MENU 2007
11th International Conference
on Meson-Nucleon Physics and
the Structure of the Nucleon
September10-14, 2007
IKP, Forschungzentrum Jülich, Germany

RHO-LIKE-MESON FAMILY IN THE PION-PION SCATTERING

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Abstract

Experimental data on the isovector P-wave of $\pi\pi$ scattering have been analyzed to study ρ-like mesons. The analysis, performed in the model-independent and Breit–Wigner approaches, indicates evidently a presence of three ρ-like mesons: $\rho(1250)$, $\rho(1450)$ and $\rho(1600)$ in the 1200-1800-MeV region. The obtained P-wave $\pi\pi$ scattering length agrees mostly with the result of the local Nambu–Jona-Lasinio model.

1 The model-independent analysis

Initially we outline the application of the model-independent approach (MIA) [1] to studying the 2-channel $\pi\pi$ scattering. Let the $\pi\pi$-scattering S-matrix be determined on the 4-sheeted Riemann surface with the right branch-points at $s = 4m_{\pi^0}^2$ and $(m_\omega + m_{\pi^0})^2$ and with the left one at $s = 0$. The surface sheets are numbered according to the signs of analytic continuations of the channel momenta $k_1 = [s - 4m_{\pi^0}^2]^{1/2}/2$ and $k_2 = [s-(m_\omega + m_{\pi^0})^2]^{1/2}/2$ as follows: signs$(\mathrm{Im}k_1, \mathrm{Im}k_2) = ++, -+, --,$ and $+-$ correspond to sheets I, II, III, and IV, respectively. An explicit allowance for the $(m_\omega+m_{\pi^0})^2$ branch point permit us to describe transitions between the $\pi\pi$ and $\omega\pi$ initial and final states using only the Jost matrix determinant $d(k_1, k_2)$ in the Le Couteur–Newton relations (CNR) [2]. The 2-channel resonances are represented by the three possible types of pole clusters (poles and zeros on the Riemann surface) [1] (**a**), (**b**) and (**c**): (**a**) the resonance is described by a pair of complex-conjugate poles in the S-matrix element on sheet II and by a pair of conjugate shifted poles on sheet III; (**b**) by a pair of conjugate poles on sheet IV and by a pair of conjugate shifted poles on sheet III; (**c**) by a pair of conjugate poles on sheet II and by other on sheet IV, and by two pairs of conjugate poles on sheet III. Due to unitarity, the zeros on sheets I, IV, and III are situated in the same energy points as the corresponding poles on sheets II, III, and IV, respectively.

The considered Riemann surface is mapped by the uniformizing variable $v = [(m_\omega + m_{\pi^0})k_1 + 2m_{\pi^0}k_2]/[s(((m_\omega + m_{\pi^0})/2)^2 - m_{\pi^0}^2)]^{1/2}$ onto the v-plane [3]. In the S-matrix, taken as $S = S_{res}S_{bg}$, the resonance part is $S_{res} = d(-v^{-1})/d(v)$, where $d = v^{-M}\prod_{n=1}^{M}(1-v_n^* v)(1+v_n v)$ with M the number of pairs of the conjugate resonance zeros. The background part is $S_{bg} = \exp[2i(2k_1/\sqrt{s})^3(\alpha_1+\alpha_2\theta(s-s_1)(s-s_1)/s+\alpha_3\theta(s-s_2)(s-s_2)/s)]$ where $\alpha_i = a_i + ib_i$, and s_1 and s_2 are the thresholds of the 4π and $\rho 2\pi$ channels, respectively. Due to allowing for the left branch-point in the variable v, $a_1 = b_1 = 0$. From the experimental data $b_2 = 0$.

In our analysis of data [4, 5] for η and δ ($S(\pi\pi \to \pi\pi) = \eta\exp(2i\delta)$), different number of resonances was considered: three ($\rho(770)$, $\rho(1250-1580)$, and $\rho(1550-1780)$); four (the previous three and $\rho(1860-1910)$); and five (the previous four and $\rho(1450)$). Satisfactory description of data was obtained. The values of χ^2/NDF and the constant systematic error of δ in data [5] (see [3] for discussion) are, respectively, 1.72 and $-1.885°$ for three, 1.68 and $-1.897°$ for four, 1.65 and $-1.876°$ for five resonances. The analyses prefer the case in which $\rho(770)$ is described by the cluster of type (**a**) and the others by type (**b**). The cluster poles for the 5-resonance description, located on the lower $\sqrt{s}$-half-plane, are (in MeV) $765.8-i73.3$ (sheet II) and $778.2-i68.9$ (sheet III) for $\rho(770)$, $1250-i131.4$ (sheet III) and $1249.4-i130.7$ (sheet IV) for $\rho(1250)$, $1469.2-i89.3$ (sheet III) and $1465.4-i100.4$ (sheet IV) for $\rho(1450)$, $1634.8-i145.9$ (sheet III) and $1593.4-i72.9$ (sheet IV) for $\rho(1600)$, and $1883-i106.5$ (sheet III) and $1893.4-i87.6$ (sheet IV) for $\rho(1900)$. The background parameters are: $a_2 = 0.0248$, $a_3 = 0.0841$ and $b_3 = 0.0019$. The pole clusters and background parameters for the 3- and 4-resonance descriptions are given in [3]. Though the description is practically the same in all three cases, careful consideration of the obtained parameters and energy dependence of the fitted quantities suggests that $\rho(1900)$ is strongly desired and that $\rho(1450)$ should be also present improving slightly the description.

Masses and total widths, calculated from the poles on sheet II for the type-(**a**) resonances and on sheet IV for the type-(**b**) ones, using the resonance part of the amplitude in the form $T^{res} = \sqrt{s}\,\Gamma_{el}/(m_{res}^2 - s - i\sqrt{s}\,\Gamma_{tot})$, are, respectively (in MeV): 769.3 and 146.6 for $\rho(770)$, 1256.2 and 261.4 for $\rho(1250)$, 1468.8 and 200.8 for $\rho(1450)$, 1595.1 and 145.8 for $\rho(1600)$, and 1895.4 and 175.2 for $\rho(1900)$.

2 The Breit–Wigner analysis

. It was shown in [6], that the ρ-like resonances possess also other important decay channels in addition to those considered above. The $\rho(1450)$ and/or a possible $\rho(1250)$ can decay also to $\eta\rho^0$, 4π and $\phi\pi$. The $\rho(1700)$ has a large branching to the 4π, $\rho 2\pi$ and $\eta\rho^0$ channels. To obtain information about couplings with some selected channels from analysis of the $\pi\pi$-scattering data, we used the 5-channel Breit–Wigner (BW) forms in constructing the $d(k_1,\cdots,k_5)$-function in CNR: $S_{res} = d(-k_1,\cdots,k_5)/d(k_1,\cdots,k_5)$, where k_1, k_2, k_3, k_4 and k_5 are the $\pi\pi$-, $\pi^+\pi^-2\pi^0$-, $2(\pi^+\pi^-)$-, $\eta 2\pi$- and $\omega\pi^0$-channel momenta, respectively. The d-function is $d = d_{res}d_{bg}$ with $d_{res}(s) = \prod_r [M_r^2 - s - i\sum_{j=1}^{5}\rho_{rj}^3\ R_{rj}\ f_{rj}^2]$, where $\rho_{rj} = k_i(s)/k_i(M_r^2)$, f_{rj}^2/M_r is the partial width of resonance, $R_{rj}(s,s_j,r_{rj},M_r)$ is a Blatt–Weisskopf barrier factor [7] with s_j the j-channel threshold and radii $r_{rj} = 0.7035$ fm for all resonances in all channels. We have assumed that $f_{r2} = f_{r3}/\sqrt{2}$. The background part $d_{bg} = \exp[-i(2k_1/\sqrt{s})^3(\alpha_1 + \alpha_2\theta(s-s_1)(s-s_1)/s)]$, where $\alpha_i = a_i + ib_i$ $(b_1 = 0)$ and s_1 is the $\rho 2\pi$ channel threshold.

The data were analyzed as in MIA considering three, four and five resonances. A reasonable description of all three cases was obtained: $\chi^2/\mathrm{NDF} = 1.87$ for three, 1.92 for four and 1.91 for five resonances. The resonance parameters for the last case are shown in Table 1. The systematic error of data [5] is $-1.987°$. The background parameters are: $a_1 = -0.00121\pm0.0018$, $a_2 = -0.1005\pm0.011$, and $b_2 = 0.0012\pm0.006$.

Table 1: The ρ-like resonance parameters (in MeV).

State	$\rho(770)$	$\rho(1250)$	$\rho(1450)$	$\rho(1600)$	$\rho(1900)$
M	777.69±0.32	1249.8±15.6	1449.9±12.2	1587.3±4.5	1897.8±38
f_{r1}	343.8±0.73	87.7±7.4	56.9±5.4	248.2±5.2	47.3±12
f_{r2}	24.6±5.8	186.3±39.9	100.1±18.7	240.2±8.6	73.7
f_{r3}	34.8±8.2	263.5±56.5	141.6±26.5	339.7±12.5	104.3
f_{r4}		231.8±111	141.2±98	141.8±33	9
f_{r5}		231±115	150±95	108.6±40.4	10
Γ_{tot}	≈154.3	>175	>52	>168	>10

We have calculated also the isovector P-wave length of $\pi\pi$ scattering: $a_1^1 = 33.9\pm2.02[10^3 m_{\pi^+}^{-3}]$. It can be compared with values evaluated by using the local [8] (34) and non-local [9] (37) Nambu–Jona-Lasinio (NJL) model and Roy's equations: [10] (38.4 ± 0.8), [11] (37.9 ± 0.5), and [12] (39.6 ± 2.4).

3 Summary

The reasonable description of the accessible experimental data on the isovector P-wave of $\pi\pi$ scattering [4, 5] has been obtained up to 1.88 GeV in MIA (χ^2/NDF = 1.654) and applying the BW forms (χ^2/NDF = 1.906).

For $\rho(770)$, the mass is a little smaller in MIA (769.3 MeV) and a little bigger in the BW analysis (777.69 ± 0.32 MeV) than the averaged mass (775.5 ± 0.4 MeV) in the PDG tables [6]. The total width in MIA (146.6 MeV) coincides with the PDG one (146.4 ± 1.1 MeV) but the width in the BW analysis is a little bit larger (≈ 154.3 MeV).

The 2nd ρ-like meson has the mass 1256.2 MeV in MIA and 1249.8 ± 15.6 MeV in the BW analysis. This differs significantly from the PDG value (1459 ± 11 MeV) [6]. The $\rho(1250)$ meson was discussed some time ago and it was confirmed relatively recently in some analyses [13]. If $\rho(1250)$ is the 1st radial excitation of the 1^+1^{--}-state, then it lies down well on the corresponding linear trajectory on the (n, M^2)-plane [14] (n is the radial quantum number of the $q\overline{q}$ state).

It was shown [15] that the 1600-MeV region contains in fact two ρ-like mesons. Then there are three ρ-like mesons in the 1200-1800-MeV region, the 3rd meson having the mass about 1450 MeV. This does not contradict the data. In MIA, the description is even slightly improved if this state is included.

The 4th ρ-like meson turns out to have the mass 1590 MeV rather than 1720 MeV cited in the PDG tables, though in some analyses, the resonance with the mass near 1590 MeV was also found [6]. Note that a rather large coupling of these ρ-like mesons with the 4π channels was obtained.

In the case of $\rho(1900)$, there are practically no data in this energy region. Whereas MIA testifies in favor of existence of this state, the BW analysis gives equivalent results with and without the $\rho(1900)$.

The P-wave $\pi\pi$-scattering length, $a_1^1 = 33.9 \pm 2.02[10^3 m_{\pi^+}^{-3}]$, obtained in the BW analysis, mostly matches the result of the local NJL model [8].

Yu.S. acknowledges support provided by the Votruba-Blokhintsev Program. P.B. thanks the Grant Agency of the Czech Republic, Grant No.202/05/2142.

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