PARTIAL WAVE ANALYSIS OF $\pi N$ SCATTERING WITH FIXED-$t$ CONSTRAINTS

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Abstract

We have finished the forward analysis of $\pi N$ scattering and are proceeding with non-zero angles to achieve a full fixed-$t$ constrained partial wave analysis. The outcome of the forward analysis is a set of coefficients of Pietarinen's expansion together with a full covariance matrix, which can be used for calculating some forward observables with proper error bars. As an illustration, we calculated some of the subthreshold expansion coefficients $c_{n0}^{\pm}$ and their errors.

1 Introduction

As is well known, the experimental data alone is not enough to give a unique partial wave solution [1, 2], and some theoretical constraints are required. Besides the standard constraints of unitarity, crossing symmetry and isospin symmetry, we are incorporating the fixed-$t$ analyticity constraint in the form of Pietarinen’s expansions [3]

$$C^{\pm}(\nu, t) = C_N^{\pm}(\nu, t) + H^{\pm}(Z, t) \sum_{k=1}^{N} c_k^{\pm} \left[ \frac{\alpha - \sqrt{\nu_T^2 - \nu^2}}{\alpha + \sqrt{\nu_T^2 - \nu^2}} \right]^k . \quad (1)$$

Here $\alpha$ is controlling the distribution of the experimental data along the unit circle of the conformal mapping and at $t = 0$ the threshold value $\nu_T = \mu$ (mass of the charged pion). The functions $H^{\pm}(Z, t)$ are taking care of the assumed asymptotic behaviour of the amplitudes and the nucleon pole terms $C_N^{\pm}(\nu, t)$ are treated separately. The coefficients $c_k^{\pm}$ of Eq. (1) are fixed by fitting to the experimental data and to a partial wave solution of the earlier
iteration step. The simple form (1) of the expansion is valid for \( t \)-values from 0 to -0.52 GeV\(^2\), below that the \( s \)- and \( u \)-channel cuts are overlapping and they should be treated separately.

2 Input data

For forward analysis, we used all \( \pi N \) forward scattering data available on the Particle Data Group data bases [4] except a few data sets, which are listed in ref. [5]. In addition, we used the scattering lengths from pionic hydrogen experiments [6] and from a discrete phase shift analysis [7]. For the partial wave analysis, all elastic \( \pi N \) data available at the Durham HEP database [8] have been used. Before the analysis, the electromagnetic effects and delta splitting were removed from the data as described in [5,9], and the corrected data were assumed to be purely hadronic and isospin invariant.

3 Forward Analysis

Our forward analysis is described in ref. [5]. As the end result of the analysis, the coefficients of the expansion (1) for \( t = 0 \) were obtained together with the full correlation matrix. These can be used to calculate any desired forward scattering observable with proper statistical errors, and as illustration, we give some of the subthreshold expansion coefficients \( c_{n0}^\pm \) in [5]. The shape of the solution is much better known than the overall normalization, and respectively the \( c_{00}^\pm \) coefficients can not be pinned down with the same accuracy as the higher ones. The coefficients \( c_{nm}^\pm \) with non-zero \( m \) are not accessible in a forward analysis. The asymptotic behaviour of the total cross sections, calculated from our solution, is presented in Figs. 1 and 2.

4 Upcoming PWA

After fixing the coefficients \( c_k \) of Eq. (1) for all desired \( t \)-values, the partial waves are found by fitting both to the expansions (1) and to the experimental data. In fitting, the \( \chi^2 \)-function includes terms taking care of the unitarity constraint as well as a term measuring the deviation from the old solution. The latter is essential, because in order to smooth out the statistical fluctuations in the experimental data, the iteration should be carried out almost in "adiabatic" manner, \( i.e. \), the iteration step size should be as small as the CPU time allows. For the same reason, the data are renormalized to the earlier iteration solution in every iteration step. The procedure is continued
Figure 1: The asymptotic behaviour of $\sigma_{\pi^+p}^{\text{Tot}}$. The Carroll 1976 data are marked with dashes, Brick 1982 data with asterisks and Carroll 1979 data with circles. All data points are available at PDG web site [4].

Figure 2: The asymptotic behaviour of $\sigma_{\pi^-p}^{\text{Tot}}$. The Carroll 1976 data are marked with dashes, Carroll 1979 data with asterisks and Dersch 1999 data with circles. All data points are available at PDG web site [4].
iteratively until all parts will agree within a few percent. This kind of fitting procedure should be able to fix the phase shifts at least up to $f$-waves.

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**References**


