

# $\Lambda(1405)$ AS A RESONANCE IN THE BARYON-MESON SCATTERING COUPLED TO THE $q^3$ STATE IN A QUARK MODEL

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## Abstract

To describe  $\Lambda(1405)$  as a resonance in the baryon-meson scattering by a quark model, we have investigated  $q^3$ - $q\bar{q}$  scattering systems with the flavor-singlet  $q^3$  ( $0s$ )<sup>2</sup>( $0p$ ) state, which is treated as a bound state embedded in the continuum. We found that the peak appears below the  $N\bar{K}$  threshold in the spin  $\frac{1}{2}$ , isospin 0 channel with an appropriate width. We also compare the mechanism to form such a resonance with that of the chiral-unitary-type model.

## 1 Introduction

Constituent quark models are known to reproduce most of the observed low-energy features of baryons. The picture that the baryon mass comes mainly from the masses of the three constituent quarks, which interact with each other by exchanging gluons and/or Goldstone bosons, seems appropriate [1, 2]. There, however, are some exceptions. For example, such quark models cannot give the observed light mass of  $\Lambda(1405)$  nor the large mass difference between  $\Lambda(1405, \frac{1}{2}^-)$  and  $\Lambda(1520, \frac{3}{2}^-)$ . In the conventional constituent quark model, each of these baryons is treated as a system of three quarks in the flavor-singlet ( $0s$ )<sup>2</sup>( $0p$ ) state. Then, the hyperfine interaction only gives a part ( $\sim 150$  MeV) of the observed mass difference between the flavor-singlet and the flavor-octet spin- $\frac{1}{2}$  baryons,  $\sim 200$  MeV. Moreover, in order to give the large mass difference between  $\Lambda(1405)$  and  $\Lambda(1520)$ , one has to assume a strong spin-orbit force between quarks, which is absent in other negative-parity baryons.

$\Lambda(1405)$  is a meson-baryon resonance found in the  $\Sigma\pi$  scattering state. To investigate this baryon as a negative-parity baryon embedded in the  $\Sigma\pi$  continuum is most appropriate. In ref [3], the idea that  $\Lambda(1405)$  can be a  $N\bar{K}$  bound state has been presented. A more systematic treatment may be found in the chiral unitary approach [4].

In the present work [5], we perform the  $\Sigma\pi$ - $N\bar{K}$ - $\Lambda\eta_{ud}$  coupled channel scattering calculation by the quark cluster model (QCM). Each baryon is treated as a  $q^3(0s)^3$  state, and the meson is assumed to be a  $q\bar{q}(0s)$  state. Moreover, we have introduced the conventional  $\Lambda^1$ , the flavor-singlet  $q^3(0s)^2 0p$  state, as a bound state embedded in the continuum (the  $\Lambda^1$  pole).

## 2 Model

In QCM, the wave function of a baryon-meson system is written as a product of the single baryon and meson wave functions,  $\phi_B$  and  $\phi_M$ , and the relative wave function between these two clusters,  $\chi(\mathbf{R}_{BM})$ :

$$\Psi = \mathcal{A}_q[ \phi_B \phi_M \chi(\mathbf{R}_{BM}) ]. \quad (1)$$

A baryon consists of three quarks and a meson consists of quark and an antiquark;  $\mathcal{A}_q$  in eq. (1) is an antisymmetrization operator among these four quarks. By integrating out the internal coordinates, we obtain the following RGM equation to determine the relative wave function:

$$\int [ H_{RGM}(\mathbf{R}, \mathbf{R}') - EN_{RGM}(\mathbf{R}, \mathbf{R}') ] \chi(\mathbf{R}') d\mathbf{R}' = 0, \quad (2)$$

where  $H [N]$  is the hamiltonian [norm] kernel.

The hamiltonian kernel can be obtained from the hamiltonian for quarks,  $H_q$ , which consists of the kinetic term, the one-gluon-exchange term (OGE), the confinement term (Conf), the collective term (Coll), and the instanton-induced interaction term (INS) [5]:

$$H_q = \sum_i \left( \frac{p_i^2}{2m_i} + V_0 \right) - K_G + V_{\text{Conf}} + V_{\text{OGE}} + V_{\text{Coll}} + V_{\text{INS}}. \quad (3)$$

In the present work, we assume that the coupling between the scattering state and the  $\Lambda^1$  pole comes from OGE, which can be written as:

$$\langle V_{3q-5q} \rangle = 6 \langle B^*(q^3)_{123} | V_{3;4\bar{5}} \frac{1}{\sqrt{4}} (1 - 2P_{24} - P_{34}) | B(q^3)_{123} M(q_4\bar{q}_5) \rangle \quad (4)$$

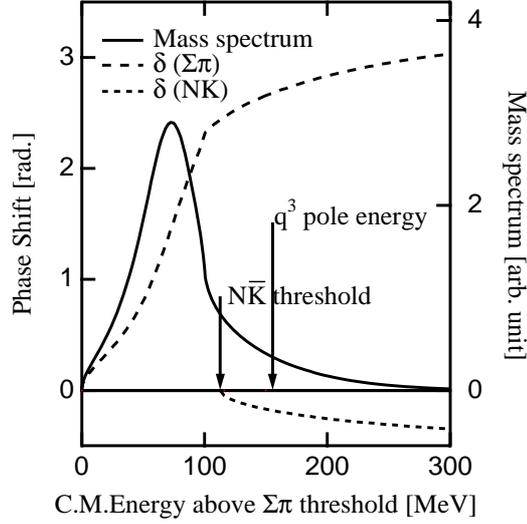


Figure 1: Mass spectrum and the phase shift ( $\delta$ ) of the  $\Sigma\pi$  and  $N\bar{K}$  coupled channel QCM.

with

$$V_{i;j\bar{k}} = \lambda_i \cdot \lambda_{\bar{k}j} \frac{\alpha_s \pi}{4 m_a^2} \left\{ \left( \frac{\mathbf{k}}{2m_a} - \frac{\mathbf{p}_i + \mathbf{p}'_i + i\boldsymbol{\sigma}_i \times \mathbf{k}}{2m_i} \right) \cdot \boldsymbol{\sigma}_{\bar{k}j} \right\} \delta_{\bar{k}j}^f, \quad (5)$$

where  $m_a$  is the quark mass in the annihilating  $q_j\bar{q}_{\bar{k}}$  pair,  $m_i$  is the mass of the  $i$ -th quark, and  $\mathbf{k} = \mathbf{p}_j + \mathbf{p}_{\bar{k}}$ .

### 3 Results

When the  $\langle V_{3q-5q} \rangle$  term is switched off, there is no structure in the  $\Sigma\pi$  phase shift below the  $N\bar{K}$  threshold in the  $T=0$  nor in the  $T=1$  channels. It is because the kinetic energy of the system is large due to the light pion mass. It cancels the short-ranged attraction of OGE in the  $\Sigma\pi$  channel.

A resonance, however, appears when the conventional  $\Lambda^1$ , the  $q^3(0s)^2 0p$  state, is introduced into the system as a pole in the scattering. The energy of this  $\Lambda^1$  pole is assumed to be 160 MeV above  $\Sigma\pi$  threshold without the coupling. When the coupling is switched on, the pole energy is reduced by about 85 MeV, which corresponds to 1404 MeV; it becomes a resonance below the  $N\bar{K}$  threshold (Fig. 1). The width of the peak in the mass spectrum is about 55 MeV, which also agrees reasonably with the observed one. The  $N\bar{K}$  phase shift goes to negative: the scattering length is  $-0.75 + i0.38$  fm,

Table 1: Matrix elements of the flavor-type operator,  $-\langle \sum (F_i \cdot F_j) \rangle$ , and the color-spin-type operator,  $-\langle \sum (\lambda_i \cdot \lambda_j)(\sigma_i \cdot \sigma_j) \rangle$  for  $T = 0$ .

	flavor-type					color-spin-type			
	$\Sigma\pi$	$N\bar{K}$	$\Lambda\eta$	$\Xi K$		$\Sigma\pi$	$N\bar{K}$	$\Lambda\eta$	$\Xi K$
$\Sigma\pi$	-8	$\sqrt{6}$	0	$-\sqrt{6}$	$\Sigma\pi$	$-\frac{16}{3}$	$\frac{116\sqrt{7}}{21}$	$-\frac{16\sqrt{105}}{105}$	0
$N\bar{K}$		-6	$3\sqrt{2}$	0	$N\bar{K}$		0	$\frac{28\sqrt{15}}{15}$	0
$\Lambda\eta$			0	$-3\sqrt{2}$	$\Lambda\eta$			$\frac{112}{15}$	$-\frac{40\sqrt{70}}{21}$
$\Xi K$				-6	$\Xi K$				$-\frac{160}{21}$

which, as a simple model, agrees with the experimental value,  $(-1.70 \pm 0.07) + i(0.68 \pm 0.04)$  fm [6], reasonably well.

In the baryon-meson picture where the flavor-flavor type interaction,  $-F \cdot F$ , is employed, the peak appears because of the attraction in the  $N\bar{K}$  channel [7]. (See Table 1.) In the present scheme of the color-spin interaction, where the hyperfine interaction is proportional to  $-(\lambda \cdot \lambda)(\sigma \cdot \sigma)$ , there is no such an attraction in the  $N\bar{K}$  channel. Thus, introducing the  $\Lambda^1$  pole is necessary to reproduce the resonance. Actually, in the present case, the  $\Lambda^1$  component of the wave function at the resonance is by about 2.8 larger than the  $N\bar{K}$  component.

In summary, our results show that (1) there is a strong attraction in the  $\Sigma\pi$  channel but not in the  $N\bar{K}$  channel, (2) no peak is found in the  $\Sigma\pi$ - $N\bar{K}$ - $\Lambda\eta_{ud}$  coupled channel QCM calculation if we employ the realistic reduced mass for the kinetic energy, and (3) a reasonable peak appears if the  $\Lambda^1$  pole is included above the  $N\bar{K}$  threshold.

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