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RESONANCE POLE BY SPEED PLOT AND TIME DELAY

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Abstract

The speed plot and the time delay methods are examined using a two-channel Breit-Wigner formula. We show that the pole of the S-matrix given by using Speed plot is on the nearest unphysical Riemann sheet, while the pole given by the time delay is on the 2nd or 4th Riemann sheet. The validity of those methods is also discussed.

1 Introduction

It is an important issue how to determine the mass and the width of the nucleon resonance from the meson-baryon scattering amplitudes. The 'Speed plot' (SP) proposed by Höhler [1] and time delay methods using 'time delay' (TD) introduced by Eisenbud and Wigner [2] are frequently used to find poles of the S-matrix from the empirical partial wave amplitudes. The time delay method of Ref. [3] is not the 'eigen phase' of the coupled channel S-matrix as pointed out in Ref. [4]. Unfortunately, only the amplitude of the elastic scattering is empirically determined in most of the case and the S-matrix elements of the other open channels are not available. There can be poles on various Riemann sheets associated with a resonance when several reaction channels are open. It is interesting to study a possibility to probe poles on the various Riemann sheets. In this work, we examine the speed plot and the time delay methods using simple two-channel Breit-Wigner amplitudes to clarify the nature of the resonance information provided from those methods and the validity of those methods. We applied those methods to the πN scattering amplitudes of SAID [5].

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2 Speed plot and time delay

The time delay associated to the scattering process has been introduced by Eisenbud and Wigner [2] by using wave-packet analysis. For the coupled-channels case, the time delay is related to the following complex number

$$\Delta t_{ij} = -i\frac{1}{S_{ij}}\frac{dS_{ij}}{dE}, \tag{1}$$

where S_{ij} is the S-matrix element for channels i, j and E is the scattering energy. Taking the real part of Δt_{ii} for elastic scattering, the time delay (TD) is defined in Ref. [3] as the energy derivative of the phase shift δ

$$TD = Re(\Delta t_{ii}) = 2\frac{d\delta}{dE}, \tag{2}$$

where we parametrized the S-matrix using phase shift δ and inelasticity η as $S = \eta e^{2i\delta}$. In the time delay method, the resonance position is extracted from the stationary point of the time delay TD. The speed (SP) is defined from the energy derivative of the T-matrix as

$$SP = |\frac{dT_{ii}}{dW}| = \frac{1}{2}|\frac{dS_{ii}}{dW}| = |\eta\frac{d\delta}{dW} + \frac{1}{2i}\frac{d\eta}{dW}|, \tag{3}$$

where $T_{ii} = (S_{ii} - 1)/2i$. The resonance position is determined at the maximum of the speed SP. From Eqs. (2) and (3), the speed plot and the time delay method may give different resonance energy when the inelastic reaction takes place.

To understand the meaning of this difference, we studied the two-channel Breit-Wigner formula

$$T_{11} = \frac{-\gamma_1 p_1}{E - M + i\gamma_1 p_1 + i\gamma_2 p_2}. \tag{4}$$

Here p_1, p_2 are the on-shell momenta defined by $E = p_1^2/2\mu_1 = p_2^2/2\mu_2 + \Delta$. γ_1, γ_2 are constants related to the partial width. Here we take a simple example with $\Delta = 0,\ \mu_1 = \mu_2 = \mu$.

The analytic structure of the two particles scattering in two channels can be studied in four Riemann sheets. The 1st, 2nd, 3rd and 4th sheets correspond to [tt], [bt], [bb] and [tb] sheets, respectively. Here [t] and [b] represents top (physical) and bottom (unphysical) sheets for each channel. For the two-channel resonance, two poles can exist on the different Riemann sheets at the same time. One is a resonance pole on the 3rd sheet and the other one is a shadow pole [6] on the 2nd or 4th sheet.

The explicit expression of the poles of the two-channel Breit-Wigner formula can be obtained as follows. The pole on the 3rd sheet is given as

$$E = M - \mu\Gamma^2 - i\sqrt{\mu\Gamma^2(2M - \mu\Gamma^2)} \tag{5}$$

with $\Gamma = \gamma_1 + \gamma_2$. We can also find the 'shadow pole' on the 2nd or the 4th Riemann sheet depending on the sign of $\gamma = \gamma_1 - \gamma_2$ as

$$E = M - \mu\gamma^2 - i\sqrt{\mu\gamma^2(2M - \mu\gamma^2)}. \tag{6}$$

Now we can apply the speed plot and time delay methods to the above two-channel Breit-Wigner formula. The speed plot method gives the following resonance pole

$$E_{SP} = M - (\mu\Gamma^2 + i\sqrt{\mu\Gamma^2(2M - \mu\Gamma^2)})(1 + O(\mu\Gamma^2/M))\,. \tag{7}$$

Therefore the speed plot gives the approximate resonance pole on the 3rd sheet. The same exercise for the time delay method gives

$$E_{TD} = M - (\frac{1}{1 - x + x^2}\mu\gamma^2 + i\sqrt{\mu\gamma^2(2M - \mu\gamma^2)})(1 + O(\mu\gamma^2/M)), \tag{8}$$

with $x = \gamma/\Gamma$. The time delay method gives the approximate pole position on the 2nd or 4th sheet.

3 Application for πN scattering amplitudes

We apply the speed plot and the time delay for the S_{11} πN amplitude of SAID analysis [5]. In PDG, two resonances $N^*(1535)$ and $N^*(1650)$ are listed and their mass and the width are given in Table 1. The results of the SP and TD are also shown in Table 1. We could not find a pole corresponding to $N^*(1535)$ from SP. This might be due to the fact that $N^*(1535)$ is too close to the ηN threshold. Therefore the condition discussed in the previous section is not satisfied. The TD has a minimum at $W = 1535$MeV, which suggests the pole is on the 4th sheet but not on the 2nd sheet. The pole of $N^*(1650)$ appears both in the SP and the TD. The SP gives a resonance pole on the 3rd sheet and TD gives a shadow pole probably on the 2nd sheet, which corresponds to the zero of the S-matrix on the physical sheet.

In summary, the SP and the TD are both approximate methods to probe the pole of the resonance. We find the speed plot gives the position of the resonance pole and time delay leads to the shadow pole/or zero of the S-matrix.

Partial wave	PDG	Speed plot	Time delay
S11(1535)	1490 to 1530 -i 45 to 125		1535 -i 33
S11(1650)	1640 to 1670 -i 75 to 90	1649 -i 65	1651 -i 51

Table 1: Pole given by the SP and the TD for the S_{11} πN partial wave.

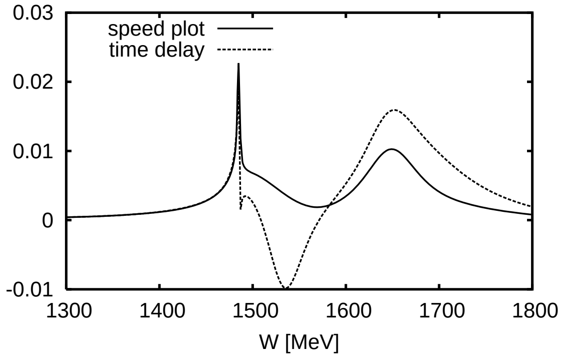

Figure 1: The SP and the TD for S_{11} partial wave of πN elastic scattering.

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