

## $Q^2$ -dependent of PPDFs in Valon model

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### Abstract

In this paper, we want to calculate  $Q^2$ -dependent of polarized parton distribution functions in the valon framework using the appropriate initial input densities to solve the DGLAP equations. A valon is characterized by its valence quark and its associated sea quarks and gluons. The structure of a valon arises from the perturbative dressing of the valence quark in QCD. We find  $Q^2$ -dependent of these functions and reproduce PPDFs in the range of  $1\text{Gev}^2 < Q^2 < 10^6\text{Gev}^2$ . This procedure helps us to understand the validity of valon model for studying the hadron structure. It means we can use valon model as a valid model for calculating PPDFs as other theoretical models such as AAC, BB and GRSV. The results with using this model are in good agreement with the experimental data and other theoretical models.

## 1 Polarized valon model

In the valon model, the structure of any hadron is obtained by convolution of two distributions: The valon distribution in the hadron and the structure function to each valon. Then polarized hadron structure in the valon model is:

$$g_1^p(x, Q^2) = \sum_v \int_x^1 \frac{dy}{y} \delta G_{v/p}(y) \mathcal{G}_1^v\left(\frac{x}{y}, Q^2\right), \quad (1)$$

Here,  $\delta G_{v/p}(y)$ , is the polarized valon distribution function. It refers to the probability of finding the  $\nu$  valon to have momentum fraction  $y$  in the hadron with spin aligned/anti-aligned to the spin of the proton. The polarized parton distribution functions are given by:

$$\delta q_{\frac{i}{h}}^p(x, Q^2) = \sum_v \int_x^1 \frac{dy}{y} \delta G_{v/p}(y) \delta q_{\frac{i}{v}}^v\left(\frac{x}{y}, Q^2\right), \quad (2)$$

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We can calculate polarized parton distribution functions from solving the DGLAP equations. At first, let us introduce mellin transformation for function  $f(x, Q^2)$  as:

$$\delta f^n(Q^2) = \int_0^1 x^{n-1} \delta f(x, Q^2) dx. \quad (3)$$

Now, DGLAP equations in n-moment space give us these two equations for extracting PPDFs : The non-singlet (NS) moments evolve according to:

$$\delta q_{NS\pm}^n(Q^2) = \left( 1 - \frac{\alpha_s(Q^2) - \alpha_s(Q_0^2)}{2\pi} \left( \frac{-2}{\beta_0} \right) (\delta P_{NS\pm}^{(1)n} - \frac{\beta_1}{\beta_0} \delta P_{qq}^{(0)n}) \right) L^{(\frac{-2}{\beta_0})\delta P_{qq}^{(0)n}} \delta q_{NS\pm}^n(Q_0^2), \quad (4)$$

and The evolution in the flavor singlet and the gluon sector of the moments are governed by a  $2 \times 2$  splitting function matrix, with the following explicit solution:

$$\begin{pmatrix} \delta \Sigma^n(Q^2) \\ \delta g^n(Q^2) \end{pmatrix} = \begin{pmatrix} L^{(\frac{-2}{\beta_0})\delta \hat{P}^{(0)n}} + \frac{\alpha_s(Q^2)}{2\pi} \hat{U} L^{(\frac{-2}{\beta_0})\delta \hat{P}^{(0)n}} - \frac{\alpha_s(Q_0^2)}{2\pi} L^{(\frac{-2}{\beta_0})\delta \hat{P}^{(0)n}} \hat{U} \end{pmatrix} \begin{pmatrix} \delta \Sigma^n(Q_0^2) \\ \delta g^n(Q_0^2) \end{pmatrix}, \quad (5)$$

where  $\delta g^n(Q^2)$  indicates the spin dependent quark-to-gluon evolution function.  $\delta q_{NS\pm}^n$  correspond to the NS combinations:  $\delta u - \delta \bar{u} \equiv \delta u_v$ ;  $\delta d - \delta \bar{d} \equiv \delta d_v$  and  $\delta \Sigma \equiv \sum(\delta q - \delta \bar{q})$ . All the associated functions in the above equations have been defined in [1]. To solve the DGLAP equations, we have to know ppdfs at the initial input scale of  $Q_0^2$ . In this work, we turn out sea quark polarization. we work with perturbative QCD. In such processes with massless quarks, helicity is conserved and therefore the hard gluons cannot produce sea quark polarization perturbatively. This fact that sea quark polarization is consistent with zero, is in good agreement with HERMES data [2, 3].

## 1.1 Determination of the initial input densities in valon model to solve the DGLAP equations

Valon is a cluster of valence quark and its associated sea quarks and gluons. It is assumed that the internal structure of the valon cannot be resolved at the initial scale of  $Q_0^2$  and the proton behaves as a bound state of three constituents. It means the photon can only realize the valence quark in the valon. As  $Q^2$  increase, other partonic components in the valon can be resolved then  $\delta q_{NS}(z = \frac{x}{y}, Q_0^2) = \delta \Sigma(z = \frac{x}{y}, Q_0^2) = \delta(z - 1)$ . We absolutely can find only a valence quark at  $Q^2 = Q_0^2$  and no other things. So, in moment space, we obtain  $\delta q_{NS}^n(Q_0^2) = \delta \Sigma^n(Q_0^2) = \int_0^1 z^{n-1} \delta(z - 1) dx = 1$  and  $\delta g^n(Q_0^2) = 0$  [4], because there is no gluon at  $Q^2 = Q_0^2$ . Our choice for  $Q_0^2$  is  $Q_0^2 = 0.283 \text{ GeV}^2$  [5] and it can compare with the GRSV group choice :  $Q_0^2 = 0.34 \text{ GeV}^2$  [1].

## 1.2 Calculating the valon distribution function

we choose a general form for polarized valon distribution function as:

$$\delta G^\nu(y) = \delta f^\nu(y)G^\nu(y) \quad (6)$$

Where  $\nu$  refers to U and D valons and  $G^\nu(y)$  is the unpolarized valon distribution function for each valon [6,7].  $\delta f^\nu(y)$  is a function with free parameters that should be determined for  $U$  and  $D$  valons:

$$\delta f^\nu(y) = ay^b(1-y)^c(1+dy^{0.5}+ey+fy^{1.5}+gy^2) \quad (7)$$

We extract the free parameters a,b,...through fitting the valence distribution in our model with the valence distribution of GRSV at the same  $Q^2$ .

| $\nu$ | $a_\nu$ | $b_\nu$ | $c_\nu$ | $d_\nu$ | $e_\nu$ | $f_\nu$ | $g_\nu$ |
|-------|---------|---------|---------|---------|---------|---------|---------|
| $U$   | 3.4443  | 0.33905 | 3.5814  | -2.476  | 5.0713  | -1.8598 | 2.7805  |
| $D$   | -0.5680 | -0.3740 | 4.1419  | -2.844  | 11.695  | -10.096 | 14.469  |

**Table I.** Numerical values of fitting parameters in Eq.(7).

## 2 $Q^2$ -dependent of PPDFs

$Q^2$ -dependent of spin densities in valon model calculated and are given by the following functions as a function of  $t = Ln[Q^2]$ . These partonic evolution functions give PPDFs in the range of  $1GeV^2 < Q^2 < 10^6GeV^2$ .

As mentioned before each type of PPDFs can parameterized as :

$$x\delta q(x, Q^2) = a(Q^2)x^{b(Q^2)}(1-x)^{c(Q^2)}(1+d(Q^2)x+e(Q^2)x^{0.5}+f(Q^2)x^2) \quad (8)$$

where  $q$  refers to polarized valence distributions for u and d type of partons and polarized gluon distribution in the proton. We calculated the evolution of each parameter in the above equation and show that with using these evolution functions we can calculate PPDFs in the range of  $1GeV^2 < Q^2 < 10^6GeV^2$  very good.

## 3 Results

In this part we show the results of PPDFs and proton structure functions with using the evolution functions. We compare our results with other theoretical models [8–10] and they are in good agreement with them.

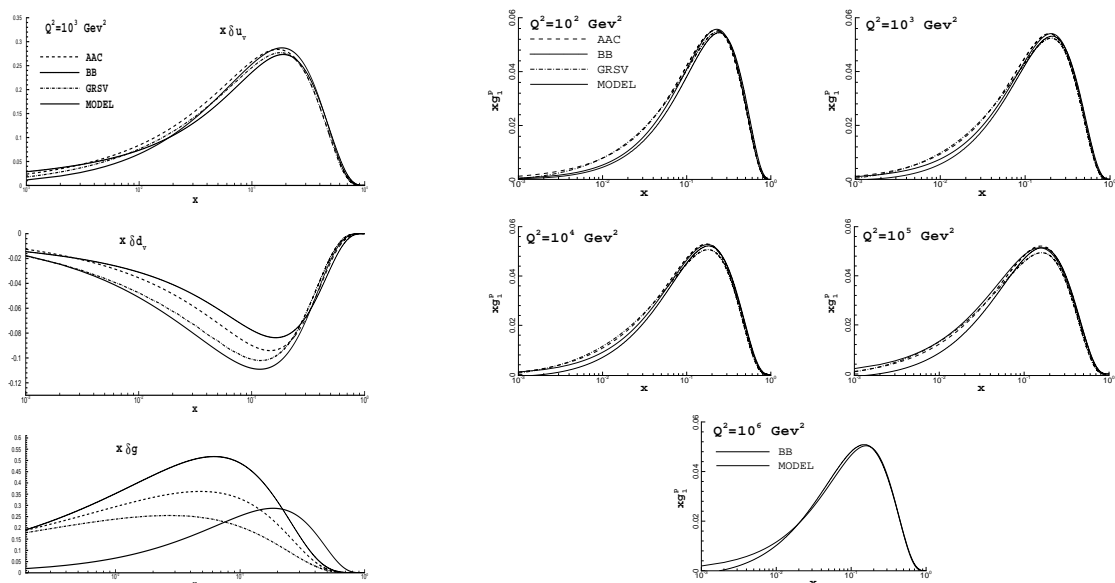


Figure 1: PPDFs for  $Q^2 = 10^3 \text{ GeV}^2$  (Left) .Proton structure function for some  $Q^2$ s .(Right)

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