Dynamically generated resonances in two meson-one baryon systems

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Abstract

We study the systems of two mesons and one baryon with total strangeness \( S = -1 \) (\( \pi K \bar{N} \), \( \pi \pi \Sigma \), etc.) by solving Faddeev equations in the coupled channel approach. The calculations have been carried out for \( L = 0 \), leading to total spin parity \( 1/2^+ \) for the three body system. The study resulted into dynamical generation of many low-lying \( \Sigma \) and \( \Lambda \) resonances which can be associated to those in the Particle Data Book.

1 Introduction

After the finding of a narrow peak in the invariant mass of the \( K^+n \) system [1], which was associated to a strangeness +1 pentaquark state, i.e., the \( \theta^+ \), a series of experimental groups reported findings of the same state but in the \( pK_S \) invariant mass (see discussion in [2] for example). Since the \( K_s \) is a mixed state of \( K^0 \) and \( \bar{K}^0 \), the \( pK_S \) system can possess strangeness +1 or −1. Therefore, the narrow state found in the \( pK_S \) system could be, for example, a \( \Sigma^*+ \) instead of a \( \theta^+ \). A search for a \( \Sigma \) or \( \Lambda \) resonance in this mass region is thus interesting.

In fact, there exist \( \Lambda \) and \( \Sigma \) resonances in this energy region but with poor or controversial status [3]. Also some of them seem to remain unexplained in terms of two-body dynamics, e.g., a study of the \( K^-p \to \pi^0\pi^0\Lambda \) reaction in [4], where the \( \Lambda(1520) \) gets dynamically generated and which explains the bulk of the data [5], fails to explain the bump in the \( \Lambda(1600) \) region. Additionally, in [6] the excitation of the \( \Sigma(1660) \) has been indicated in the data on the \( K^-p \to \pi^0\pi^0\Sigma^0 \) reaction. These findings naturally demand a study of three-body system(s) such as \( \pi\pi\Sigma, \pi\pi\Lambda, \pi\bar{K}N \), etc.

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2 Formalism

The first striking thought is that adding a $\pi$ to the $\Lambda(1405)$, which is a resonance in the $\pi\Sigma$ system, one obtains a total mass $\sim 1542$ MeV. This is exactly the energy region where the narrow state in the $pK_S$ system was found and where the $S=-1$ resonances are not well understood. The $\Lambda(1405)$ has been studied extensively using chiral dynamics [7–9] where it gets dynamically generated in the $KN$ system and its coupled channels and its properties are well reproduced. The idea, thus, is to exploit this knowledge and extend it to a study of the three-body systems formed by adding a pion to all the combinations of $0^-$ mesons and $1/2^+$ baryons which couple to strangeness $-1$. For the net charge zero, we end up with 22 coupled channels [10].

The input two-body $t$-matrices required to solve the Faddeev equations

\[ T^i = t^i + t^i G [T^j + T^k], \quad i \neq j \neq k = 1, 3 \]

have been calculated by solving the Bethe-Salpeter equation using the potentials obtained from chiral Lagrangians as in [7,9]. The form of the potentials in this approach is such that it can be divided into an on-shell part, which depends only on the center of mass (C.M) energy of the interacting particles, and an off-shell part, which is inversely proportional to the propagator of the particle. This gives rise to three body forces [10] which have been found, in some approximations, to cancel with those originating directly from the chiral Lagrangian [10]. For a realistic case, the total contribution of three-body forces from all the sources has been found to be about 5% of the total on-shell contribution [10]. Hence, the calculations have been done using on-shell two body $t$-matrices. In this case, the terms with two successive interaction can be expressed as $t^i g^{ij} t^j$, with

\[ g^{ij} = \left( \prod_{r=1}^{D} \frac{N_r}{2E_r} \right) \frac{1}{\sqrt{s} - E_i(k_i) - E_j(k_j) - E_k(k_i + k_j) + i\epsilon} \]

where $\sqrt{s}$ is the total energy in the global CM system, $E_l = \sqrt{k_l^2 + m_l^2}$ is the energy of the particle $l$, $N_l$ is a normalization constant and $D$ is the number of particles propagating between two interactions [10]. These terms correspond to the situation where there are no loops and hence the $g^{ij}$ propagators depend on the on-shell variables only. The terms with three $t$-matrices involve a loop function of three particle propagators, e.g.

\[ \int \frac{d^3q}{(2\pi)^3} t^1(s_{23}) g^{13}(q) t^3(s_{12}(q)) g^{32}(q) t^2(s_{13}) \]

(1)
where $s_{13}, s_{23}$ are defined in terms of on-shell variables and $s_{12}(q)$ denotes the dependence of the $t^3$-matrix on the loop variable. Therefore, $t^1$ and $t^2$ can be extracted out of the integral, but not $t^3$. However, if we re-arrange the expression (1) as

$$t^1(s_{23}) \left[ \int \frac{d^3q}{(2\pi)^3} g^{13}(q)t^3(s_{12}(q))g^{32}(q)[g^{32}]^{-1}[t^3(s_{12})]^{-1} \right] t^3(s_{12})g^{32}t^2(s_{13})$$

where an identity expression depending on the on-shell variables, i.e., $[g^{32}]^{-1} [t^3(s_{12})]^{-1} t^3(s_{12})g^{32}$, has been introduced, and define

$$G^{132} = \int \frac{d^3q}{(2\pi)^3} g^{13}(q)t^3(s_{12}(q))g^{32}(q)[g^{32}]^{-1}[t^3(s_{12})]^{-1}$$

the equation (1) can be expressed as $t^1(s_{23}) G^{132} t^3(s_{12}) g^{32} t^2(s_{13})$. Now the $t^3(s_{12})g^{32}l^2(s_{13})$ term depends only on the on-shell variables, the dependence of $t^3$ and $g^{32}$ on the running variable has been taken care of when defining the $G$ function. With this, the Faddeev equations can be written as [10]

$$T^i_R = t^i g^{ij} t^j + t^i \left[ G^{ij} T^j_R + G^{ijk} T^j_R T^k_R \right] i \neq j, j \neq k = 1, 3,$$

which are algebraic equations.

### 3 Results and discussion

We plot the squared $\tilde{T}_R$-matrix as a function of $\sqrt{s}$ and $\sqrt{s_{23}}$ after subtracting the $t^i g^{ij} t^j$ terms ($\tilde{T}_R = \sum_{ij} T^i_R$), which evidently do not have a resonance structure and just provide a background to the amplitudes.

In order to identify the nature of the resulting states, the $T$-matrix has been projected on the isospin base defined in terms of the total isospin of the three particles, $I$, and the total isospin of the two mesons, $I_m$.

In Fig.1 we show our results for the total isospin one, obtained by keeping the two pions in the isospin 2 configuration. We see two peaks: one at $\sqrt{s} = 1656$ MeV with a width of 30 MeV, which we correlate with the well-established $\Sigma(1660)$ [3], and another at 1630 MeV with a width of 39 MeV [10]. This is the first time when an evidence for the three body structure (two mesons and one baryon) of the $\Sigma(1660)$ has been found. We associate the peak at $\sqrt{s} = 1630$ MeV with the $\Sigma(1620)$, for which the experimental and partial wave analyzes results are listed separately, with a note saying that its not clear if there are one or more resonances in this energy region.
Figure 1: Two resonances in the $\pi\pi\Sigma$ amplitude in $I = 1$, $I_\pi = 2$ configuration.

Our calculations resulted also into dynamical generation of more strangeness $= -1$ resonances which are not well-understood, for example, $\Sigma(1560)$ with $J^\pi$ unknown [3] and for which our results predict $J^\pi = 1/2^+$, $\Sigma(1770)$, which is listed as a one star resonance in [3]. Finally, in the isospin zero sector we find evidence for two peaks in $\Lambda(1600)$ region and one corresponding to $\Lambda(1810)$ [10].

References