

# SCALAR MESONS: A CHIRAL LAGRANGIAN FRAMEWORK FOR THEIR MIXING AND SUBSTRUCTURE

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## Abstract

The highlights of studies of mixing among scalar mesons below and above 1 GeV within a nonlinear chiral Lagrangian framework is briefly presented. Two scalar meson nonets are introduced to explore the mass spectrum and decay properties of the  $I=1/2$  and  $I=1$  scalar states. For the  $I=0$  states, in addition to these two nonets a scalar glueball component is also taken into account, and together with the constraints from the  $I=1/2$  and  $I=1$  sectors, their mass spectrum is studied. The fact that an ideally mixed  $q\bar{q}$  scalar nonet has a mass ordering which is opposite to that of an ideally mixed four-quark scalar nonet is exploited to gain some insight into the quark substructure of the  $I=1/2$ ,  $I=1$  and  $I=0$  states below and above 1 GeV. Consequently, numerical estimates of various components of these states (two quark and four quark components of  $I=1/2$  and  $I=1$  states, and two quark, four quark and glue component of  $I=0$  states) are determined.

Scalar states below and above 1 GeV are shown in Fig. 1, and are all listed/discussed in PDG [1]. Not all of these states are well-established: Among these the  $f_0(600)$  [or  $\sigma$ ] and the  $f_0(1370)$  have large uncertainties on their mass and decay widths, as well as the  $K_0^*(800)$  [or  $\kappa$ ] which has been particularly under a special scrutiny and debate. It is now generally believed that the states below 1 GeV are something other than pure  $q\bar{q}$  states, as opposed to those above 1 GeV which have been the favored candidates for a  $q\bar{q}$  nonet, even though some of their properties do not quite follow a  $q\bar{q}$  assignment. Possible solutions for the status of the lowest-lying scalar states include the MIT bag model,  $K\bar{K}$  molecule and unitarized quark model, as well as many recent investigations (see [2] for a selection of refs.). There are reasons to investigate the mixing between the scalar mesons below and above

1 GeV. First, intuitively this is not inconceivable as some of these states [such as  $f_0(600)$  and  $f_0(1370)$  as well as  $K_0^*(800)$ ] are broad and their masses spread over a wide range, therefore one may expect that some of their properties may overlap. Second, the available experimental data may already be pointing to such mixings. For example, a close look at some of the properties of the  $a_0(1450)$  and  $K_0^*(1430)$  [which are expected to be two members of the same  $q\bar{q}$  scalar meson nonet (see PDG [1])] shows surprising deviations from a  $q\bar{q}$  nonet properties. Clearly, their masses are rather puzzling [1]: If these two states belong to the same  $q\bar{q}$  nonet, then why should  $a_0(1450)$  (which does not contain a strange quark) be heavier than  $K_0^*(1430)$  (which does contain a strange quark)? There are also decay properties of these states that cannot be understood based on a pure  $q\bar{q}$  picture. As a possible solution, a description of the  $I = 1/2$  and  $I = 1$  scalar states below and above 1 GeV in terms of two nonets of scalars and within a nonlinear chiral Lagrangian framework was explored in ref. [3]. In that work, it was shown that if an underlying “bare” four-quark nonet  $N$  lies beneath an underlying “bare” two-quark nonet  $N'$ , then as a result of mixing of  $N$  and  $N'$  we can easily understand why  $a_0(1450)$  becomes heavier than  $K_0^*(1430)$  (in addition, the decay properties of these states can be understood in this scenario). Fig. 1 shows how this mechanism works. It was also found in [3] that the  $I = 1$  states are close to equal admixtures of two and four-quark states, whereas the  $I = 1/2$  states are less mixed, with  $K_0^*(800)$  containing close to 75% four-quark and 25% two-quark [and vice versa for  $K_0^*(1430)$ ]. What does this scenario say about the  $I = 0$  states? This question was studied in [4] in which the implications of such underlying mixing of nonets  $N$  and  $N'$  on the  $I = 0$  states was investigated. Fig. 1 summarizes the results and shows how the  $I = 0$  states originate from the four-quark nonet  $N$ , two-quark nonet  $N'$  and a scalar glueball  $G$ . The mass part of the Lagrangian for  $N$ ,  $N'$  and  $G$  is (in the leading order of mixing):

$$\begin{aligned} \mathcal{L}_{mass} = & -a\text{Tr}(NN) - b\text{Tr}(NN\mathcal{M}) - a'\text{Tr}(N'N') - b'\text{Tr}(N'N'\mathcal{M}) \\ & -c\text{Tr}(N)\text{Tr}(N) - d\text{Tr}(N)\text{Tr}(N\mathcal{M}) - c'\text{Tr}(N')\text{Tr}(N') - d'\text{Tr}(N')\text{Tr}(N'\mathcal{M}) \\ & -\gamma\text{Tr}(NN') - \rho\text{Tr}(N)\text{Tr}(N') - gG^2 - eG\text{Tr}(N) - fG\text{Tr}(N') \end{aligned} \quad (1)$$

in which  $\mathcal{M}$  is the usual quark mass spurion. The mass of the  $I = 1/2$  and  $I = 1$  states involve terms  $a$ ,  $b$ ,  $a'$ ,  $b'$  and  $\gamma$  only. The mass of  $I = 0$  states involve all 13 parameters. The mixing of  $I = 0$  states is clearly much more complicated and amounts to  $5 \times 5$  rotation matrices among  $N$ ,  $N'$  and  $G$ . The result of the numerical analysis of [4] for the prediction of the substructure of the  $I = 0$  states are given in Fig. 2, in which, in the middle, the dashed lines represent nonet  $N$  (that has a mass ordering consistent with an ide-

ally mixed four-quark nonet), the solid lines represent nonet  $N'$  (that has a mass ordering consistent with an ideally mixed two-quark nonet) and the box represents the scalar glueball predicted in this model. Identifying the components of the two bare nonets with the corresponding members of an ideally mixed four-quark nonet and ideally mixed two-quark nonet results in conclusion that the bare masses in nonet  $N$  are (from bottom to top):  $m(\bar{u}\bar{d}ud) = 0.83$  GeV,  $m(\bar{d}\bar{s}ud) = 1.06$  GeV,  $m[(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)/\sqrt{2}] = 1.24$  GeV; and the bare masses in nonet  $N'$  are (from bottom to top):  $m[(\bar{u}u + \bar{d}d)/\sqrt{2}] = 1.24$  GeV,  $m(\bar{u}s) = 1.31$  GeV and  $m(\bar{s}s) = 1.38$  GeV. The uncertainty of the glueball mass (shown by the height of the box, approximately between 1.5 GeV to 1.7 GeV) is due to the uncertainty of the input masses of  $f_0(600)$  and  $f_0(1370)$ . In Fig. 1, on the right, the  $I = 0$  physical states are shown, and the height of the two boxes represent the prediction of the present model for the uncertainties of the masses of  $f_0(600)$  and  $f_0(1370)$ , which are (in this model) approximately in ranges  $0.4 - 0.7$  GeV and  $1.3 - 1.45$  GeV, respectively. On the left, the  $I = 1/2$  and  $I = 1$  physical states are shown [note the level-crossing that explains the properties of  $a_0(1450)$  and  $K_0^*(1430)$ ]. The arrows show the dominant component of each physical state. Finally, the detailed numerical analysis of [4] predicts the substructure of the  $I = 0$  scalars (in terms of two quark, four quark and glueball components) which are given in Fig. 2.

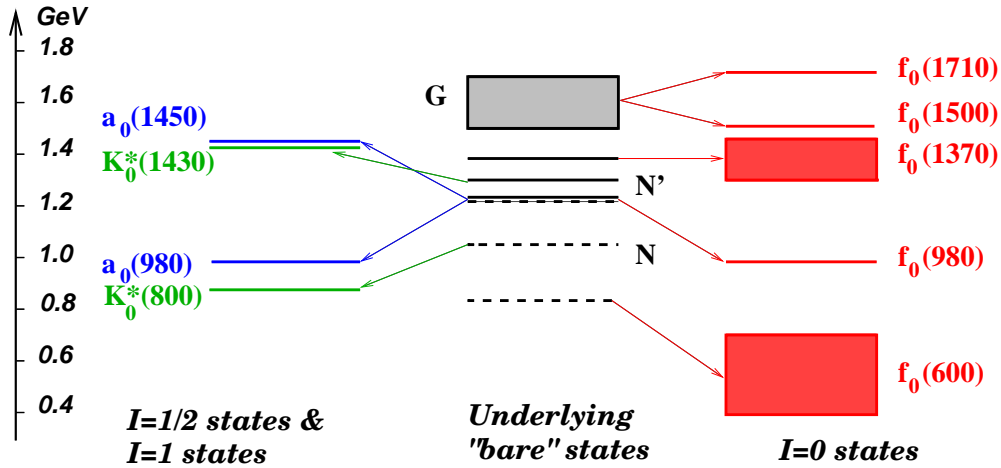


Figure 1: Prediction of the present model for the substructure of the  $I = 1/2$ ,  $I = 1$  scalar states below 2 GeV (left) and for the  $I = 0$  scalar states below 2 GeV (right) in terms of the underlying “bare” states (middle).

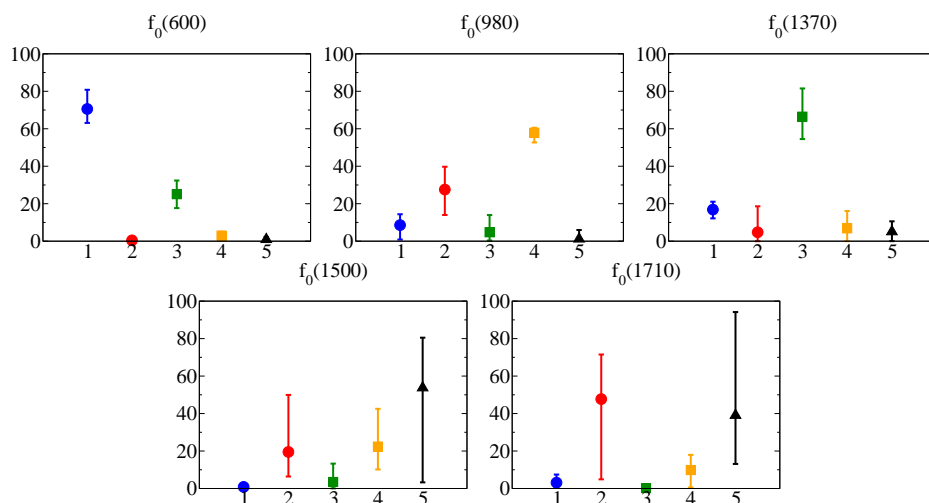


Figure 2: Components 1 to 5 respectively represent  $\bar{u}\bar{d}ud$ ,  $(\bar{s}\bar{d}ds + \bar{s}uus)/\sqrt{2}$ ,  $\bar{s}s$ ,  $(\bar{u}u + \bar{d}d)/\sqrt{2}$ , and glueball. The symbols represent the averaged values of each component and the error bars reflect the uncertainties of  $m^{\text{exp}}[f_0(600)]$  and  $m^{\text{exp}}[f_0(1370)]$ .

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