SCALAR MESONS: A CHIRAL LAGRANGIAN FRAMEWORK FOR THEIR MIXING AND SUBSTRUCTURE

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Abstract

The highlights of studies of mixing among scalar mesons below and above 1 GeV within a nonlinear chiral Lagrangian framework is briefly presented. Two scalar meson nonets are introduced to explore the mass spectrum and decay properties of the $I=1/2$ and $I=1$ scalar states. For the $I=0$ states, in addition to these two nonets a scalar glueball component is also taken into account, and together with the constraints from the $I=1/2$ and $I=1$ sectors, their mass spectrum is studied. The fact that an ideally mixed $q\bar{q}$ scalar nonet has a mass ordering which is opposite to that of an ideally mixed four-quark scalar nonet is exploited to gain some insight into the quark substructure of the $I=1/2$, $I=1$ and $I=0$ states below and above 1 GeV. Consequently, numerical estimates of various components of these states (two quark and four quark components of $I=1/2$ and $I=1$ states, and two quark, four quark and glue component of $I=0$ states) are determined.

Scalar states below and above 1 GeV are shown in Fig. 1, and are all listed/discussed in PDG [1]. Not all of these states are well-established: Among these the $f_0(600)$ [or $\sigma$] and the $f_0(1370)$ have large uncertainties on their mass and decay widths, as well as the $K_0^*(800)$ [or $\kappa$] which has been particularly under a special scrutiny and debate. It is now generally believed that the states below 1 GeV are something other than pure $q\bar{q}$ states, as opposed to those above 1 GeV which have been the favored candidates for a $q\bar{q}$ nonet, even though some of their properties do not quite follow a $q\bar{q}$ assignment. Possible solutions for the status of the lowest-lying scalar states include the MIT bag model, $K\bar{K}$ molecule and unitarized quark model, as well as many recent investigations (see [2] for a selection of refs.). There are reasons to investigate the mixing between the scalar mesons below and above 1 GeV.
1 GeV. First, intuitively this is not inconceivable as some of these states [such as $f_0(600)$ and $f_0(1370)$ as well as $K^*_0(800)$] are broad and their masses spread over a wide range, therefore one may expect that some of their properties may overlap. Second, the available experimental data may already be pointing to such mixings. For example, a close look at some of the properties of the $a_0(1450)$ and $K^*_0(1430)$ [which are expected to be two members of the same $q\bar{q}$ scalar meson nonet (see PDG [1])] shows surprising deviations from a $q\bar{q}$ nonet properties. Clearly, their masses are rather puzzling [1]: If these two states belong to the same $q\bar{q}$ nonet, then why should $a_0(1450)$ (which does not contain a strange quark) be heavier than $K^*_0(1430)$ (which does contain a strange quark)? There are also decay properties of these states that cannot be understood based on a pure $q\bar{q}$ picture. As a possible solution, a description of the $I = 1/2$ and $I = 1$ scalar states below and above 1 GeV in terms of two nonets of scalars and within a nonlinear chiral Lagrangian framework was explored in ref. [3]. In that work, it was shown that if an underlying “bare” four-quark nonet $N$ lies beneath an underlying “bare” two-quark nonet $N'$, then as a result of mixing of $N$ and $N'$ we can easily understand why $a_0(1450)$ becomes heavier than $K^*_0(1430)$ (in addition, the decay properties of these states can be understood in this scenario). Fig. 1 shows how this mechanism works. It was also found in [3] that the $I = 1$ states are close to equal admixtures of two and four-quark states, whereas the $I = 1/2$ states are less mixed, with $K^*_0(800)$ containing close to 75% four-quark and 25% two-quark [and vice versa for $K^*_0(1430)$]. What does this scenario say about the $I = 0$ states? This question was studied in [4] in which the implications of such underlying mixing of nonets $N$ and $N'$ on the $I = 0$ states was investigated. Fig. 1 summarizes the results and shows how the $I = 0$ states originate from the four-quark nonet $N$, two-quark nonet $N'$ and a scalar glueball $G$. The mass part of the Lagrangian for $N$, $N'$ and $G$ is (in the leading order of mixing):

$$L_{\text{mass}} = -a \text{Tr}(NN) - b \text{Tr}(NN'M) - a' \text{Tr}(N'N') - b' \text{Tr}(N'N'M')$$

$$- c \text{Tr}(N')\text{Tr}(N) - d \text{Tr}(N')\text{Tr}(NM) - c' \text{Tr}(N')\text{Tr}(N') - d' \text{Tr}(N')\text{Tr}(N'M)$$

$$- \gamma \text{Tr}(NN') - \rho \text{Tr}(N')\text{Tr}(N') - gG^2 - eG\text{Tr}(N) - fG\text{Tr}(N')$$

in which $M$ is the usual quark mass spurion. The mass of the $I = 1/2$ and $I = 1$ states involve terms $a$, $b$, $a'$, $b'$ and $\gamma$ only. The mass of $I = 0$ states involve all 13 parameters. The mixing of $I = 0$ states is clearly much more complicated and amounts to $5 \times 5$ rotation matrices among $N$, $N'$ and $G$. The result of the numerical analysis of [4] for the prediction of the substructure of the $I = 0$ states are given in Fig. 2, in which, in the middle, the dashed lines represent nonet $N$ (that has a mass ordering consistent with an ide-
ally mixed four-quark nonet), the solid lines represent nonet $N'$ (that has a mass ordering consistent with an ideally mixed two-quark nonet) and the box represents the scalar glueball predicted in this model. Identifying the components of the two bare nonets with the corresponding members of an ideally mixed four-quark nonet and ideally mixed two-quark nonet results in conclusion that the bare masses in nonet $N$ are (from bottom to top): $m(\bar{u}dud) = 0.83 \text{ GeV}$, $m(\bar{d}ss) = 1.24 \text{ GeV}$; and the bare masses in nonet $N'$ are (from bottom to top): $m[(\bar{u}u + \bar{d}d)/\sqrt{2}] = 1.24 \text{ GeV}$, $m(\bar{u}s) = 1.31 \text{ GeV}$ and $m(\bar{s}s) = 1.38 \text{ GeV}$. The uncertainty of the glueball mass (shown by the height of the box, approximately between 1.5 GeV to 1.7 GeV) is due to the uncertainty of the input masses of $f_0(600)$ and $f_0(1370)$. In Fig. 1, on the right, the $I = 0$ physical states are shown, and the height of the two boxes represent the prediction of the present model for the uncertainties of the masses of $f_0(600)$ and $f_0(1370)$, which are (in this model) approximately in ranges $0.4 - 0.7 \text{ GeV}$ and $1.3 - 1.45 \text{ GeV}$, respectively. On the left, the $I = 1/2$ and $I = 1$ physical states are shown [note the level-crossing that explains the properties of $a_0(1450)$ and $K^*_0(1430)$]. The arrows show the dominant component of each physical state. Finally, the detailed numerical analysis of [4] predicts the substructure of the $I = 0$ scalars (in terms of two quark, four quark and glueball components) which are given in Fig. 2.

![Figure 1](https://example.com/figure1.png)

**Figure 1:** Prediction of the present model for the substructure of the $I = 1/2$, $I = 1$ scalar states below 2 GeV (left) and for the $I = 0$ scalar states below 2 GeV (right) in terms of the underlying “bare” states (middle).
Figure 2: Components 1 to 5 respectively represent $\bar{u}d$, $(\bar{s}d + \bar{s}u)/\sqrt{2}$, $\bar{s}s$, $(\bar{u}u + \bar{d}d)/\sqrt{2}$, and glueball. The symbols represent the averaged values of each component and the error bars reflect the uncertainties of $m_{\text{exp}}[f_0(600)]$ and $m_{\text{exp}}[f_0(1370)]$.

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References


