THE RULE OF OAM AND POLARIZED VALON MODEL

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Abstract

In this paper, we study the rule of orbital angular momentum into the spin of proton with using the polarized Valon model. Each valence quark plus its associated sea quarks and gluons describe a valon in the dressing process of QCD. These valons have the quantum numbers of the valence quarks. In valon model we can write the Parton distributions into the hadron as a convolution of two parts: parton distribution in a valon , and valon distribution in hadron. To extract PPDFs in the proton, we have to know these function in each valon. So we need to solve the DGLAP equations into the valon. We use a simple approach to the polarized valon model and use certain criterion to initial input densities to solve the DGLAP equations in valon framework then calculate PPDFs into the proton. After calculating the first moment of each polarized parton distribution, we extract the rule of angular momentum into the valon and proton.

1 Polarized valon model

In the valon model, the structure of any hadron is obtained by convolution of two distribution: The valon distribution in the hadron and the structure function to each valon. Then polarized hadron structure in the valon model is:

\[ g_1^p(x, Q^2) = \sum_v \int_x^1 \frac{dy}{y} \delta G_{v/p}(y) G_1^v\left(\frac{x}{y}, Q^2\right), \]  

(1)

Here, \(\delta G_{v/p}(y)\) is the polarized valon distribution function. It refers to the probability of finding the \(v\) valon to have momentum fraction \(y\) in the hadron.

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with spin aligned/anti-aligned to the spin of the proton. The polarized parton distribution function are given by:

$$\delta q_i^s(x, Q^2) = \sum_v \int_x^1 \frac{dy}{y} \delta G_{v/p}(y) \delta q_v^s(x/y, Q^2),$$

(2)

We can calculate polarized parton distribution functions from solving the DGLAP equations. At first, let us introduce mellin transformation for function $f(x, Q^2)$ as:

$$\delta f^n(Q^2) = \int_0^1 x^{n-1} \delta f(x, Q^2) dx,$$

(3)

then the first moment of ppdf is:

$$\delta q^{n=1}(Q^2) \equiv \Delta q(Q^2) = \int_0^1 \delta q(x, Q^2) dx,$$

(4)

This quantity measures the spin contribution of parton $q$ to the spin of proton. Now, DGLAP equations in n-moment space give us these two equations for extracting PPDFs: The non-singlet (NS) moments evolve according to:

$$\delta q^{n}_{NS \pm}(Q^2) = \left( 1 - \frac{\alpha_s(Q^2) - \alpha_s(Q_0^2)}{2\pi} \left( \frac{-2}{\beta_0} \delta P^{(1)n}_{NS \pm} - \frac{\beta_1}{\beta_0} \delta P^{(0)n}_{qq} \right) \right) L^{(-2)\delta \hat{P}^{(0)n}} \delta q^{n}_{NS \pm}(Q_0^2),$$

(5)

and the evolution in the flavor singlet and the gluon sector of the moments are governed by a $2 \times 2$ splitting function matrix, with the following explicit solution:

$$\begin{pmatrix} \delta \Sigma^n(Q^2) \\ \delta g^n(Q^2) \end{pmatrix} = \begin{pmatrix} \frac{\alpha_s(Q^2)}{2\pi} + \frac{\alpha_s(Q_0^2)}{2\pi} \hat{U} L^{(-2)\delta \hat{P}^{(0)n}} \hat{U} & -\frac{\alpha_s(Q_0^2)}{2\pi} L^{(-2)\delta \hat{P}^{(0)n}} \hat{U} \\ \delta \Sigma^n(Q_0^2) & \delta g^n(Q_0^2) \end{pmatrix},$$

(6)

where $\delta q^n(Q^2)$ indicates the spin dependent quark-to-gluon evolution function. $\delta q^n_{NS \pm}$ correspond to the NS combinations: $\delta u - \delta \bar{u} \equiv \delta u_\uparrow; \delta d - \delta \bar{d} \equiv \delta d_\uparrow$ and $\delta \Sigma = \sum (\delta q - \delta \bar{q})$. All the associated functions in the above equations have been defined in [1]. To solve the DGLAP equations, we have to know ppdfs at the initial input scale of $Q_0^2$. It is assumed that the internal structure of the valon cannot be resolved at the initial scale of $Q_0^2$ and the proton behaves as a bound state of three constituents. It means the photon can only realize
the valence quark in the valon. So, in moment space, our initial input densities are: \( \delta q_{NS}^{n}(Q_0^2) = \delta \Sigma^{n}(Q_0^2) = \int_0^1 z^{-1} \delta(z - 1) dx = 1 \) and \( \delta g^{n}(Q_0^2) = 0 \) [2].

Our choice for \( Q_0^2 \) is \( Q_0^2 = 0.283 \text{GeV}^2 \) [3] and it can compare with the GRSV group choice: \( Q_0^2 = 0.34 \text{GeV}^2 \) [1]. Also, in this work, we turn out sea quark polarization. We work with perturbative QCD. In such processes with massless quarks, helicity is conserved and therefore the hard gluons cannot produce sea quark polarization perturbatively. This fact that sea quark polarization is consistent with zero, is in good agreement with HERMES data [4, 5].

## 2 Calculating the first moment of PPDFs in the Valon and proton

Now we calculate the first moment of PPDFs in the valon using the DGLAP equations. They are related to the total \( z \) component of quark and gluon spins, thus:

\[
< S_z >_q = \frac{1}{2} \Delta \Sigma, \quad < S_z >_g = \Delta g
\]  

(7)

We have this spin sum rule:

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta g + L_z
\]  

(8)

where, \( L_z \) is the orbital angular momentum carried by the sea partons (\( q - \bar{q} \) pairs and gluons) within the valon. The size of this orbital angular momentum turns out to be large and negative, mainly canceling out the gluon contribution. Ratcliffe [6] was the first to point out the necessity of including orbital angular momentum dependence of the evolution equation and predicted a negative value for \( < L >_z \) of the sea partons in the proton. In Fig.1 and Fig.2 the evolution of PPDFs and \( < L >_z \) in the Valon and proton are plotted (with using the polarized valon distributions [2] and using the evolution integrate in eq.2).

So, the role of Orbital angular momentum, \( L_z \) in a valon is to cancel out the gluon polarization completely, but this cancelation in proton is partial. Therefore, it is reasonable to speculate that about 60% of the spin of proton comes from gluon. It should be mentioned that the value of polarized gluon in our model is smaller than the value that the other theoretical models obtained [7–9]. Our value is \( \Delta g(Q^2 = 3 \text{GeV}^2) = 0.47 \) and it is close to recently COMPASS result: \( \Delta g(Q^2 = 3 \text{GeV}^2) = 0.3 \) [10].
Figure 1: The evolution of PPDFs in the Valon. (Left) The evolution of $L_z$ in the Valon ($Q_0^2 = 0.283 Gev^2$). (Right)

Figure 2: The evolution of PPDFs in the Proton (Left). The evolution of $L_z$ in the Proton. (Right)
References


