT, because the two reactions are strong and weak processes, and are quite different in kinematics.

## 2 Chiral $p$-wave $\pi$-production operator

We use the following $\pi$-production operators by referring to Ref. [1] in which the operators were derived using a counting rule based on an expansion parameter, $\sqrt{m_\pi/m_N}$; the nucleon (pion) mass is denoted by $m_N$ ($m_\pi$). The leading order (LO, $\mathcal{O}(1)$) operator is the one-body direct production of the pion off the nucleon. Another LO mechanism we consider is the $\Delta$-excitation followed by the $\pi$ emission. In Ref. [1], the authors used the wave function which explicitly includes the $\Delta$ component, and considered the one-body operator which produces the pion with the $\Delta$ deexcited to the nucleon. Because we use nuclear wave functions with only the nucleonic degrees of freedom, we alternatively use a two-body operator in which the $\Delta$ is excited either by the $\pi$-exchange or by a contact interaction. Next we discuss next-to-leading order (NLO, $\mathcal{O}(m_\pi/m_N)$) terms: the recoil correction to the LO terms; a pion rescattering through the vertices whose strength are $c_3$ and $c_4$, or through the Weinberg-Tomozawa term or its Galilean correction; a pion emission from the contact term whose coupling constant is $\tilde{d}$. The unknown coupling, $\tilde{d}$, will be determined in the next paragraph.

We start with a benchmark calculation. We employ the same axial current

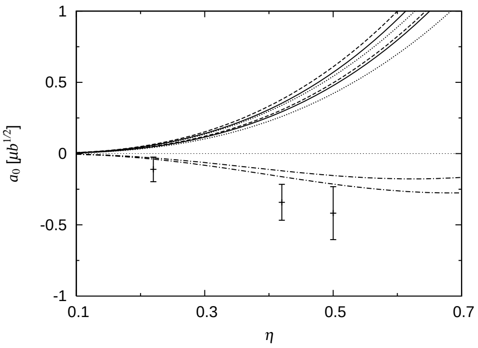


Figure 1: The amplitude $a_0$ for $pp \to pn\pi^+$. The upper (lower) solid, dashed and dotted curves correspond to $\Lambda = 500$, 600 and 800 MeV, respectively, $h_A = 2.10$ (2.68). The lower (upper) dash-dotted curve is obtained with $\tilde{d} = 0$, $\Lambda = 800$ MeV, $h_A = 2.10$ (2.68). Data are from Ref. [3].

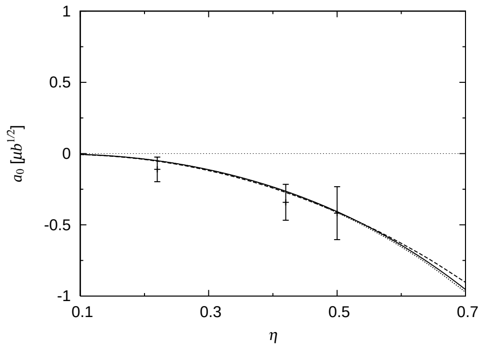


Figure 2: The amplitude $a_0$ for $pp \to pn\pi^+$. The chiral NLO $\pi$ production operator plus the $\tilde{e}$-term is used. The solid, dashed and dotted curves correspond to $\Lambda = 500$, 600 and 800 MeV, respectively, and $h_A = 2.10$. The other features are the same as Fig. 1.

operator used in Ref. [2]. The $\tilde{d}$ value in this operator has been fixed using the tritium $\beta$-decay rate. We calculate the Gamow-Teller matrix element for the low-energy $pp \to de^+\nu_e$ reaction and regard it as the benchmark. Our axial current is different from Ref. [2] in that we consider the $\Delta$ explicitly. We calculate the matrix element using our operator and fit $\tilde{d}$ to the benchmark result. We use several combinations of the $\pi N\Delta$ coupling ($h_A$), the $NN$ potential and the cutoff ($\Lambda$), and obtain the corresponding $\tilde{d}$ values with a natural strength.

# 3 Result

With the chiral operator fixed in the previous section, we calculate $a_0$ for $pp \to pn\pi^+$. In Fig. 1, we present $a_0$ as a function of $\eta \equiv q_\pi^{max}/m_\pi$, where $q_\pi^{max}$ is the maximum pion momentum. We used the CD-Bonn $NN$ potential and two (three) choices of $h_A$ ($\Lambda$). Our result is rather different from the data. For comparison, we show $a_0$ obtained without the $\tilde{d}$ term. The inclusion of the $\tilde{d}$ term makes the disagreement worse; even the sign of $\tilde{d}$ fixed by the low-energy weak process is inconsistent with the data. We change the values of $\Lambda$, $h_A$ and the $NN$ potential, however, the situation of the disagreement does not change. This result shows that the bridging program among reactions with significantly different kinematics is not necessarily successful.

The failure of the bridging program is understandable if we recall the success of the chiral nuclear force which describes the $NN$ scattering in a wide energy region. This is partly because the LECs have been fitted to data from the same energy region. In order to accurately describe the two reactions in different energy regions, data from the both energy region are necessary to fix the LECs. It is also expected that higher order terms are necessary to accurately reproduce the data from the wide energy region.

To explore, even roughly, a higher order calculation, we perform a simple extension of the previous calculation by adding $\mathcal{L}_{\rm CT}^{(2)} = \tilde{e} N^\dagger \boldsymbol{\tau}\sigma\cdot\nabla\boldsymbol{\pi} N\, (N^\dagger \nabla^2 N +$ h.c.) to the NLO operator. Now we have the two independent LECs: $\tilde{d}$ and $\tilde{e}$. We fix these two LECs to reproduce two quantities: the benchmark result for the matrix element of $pp \to de^+\nu_e$; $a_0$ for $pp \to pn\pi^+$ at $\eta = 0.5$ from data. We obtain a set of $\tilde{d}$ and $\tilde{e}$ with a natural strength in this way. We calculate $a_0$ with the extended operator and show the result in Fig. 2. By construction, the calculated $a_0$ goes through the central value of the data at $\eta = 0.5$. We examined the dependence of $a_0$ on $h_A$, $\Lambda$ and the $NN$ potential, and found that the $\eta$ dependence of $a_0$ is consistent with the data.

# 4 Summary

We determined $\tilde{d}$, the LEC of the contact term, using the weak process and then used it to predict $a_0$ for $pp \to pn\pi^+$. Through this work, we tried to explore the power of $\chi$PT that enables us to bridge different reactions. Our prediction of $a_0$ with the NLO operator does not agree with the data. The result indicates that the bridging program between reactions with significantly different kinematics is not always successful.

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