MENU 2007
11th International Conference
on Meson-Nucleon Physics and
the Structure of the Nucleon
September10-14, 2007
IKP, Forschungzentrum Jülich, Germany

S-WAVE MESON SCATTERING UP TO $\sqrt{s} \lesssim 2$ GeV FROM CHIRAL LAGRANGIANS

M. Albaladejo[⋆,1], J.A. Oller[⋆,2], C. Piqueras[†]

[⋆] Departamento de Física, Universidad de Murcia, E-30071, Murcia, Spain
[†] I.E.S. Ribera de los Molinos, E-30170 Murcia, Spain

Abstract

The problem of scalar mesons still remains a challenging puzzle, for which we do not even know which are the right pieces to set up. The proliferation of resonances (some of them are very broad and appear on top of hadronic thresholds) and of coupled channels that interact strongly among each other makes the study of this sector a hard task. Our objective is the study of the strongly interacting mesons in coupled channels with quantum numbers $J^{PC} = 0^{++}$ and $I = 0$ and $I = 1/2$, up to a center of mass energy $\sqrt{s} \lesssim$ 2 GeV. Our framework is based on Unitary Chiral Perturbation Theory. We include for $I = 0$ the channels: $\pi\pi$, $K\overline{K}$, $\eta\eta$, $\sigma\sigma$, $\eta\eta'$, $\rho\rho$, $\omega\omega$, $\eta'\eta'$, $\omega\phi$, $\phi\phi$, $K^*\overline{K}^*$, $a_1(1260)\pi$ and $\pi^\star(1300)\pi$. In addition, and in order to constrain our fits, we also study the $I = 1/2$, $3/2$ channels given by $K\pi$, $K\eta$ and $K\eta'$. We finally present the resonant content of our fits with the σ, $f_0(980)$, $f_0(1310)$, $f_(1500)$, $f_0(1710)$ and $f_0(1790)$.

1 Lagrangians. $U(3)$ symmetry

Due to the spontaneous breakdown of chiral $SU(3)$ symmetry the π, K and η are the octet of pseudo-Goldstone bosons. As it is well known, chiral symmetry strongly constrains the allowed interactions between these pseudoscalars and it is a basic ingredient in any study of strong interactions involving those mesons. If one considers higher energy regions, as it is our case here where we study the $I = 0$ and $1/2$ S-waves up to about 2 GeV, one also needs to take into account the $\eta\eta$, $\eta\eta'$ and $\eta'\eta'$ channels. Interestingly, in the large N_c limit, the η_1 becomes the ninth Goldstone boson. This fact can be used to

[1] E-mail address: albaladejo@um.es
[2] E-mail address: oller@um.es

settle down chiral Lagrangians based on $U(3)$ chiral symmetry and to include the η_1 field. The η_1-η_8 mixing angle is taken as $\sin\theta = -1/3 \to \theta \approx -20°$. The channels we include for $I=0$ are: (1) $\pi\pi$, (2) $K\overline{K}$, (3) $\eta\eta$, (4) $\sigma\sigma$, (5) $\eta\eta'$, (6) $\rho\rho$, (7) $\omega\omega$, (8) $\eta\eta'$, (9) $\omega\phi$, (10) $\phi\phi$, (11) $K^*\overline{K}^*$, (12) $a_1(1260)\pi$ and (13) $\pi^\star(1300)\pi$. For $I=1/2$ and $3/2$ we take the $K\pi$, $K\eta$ and $K\eta'$ ones. For these latter isospin channels we follow ref. [1] .

We employ the Chiral Perturbation Theory (ChPT) Lagrangians to lowest order, and the chirally invariant resonance ones [2], with the $J^{PC}=0^{++}$ singlet and octet multiplets. These Lagrangians also incorporate the r_μ and l_μ external sources by gauging the $U(3)_L\otimes U(3)_R$ chiral symmetry. In our case, as we are interested in the vector resonances, we have $r_\mu = l_\mu = g v_\mu$, where the last term is a matrix of vector fields times a coupling constant, which can be determined through the width of $\rho\to\pi\pi$, $g=4.23$.

2 Unitarization. $\sigma\sigma$ channels. Width effects

We want to calculate the amplitudes involving the $\sigma\sigma$ channel from our Lagrangians. As it was shown within UChPT [3] , the σ is made up from two pions interacting in $I=0$ S-wave, which allows us to obtain these amplitudes without any new free parameter. We first consider the amplitudes of a generic channel, say a, to four pions grouped as two $I=0$ $\pi\pi$ states ($(\pi\pi)_0$). Let us call s_1 and s_2 the total CM energy squared of each of these states. Now, every $(\pi\pi)_0$ state rescatters and gives rise to a σ pole; this is taken into account with the factor $1/D(s_1)D(s_2)$, where $D(s)=(1+t_2G(s))^{-1}$, t_2 is the elastic $(\pi\pi)_0$ S-wave amplitude at lowest order in ChPT, and $G(s)$ is the two-meson loop function. The transition amplitude is obtained by taking the limits $s_i\to s_\sigma$, with s_σ the σ pole position. If we denote the $a\to(\pi\pi)_0(\pi\pi)_0$ amplitude by $T_{a\to(\pi\pi)_0(\pi\pi)_0}$, and the one deduced from ChPT with resonances by $T^{2+R}_{a\to(\pi\pi)_0(\pi\pi)_0}$, we have:

$$T_{a\to(\pi\pi)_0(\pi\pi)_0} = T^{2+R}_{a\to(\pi\pi)_0(\pi\pi)_0}\frac{1}{D(s_1)D(s_2)}. \qquad (1)$$

The $a\to(\sigma\sigma)_0$ amplitude, $N_{a\to(\sigma\sigma)_0}$, is obtained from,

$$\lim_{s_i\to s_\sigma} T_{a\to(\pi\pi)_0(\pi\pi)_0} = \lim_{s_i\to s_\sigma}\frac{T^{2+R}_{a\to(\pi\pi)_0(\pi\pi)_0}}{D_{II}(s_1)D_{II}(s_2)} = N_{a\to(\sigma\sigma)_0}\frac{g^2_{\sigma\pi\pi}}{(s_1-s_\sigma)(s_2-s_\sigma)} \qquad (2)$$

The subscript II means that we have to calculate the corresponding function on the second Riemann sheet, where the σ pole appears. Finally, calculating this limit with an appropriate Laurent expansion around s_σ,

$$N_{a\to(\sigma\sigma)_0} = T^{2+R}_{a\to(\pi\pi)_0(\pi\pi)_0} \left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2 \quad , \quad \left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2 \simeq 9.1\cdot 10^{-3}\ \mathrm{GeV}^2\ . \quad (3)$$

Now, the general expression for a coupled channels partial wave amplitude is $T = (I + N(s)g(s))^{-1}N(s)$, where $N(s)$ is a matrix containing our amplitudes between all the channels. Each element of the diagonal matrix $g(s)$ is given by the once subtracted dispersion relation,

$$g_i(s) = g_i(s_0) - \frac{s-s_0}{8\pi^2}\int_{s_{\rm th,i}}^{\infty} \mathrm{d}s' \frac{p_i(s')/\sqrt{s'}}{(s'-s_0)(s'-s+i\epsilon)} \quad (4)$$

A remark is in order. These integrals involve the masses of the particles of the scattering states, but some of them, as the σ, ρ, $a_1(1260)$ and $\pi^\star(1300)$ have very large widths. To take these effects into account, we consider instead of eq.(4) an integral of this loop function times a mass distribution over a wide range of masses for each of these unstable particles.

3 Results and spectroscopy

With all these amplitudes, one can construct the S-matrix and calculate observables; in our case, these will be phase shifts and amplitude moduli. The curves resulting from our fit are depicted in Fig. 1. We use 13 parameters for about 373 experimental data, and a fair agreement with data is achieved. We have reduced the number of free parameters compared with other approaches in the literature which do not employ (chiral) Lagrangians.

Once the observables are fitted, we can explore the $s-$complex plane to find the relevant poles of the amplitudes, and discuss their resonance content. We present in Table 1 the masses and widths of the resonances that we find. The agreement with the ones in the PDG is remarkable.

References

[1] M. Jamin, J. A. Oller and A. Pich, Nucl. Phys. B **587** (2000) 331; M. Jamin, J. A. Oller and A. Pich, Nucl. Phys. B **622**, 279 (2002)

[2] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 465 (1985); G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321**, 311 (1989).

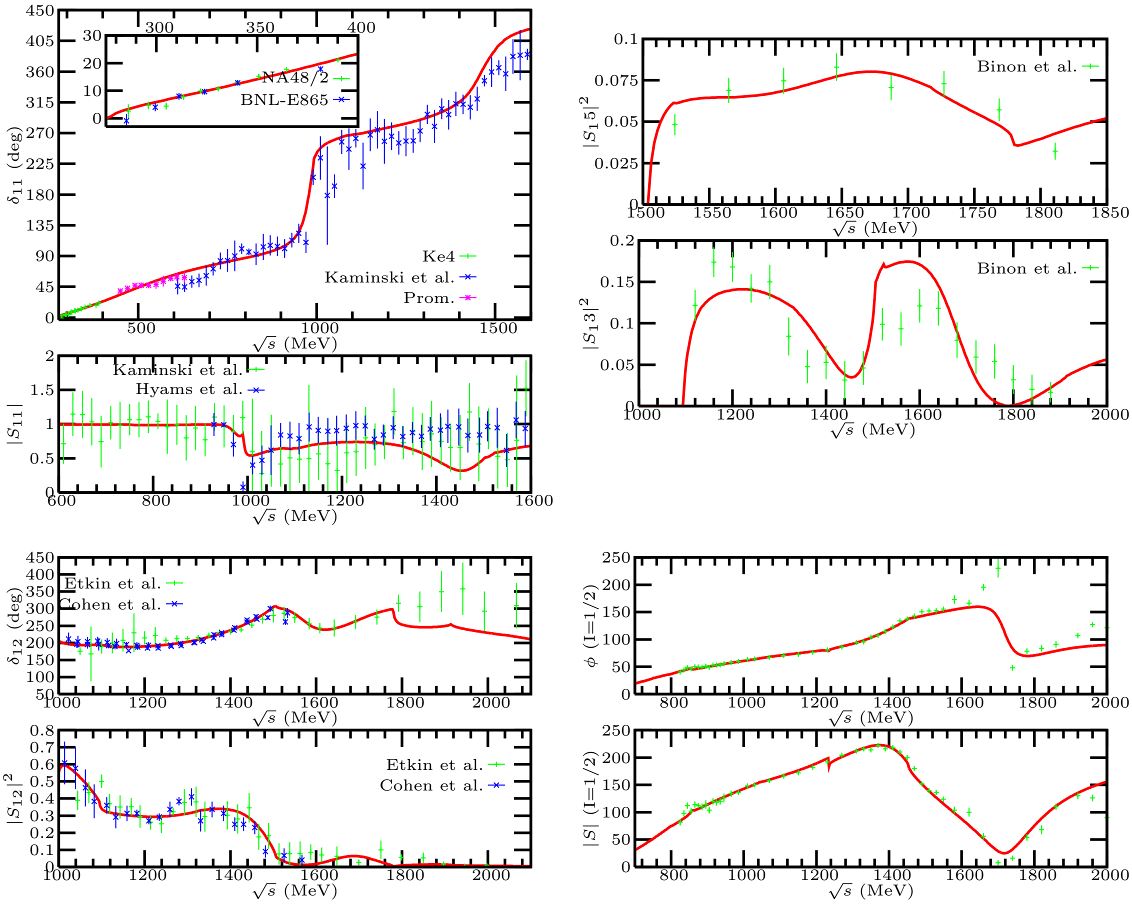

Figure 1: Left panels, from up to down: $\pi\pi \to \pi\pi$ phase shift, amplitude and the same for $\pi\pi \to K\bar{K}$. Right panels, from up to down: modulus squared of the S-matrix elements $\pi\pi \to \eta\eta'$ and $\pi\pi \to \eta\eta$. The last two figures correspond to the phase and modulus of the $K^-\pi^+ \to K^-\pi^+$ scattering.

[3] J. A. Oller and E. Oset, Nucl. Phys. A **620**, 438 (1997) [Erratum-ibid. A **652**, 407 (1999)]; J. A. Oller and E. Oset, Phys. Rev. D **60**, 074023 (1999)

Table 1: Parameters of resonances. On the left columns we have the masses and widths that we find. On the right ones, the values are given by the PDG or the BES Collaboration.

Resonance	Mass (MeV)	Width (MeV)	Mass (MeV)	Width (MeV)
σ	454	475	400-1200	600-1200
$f_0(980)$	980	44	980 ± 10	40-100
$f_0(1370)$	1380	350	1200-1500	200-500
$f_0(1500)$	≈ 1500	$100 - 170$	1507 ± 5	109 ± 7
$f_0(1710)$	≈ 1680	≈ 160	1718 ± 6	137 ± 8
$f_0(1790)$	≈ 1805	≈ 390	1790^{+40}_{-30}	270^{+60}_{-30}