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THE GMO SUM RULE REVISITED

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Abstract

A brief introduction is given for the analysis of pion-nucleon data with the Goldberger-Miyazawa-Oehme sum rule. With information on pionic hydrogen, low-energy π^+p scattering and total cross sections, a value $f^2 = 0.075 \pm 0.002$ for the coupling constant is deduced.

1 Introduction

The Goldberger-Miyazawa-Oehme sum rule [1] provides a simple relation connecting four different quantities:

- the s-wave π^-p scattering length a_{π^-p},
- the s-wave π^+p scattering length a_{π^+p},
- the πN coupling constant f^2,
- the weighted integral, J^-, of the difference of the π^-p and π^+p total cross sections.

By evaluating the forward dispersion relations at the physical threshold one obtains [2]

$$(1 + \mu/m)(a_{\pi^-p} - a_{\pi^+p})/\mu = 4f^2/(\mu^2 - \omega_B^2) + 2\, J^-, \tag{1}$$

where μ is the (charged) pion mass, m is the proton mass, $\omega_B = -\mu^2/2m$ and

$$J^- = 1/(4\pi^2) \int_0^\infty (\sigma^{\mathrm{Tot}}_{\pi^-p} - \sigma^{\mathrm{Tot}}_{\pi^+p})/\omega \, dk. \tag{2}$$

Here I'll present the results of an evaluation [3] of three of these quantities, a_{π^-p}, a_{π^+p} and J^-, and give a value for the resulting coupling constant, f^2.

2 Pionic Hydrogen

The strong interaction shift of the $1s$ level of the pionic hydrogen provides a handle to the scattering length $a_{\pi^- p}$. The connection is the Deser-type formula [4]

$$\epsilon_{1s} = -2\alpha^3 \, \mu_c^2 \, a_{\pi^- p} \, (1 + \delta_\epsilon), \tag{3}$$

where μ_c is the reduced mass of the $\pi^- p$ system and $\alpha \simeq 1/137.036$ is the fine structure constant. In Eq. (3) the identification $a_{0+}^+ + a_{0+}^- = a_{\pi^- p}$ of the sum of the isoscalar and isovector s-wave πN scattering lengths has been made. The quantity δ_ϵ evaluated next-to-leading order in isospin breaking and in the low-energy expansion has the value [4]

$$\delta_\epsilon = (-7.2 \pm 2.9) \times 10^{-2}. \tag{4}$$

The pionic hydrogen level shift has been measured [5, 6] to the precision of about 0.2 %

$$\epsilon_{1s} = -7.120 \pm 0.008 \pm 0.009 \ \text{ eV}, \tag{5}$$

where the first error is due to statistics and the second due to systematics. The measured level shift, Eq. (5), gives then for the scattering length

$$a_{\pi^- p} = 0.0933 \pm 0.0029 \ \ 1/\mu, \tag{6}$$

if errors are added linearly.

3 The $\pi^+ p$ S-wave Scattering Length

Information on angular distributions of the $\pi^+ p$ scattering observables has been used as input for a discrete phase shift analysis in the laboratory momentum range $k = 0.077 - 0.725$ GeV/c. The Tromborg corrections [7] were applied to extract the hadronic amplitudes from the experimental data. Constraints from forward dispersion relations [8] were also implemented. The resulting s-wave scattering length for $\pi^+ p$ is

$$a_{\pi^+ p} = -0.0764 \pm 0.0014 \ \ 1/\mu \tag{7}$$

in rather close agreement with the Matsinos *et al.* result [9]

$$a_{\pi^+ p} = -0.0751 \pm 0.0039 \ 1/\mu \tag{8}$$

based on a fit with a low-energy model.

4 Integral J^-

The integral J^-, given in Eq. (2), has been evaluated with the total cross section input from the forward dispersion relation analysis of Ref. [8], where corrections for the electromagnetic interaction [7] and the P_{33} splitting of $\pi^- p$ [10] were applied. The result of the evaluation [3] is displayed in the Table together with some earlier determinations. An important issue is the question

Table 1: The values of the integral J^- (mb).

Reference	J^- (mb)
Höhler-Kaiser [11]	-1.06
Koch [12]	-1.077 ± 0.047
Gibbs *et al.* [13]	-1.051 ± 0.005[a]
Ericson *et al.* [14]	-1.083 ± 0.032
Abaev *et al.* [3]	-1.060 ± 0.030

[a] Statistical error only.

of the uncertainty to be attached to the value of the integral J^-. Abaev *et al.* [3] combine linearly errors related to statistics (0.007 mb), discrepant data sets (0.012 mb), electromagnetic corrections beyond the Tromborg range (0.006 mb) and the asymptotic behavior (0.004 mb).

5 Results and Conclusions

With the values for $a_{\pi^- p}$, $a_{\pi^+ p}$ and J^- one can solve f^2 from Eq. (1). The outcome is $f^2 = 0.075 \pm 0.002$. The result depends on the techniques used to extract the hadronic scattering length from the level shift measurement, potential model calculations tend to produce larger values for f^2.

In the present analysis the largest uncertainty in f^2 is due to the uncertainty in $a_{\pi^- p}$ which, in turn, is a consequence of the largely unknown low-energy constant f_1 needed to fix δ_ϵ. Assuming isospin invariance, a precise value for Γ_{1s}, the width of the pionic hydrogen $1s$ state, would avoid this problem.

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