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NEUTRON-NEUTRON SCATTERING LENGTH FROM THE REACTION $\gamma d \to \pi^+ nn$

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Abstract

We discuss the possibility to extract the value of the neutron-neutron scattering length a_{nn} from experimental data on the reaction $\gamma d \to \pi^+ nn$. We provide a high accuracy calculation of the differential cross sections for this reaction within chiral perturbation theory. We argue that for properly chosen kinematics, the theoretical uncertainty of the method can be as low as 0.1 fm.

A precise determination of the neutron-neutron scattering length a_{nn} is important for an understanding of the effects of charge symmetry breaking in nucleon–nucleon forces. However, a direct measurement of a_{nn} in a scattering experiment is practically impossible at the moment due to the absence of a free neutron target, and the commonly used value for a_{nn} is obtained as a result of analysis of reactions where there are three particles in the final state, *e.g.* $\pi^- d \to \gamma nn$ or $nd \to pnn$. There is a large spread in the results for a_{nn} obtained by various groups. For instance, analyses of the reaction $nd \to pnn$ give significantly different values for a_{nn}: $a_{nn} = -16.1 \pm 0.4$ fm [1] and $a_{nn} = -18.7 \pm 0.6$ fm [2], whereas the value obtained from the reaction $\pi^- d \to \gamma nn$ is $a_{nn} = -18.5 \pm 0.3$ fm [3]. For the proton-proton scattering

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length, which is directly measured, the analysis gives $a_{pp} = -17.3 \pm 0.4$ fm [4] after correction for electromagnetic effects, which means that even the sign of $\Delta_a = a_{pp} - a_{nn}$ is not fixed.

Here we discuss a possibility to determine a_{nn} from differential cross sections in the reaction $\gamma d \to \pi^+ nn$. Our calculation is based on the recent work of Ref. [5], where the transition operator for the reaction $\gamma d \to \pi^+ nn$ was calculated in chiral perturbation theory (ChPT) up to order $\chi^{5/2}$ with $\chi = m_\pi/M_N$, and $m_\pi(M_N)$ being the pion (nucleon) mass. Half-integer powers of χ in the expansion arise from three–nucleon cuts. The results of Ref. [5] are in a good agreement with the experimental data for the total cross section. The main theoretical error is due to the uncertainty in the leading photoproduction multipole E_{0+}, which was fixed from the ChPT calculation [6], and is the only input parameter in the calculation. In order to estimate the theoretical uncertainty that arises from the nucleon-nucleon (NN) wave functions, we use NN wave functions constructed within the same ChPT framework—this is what a consistent field theory calls for. In our present analysis [7] we used next-to-next-to-leading order NN wave functions constructed in Ref. [8].

To examine the theoretical uncertainty for the extraction of a_{nn}, we employ our leading order calculation as a baseline result and quantify the uncertainty from the effects of the higher orders that we calculated completely.

We use in what follows the function F proportional to the five–fold differential cross section for unpolarized particles

$$F(p_r, \theta_r, \phi_r, \theta_\pi, \phi_\pi) = C\, \frac{d^5\sigma(p_r, \theta_r, \phi_r, \theta_\pi, \phi_\pi)}{d\Omega_{\mathbf{p}_r} d\Omega_{\mathbf{k}_\pi} dp_r^2}\ , \tag{1}$$

where $\mathbf{p}_r$ ($\mathbf{k}_\pi$) is the relative momentum of the two final neutrons (momentum of the final pion) in the center–of–mass frame, $\theta_r,\ \phi_r$ ($\theta_\pi,\ \phi_\pi$) are the corresponding polar and azimuthal angles, respectively. We consider only shapes of cross sections and therefore the value of the dimensionful constant C is not important for our analysis. For the differential cross section F there are two characteristic regions that are important for our considerations: the region of quasi-free production (QF peak) at large p_r, where those production diagrams dominate that do not contain the NN interaction in the final or intermediate states, and the region with prominence of the strong NN final–state interaction (FSI peak) at small p_r.

The subject of our interest, a_{nn}, is a low-energy characteristic of neutron-neutron scattering and manifests itself in the momentum dependence of the cross section at small p_r. Therefore the influence of different values of a_{nn} is significant in the FSI peak and marginal in the QF peak. On the other hand, we have shown [7] that the relative height of the QF and the FSI peaks changes if the effects of higher orders are included, whereas the individual shapes of these peaks change marginally. Therefore those angular configurations are to be preferred, where the quasi-free production is suppressed and thus the shape of the whole spectrum is only marginally influenced by higher orders. This takes place when $\mathbf{p}_r$ is perpendicular to the initial photon momentum $\mathbf{q}_\gamma$.

To quantify the theoretical uncertainty, we use the function $\mathcal{S}$, defined as

$$\mathcal{S}(a_{nn}, \Phi) = \int_0^{p_{\max}} dp_r \left(F(p_r|a_{nn}^{(0)}, \Phi^{(0)}) - N(a_{nn}, \Phi)\ F(p_r|a_{nn}, \Phi) \right)^2 w(p_r)\,, \quad (2)$$

where $p_{\max}$ is the maximal value of p_r, and $F(p_r|a_{nn}, \Phi)$ is defined in Eq. (1). In the latter we refrained from showing the angular dependence in favor of the parametric dependence of F on a_{nn} as well as the multi–index Φ, which symbolizes the dependence of F on the chosen chiral order and the wave functions used. The weight function $w(p_r)$ allows us to suppress particular regions of momenta in the analysis. We may assume $\mathcal{S}$ to be dimensionless; all dimensions can be absorbed into the constant C defined in Eq. (1).

The value $a_{nn}^{(0)}$ denotes the central value of a_{nn} (-18.9 fm) for which we estimate the theoretical uncertainty, whereas $\Phi^{(0)}$ corresponds to the baseline calculation, namely leading order with a certain choice of chiral wave functions (see Ref. [7]). The relative normalization $N(a_{nn}, \Phi)$ is fixed by demanding that $\mathcal{S}$ gets minimized for any given pair of parameters a_{nn}, Φ ($\partial\mathcal{S}/\partial N = 0$). Thus, $\mathcal{S}$ characterizes the mean-square deviation of the shape of the cross section from the baseline cross section $F(p_r|a_{nn}^{(0)}, \Phi^{(0)})$. This way to determine the theoretical uncertainty is in full analogy to the standard method of data analysis.

To quantify the theoretical uncertainty, we define $\Phi_{\max}$ as that chiral order and choice of wave function, where $\mathcal{S}(a_{nn}^{(0)}, \Phi_{\max})$ gets maximal:

$$\mathcal{S}(a_{nn}^{(0)}, \Phi_{\max}) = \max_{\Phi} \left\{ \mathcal{S}(a_{nn}^{(0)}, \Phi) \right\} \ . \quad (3)$$

Therefore $\mathcal{S}(a_{nn}^{(0)}, \Phi_{\max})$ provides an integral measure of the theoretical uncertainty of the differential cross section. Demanding that the effect of a change in the scattering length by the amount Δa_{nn} matches that by the inclusion of higher orders *etc.*, we identify Δa_{nn} as an uncertainty in a_{nn}. In terms of $\mathcal{S}$, Δa_{nn} is defined via

$$\mathcal{S}(a_{nn}^{(0)} + \Delta a_{nn}, \Phi^{(0)}) = \mathcal{S}(a_{nn}^{(0)}, \Phi_{\max}) \ . \quad (4)$$

Since the signal region is located at low p_r and, on the other hand, the theoretical uncertainty of the differential cross section is largest for large values of p_r, we use such weight functions $w(p_r)$ that suppress the contribution of large momenta. For instance, for $w(p_r) = \Theta(p^{\rm cut} - p_r)$, and $p^{\rm cut} = 30$ MeV, the value Δa_{nn} reduces to 0.07 fm. This is due to the fact that the signal changes only very little when a restriction to small values of p_r is applied. At the same time this procedure significantly reduces the value of the uncertainty $\mathcal{S}(a_{nn}^{(0)}, \Phi_{\rm max})$. We found that the ideal value for $p^{\rm cut}$ is between 25 and 40 MeV. This translates into values of Δa_{nn} between 0.05 and 0.1 fm. The value of θ_π also has some impact on the theoretical uncertainty, however, in its whole parameter range the estimated uncertainty stays below 0.1 fm for $p^{\rm cut} = 30$ MeV. We therefore argue that the reaction $\gamma d \to \pi^+ nn$ appears to be a good tool for the extraction of a_{nn} (perhaps as a complementary way to the reaction $\pi^- d \to \gamma nn$ [9]).

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